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Fast Identification Algorithm of Time-varying Modal Parameter Based on Two-layer Linear Neural Network Learning

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Abstract

The key of fast identification algorithm of time-varying modal parameter based on subspace tracking is to find efficient and fast subspace-tracking algorithm. This paper presents a new version of NIC(Novel Information Criterion) using two-layer linear neural network learning for subspace tracking. Comparing with the original algorithm, there is no need to set a key control parameter in advance. Simulation experiments show that new algorithm has a faster convergence in the initial period.

Index Terms: Subspace tracking, time-varying modal parameter, identification algorithm, neural network learning

1. Introduction

Linear time-varying (LTV) structures are widely existed in the field of aerospace, mechanics and transportation, such as expending of solar panels and mechanical arms and high-speed train[1]. For linear time-invariable(LTI) system, measuring and analyzing techniques of modal parameters have reached a mature development. However the conventional concepts of modal parameters are out of invalidation for LTV. By adopting definition of modal parameters in LTI and using "time frozen" technique, the concept of "pseudo model parameters "[2,3] is proposed. Liu[4,5] extended identification algorithms of modal parameters based on subspace in LTV. The procedure of modal parameter identification algorithm based on subspace is that: first extract signal subspace by applying input/output time-serials , then estimate system matrix, finally obtain time-varying modal parameters by modal theory. Yu[6] solved time-varying modal parameter identification of moving mass/simple-supported beam by using modal parameter identification algorithm based on subspace of ensemble data[2]. Pang[7] got a new version of the algorithm based on subspace of ensemble data by replacing input matrix

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with generalized observability matrix and applying orthogonality of singular matrix, whose compute load and noise immunity are both better than that of the original one. But the above-mentioned methods are is not suitable for tracking modal parameters for large compute load and memory space. So identification algorithms based on recursive subspace derived from batch subspace method[8] takes the advantage in on-line modal parameter identification. F. Tasker et. al[9,10] proposed on-line identification algorithm of time-varying modal parameters using TQR-SVD[11] for tracking signal subspace. Wu[12] obtained a new on-line algorithm by introducing FAST(Fast Approximate Subspace Tracking) and applied in time-varying modal parameter identification of three-link system. Pang et. al[13,14] got a fast identification algorithm of time-varying modal parameters by introducing PAST[15](Projection Approximation Subspace Tracking) and applied in two-link system and moving mass/simple-supported beam system.

The key of algorithms based on subspace tracking is to find an efficient and fast algorithm for subspace tracking. The signal subspace obtained by PAST converges asymptotically to the orthogonal subspace, by introducing orthogonal method $W = W(W^T W)^{-1/2}$ OPAST[16] is obtained. Otherwise by deflation technique of PCA(Principal Component Analysis), PASTd[15,17] is derived. Similarly, signal subspace obtained by PASTd has a strong lose of orthogonality, by introducing orthogonal method Gram-Schmit for incorporating characteristic of PASTd, a modified PASTd[18] is obtained. PAST can be viewed as a classical power iteration method. Comparing with PAST, NPI[19](Natural Power Iteration) has a faster convergence speed and ensures orthogonality of subspace vectors without compute load increasing. In both PAST and NPI, $W_{p+1} \approx W_p$ is used in the process of algorithm derivation, so they are only suitable for tracking slow subspace. Different from PAST and NPI, API[20](Approximated Power Iteration) can track rapid subspace and apply updating data in both infinite exponential window and finite moving exponential window. Different from PAST in cost function, NIC[21,22](Novel Information Criterion) is proposed, adopting in two-layer linear neural network learning for subspace tracking. By introducing orthogonal method, a modified version is obtained named FONIC[23].

This paper presents a modified version of NIC adopting in two-layer linear neural network learning for subspace tracking. Comparing with the original algorithm, there is no need to set an important parameter in advance. Simulation experiments show that it has a faster convergence in the initial period.

2. Recursive form of updating input/output data

To tracking subspace of updating input/output dates, it is unnecessary to evaluate $Y_p U_p^{\perp} Y_p^T$ every step. By adopting recursive form of updating input/output data, principle subspace tracking algorithm can be applied and then fast identification of time-varying modal parameter is achieved. Updating the input and output Hankel matrix, we have:

$$Y_{p+1} = [Y_p \quad \bar{y}_{p+1}], U_{p+1} = [U_p \quad \bar{u}_{p+1}] \tag{1}$$

Where $\bar{y}_{p+1} = [y(p+1) \quad y(p+2) \quad \cdots \quad y(p+M)]^T$,

$\bar{u}_{p+1} = [u(p+1) \quad u(p+2) \quad \cdots \quad u(p+M)]^T$,

$$U_p = \begin{bmatrix} u(1) & u(2) & \cdots & u(p) \\ u(2) & u(3) & \cdots & u(p+1) \\ \vdots & \cdots & \ddots & \vdots \\ u(M) & u(M+1) & \cdots & u(p+M-1) \end{bmatrix}, Y_p = \begin{bmatrix} y(1) & y(2) & \cdots & y(p) \\ y(2) & y(3) & \cdots & y(p+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(M) & y(M+1) & \cdots & y(p+M-1) \end{bmatrix},$$

$u(k) \in C^{m\times 1}$ the input vector at k^{th} instant, $y(k)$ the output vector.

By lemma of matrix inverse

$(A+xx^T)^{-1} = A^{-1} - A^{-1}xx^T A^{-1} / (1+x^T A^{-1}x)$, we have:

$$Y_{p+1}U^{\perp}_{p+1}Y^T_p = Y_p U^{\perp}_p Y^T_p + z_{p+1}z^T_{p+1} \tag{2}$$

Where $z_{p+1} = [Y_p U^T_p (U_p U^T_p)^{-1}\bar{u}_{p+1} - \bar{y}_{p+1}] / \sqrt{1+\alpha_{p+1}}$,

$\alpha_{p+1} = \bar{u}^T_{p+1}(U_p U^T_p)^{-1}\bar{u}_{p+1}$. Equation (2) is called rank-one updating.

3. Two-layer linear neural network for subspace tracking

After getting the recursive form of updating input/output data, the key of fast identification of time-varying modal parameters is efficient and fast algorithm for signal subspace tracking. The following is to state the algorithm based on linear two-layer neural network for subspace tracking.

The cost function[23,24] is chosen as:

$$J_{NIC}(W_k) = \frac{1}{2}\{tr[\log(W^T_k R_{zz}(k)W_k)] - tr(W^T_k W_k)\} \tag{3}$$

Where W_k is the signal subspace at k^{th} instant. $R_{zz}(k) = \sum_{i=1}^{k} \beta^{k-i} z_i z^T_i$, $0 < \beta < 1$ is the forgetting factor, $tr(\cdot)$ denotes matrix trace.

By partial differential of matrix trace, we have:

$$\nabla J_{NIC}(W_k) = R_{zz}(k)W_k(W^T_k R_{zz}(k)W_k)^{-1} - W_k \tag{4}$$

Further we have:

$$W_k = W_{k-1} + \eta_k \nabla J_{NIC}(W_k) \tag{5}$$

$$W_k = (1-\eta_k)W_{k-1} + \eta_k R_{zz}(k)W_k(W^T_k R_{zz}(k)W_k)^{-1} \tag{6}$$

Let $y_k = W_{k-1}z_k$, we get,

$$W_k = (1-\eta_k)W_{k-1} + \eta_k R_{zy}(k)R^{-1}_{yy}(k) \tag{7}$$

Where $R_{zy}(k) = \sum_{i=1}^{k} \beta^{k-i} z_i y^T_i$, $R_{ty}(k) = \sum_{i=1}^{k} \beta^{k-i} y_i y^T_i$. By lemma of matrix inverse,

$$(A+zz^T)^{-1}=A^{-1}-A^{-1}zz^TA^{-1}/(1+z^TA^{-1}z) \tag{8}$$

We get

$$W_k=(1-\eta_k)W_{k-1}+\eta_k\bar{W}_k \tag{9}$$

Where,

$$\bar{W}_k=\bar{W}_{k-1}+(z_k-\bar{W}_{k-1}y_k)y_k^TP_{k-1}/(\beta+y_k^TP_{k-1}y_k)\,,\bar{W}_{k-1}=R_{zy}(k-1)P_{k-1}\,,$$

$$P_k=(\sum_{i=1}^{k}y_iy_i^T)^{-1}=(\beta R_{yy}(k-1)+y_ky_k^T)^{-1}$$

$$=\frac{1}{\beta}(P_{k-1}-\frac{P_{k-1}y_ky_k^TP_{k-1}}{\beta+y_k^TP_{k-1}y_k}).$$

Different from the original method, η_k is not set in advance. Applying Differential segment,

$$\frac{\partial J_{NIC}(W_k)}{\partial\eta_k}=\frac{\partial J_{NIC}(W_k)}{\partial W_k}\frac{\partial W_k}{\partial\eta_k} \tag{10}$$

Let $\frac{\partial J_{NIC}(W_k)}{\partial\eta_k}=0$, we have:

$$\eta_k=\frac{\left\|(z_k-W_{k-1}y_k)y_k^TP_{k-1}/(\beta+y_k^TP_{k-1}y_k)\right\|}{\left\|\bar{W}_k-W_{k-1}\right\|} \tag{11}$$

Where $\|\square\|$ denotes Frobenius norm of matrix.

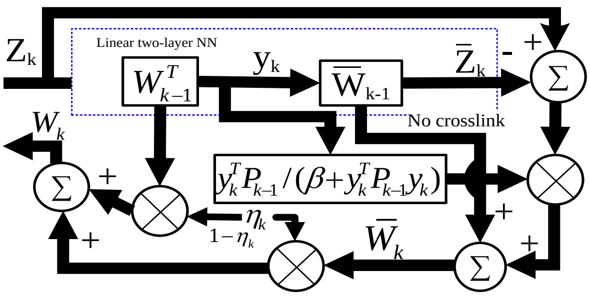

Fig. 1 Algorithm block based on linear two-layer neural network learning for subspace tracking

According to the modified version mentioned above, we get the algorithm block based on two-layer linear neural network learning for subspace tracking showed in Fig. 1. Further we get the fast identification algorithm of time-varying modal parameters.

4. Simulation and Analysis

Moving mass-beam structure is widely used in modelling bridge with moving vehicles, truss with moving crane and so on, which is a typical time-varying structure. Supposing that moving mass keeps contact with beam all the time, neglecting torsion deformation of beam, applying Euler-Bernoulli beam theory, we get the model as follows:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+m\frac{\partial^2 y(x,t)}{\partial t^2}=F(x,t) \tag{12}$$

Where E is Young's modulus, I is the second moment of beam cross-section, m is the mass per unit length of beam, $y(x,t)$ denotes the transverse deflection of beam, $F(x,t)$ is time-varying driving force. Taking the inertia load of moving mass into account[24], we have:

$$F(x,t)=\frac{1}{\varepsilon}[-m_0 g-m_0\frac{\partial^2 y(x,t)}{\partial t^2}][H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})]-f(t)\delta(x-l) \tag{13}$$

Where m_0 is the moving mass, g is the gravitational acceleration, $H(\cdot)$ is Heaviside unit function, $f(t)$ is external excitation force, $\delta(\cdot)$ is Dirac-Delta function, $\xi=Vt$, $V=const$ the moving speed of moving mass. And the other parameters are showed in Fig. 2.

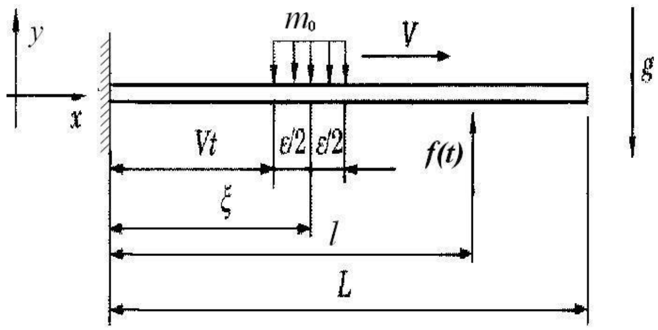

Fig. 2 Moving mass-Cantilever structure

Let $y(x,t)=\sum_{i=1}^{\infty}\phi_i(t)X_i(x)$, with $\phi_i(t)$ is unknown functions of time and $X_i(x)$ is the shape functions for the i^{th} mode of the vibration beam. Substituting them into (12) and (13) , we have:

$$M(t)\Phi''(t)+\Omega^2\Phi(t)=\bar{F}(t) \tag{14}$$

Where:

$$M(t) = I_n + diag(m_i)m_0[X(\xi)X^T(\xi) + \frac{\varepsilon^2}{24}(X(\xi)X^T(\xi)\Lambda^2 + 2\Lambda X(\xi)X^T(\xi)\Lambda + \Lambda^2 X(\xi)X^T(\xi)]$$

$\Lambda = diag(\beta_i)$, $\bar{X}(x) = [X_1(x)\cdots X_i(x)\cdots X_n(x)]^T$, $\Phi(t) = [\phi_1(t)\cdots\phi_i(t)\cdots\phi_n(t)]^T$, $I_n \in C^{n\times n}$ identity matrix, $\omega_i^2 = \beta_i^4 EI/m$, $\Omega = diag(\omega_i)$,

$X_i(x) - \beta_i^4 X_i^{(4)}(x) = 0$, $m_i = m\int_0^L X_i^2(x)dx$.

$$\bar{F}(t) = -diag(m_i)m_0 g(I_n + \frac{\varepsilon^2}{24}\Lambda^2)X(\xi) + f(t)\bar{X}(l).$$

The dimension of beam is $1.2\times 0.05\times 0.008$ m, $V = 0.2$ m/s, $EI = 418.13$ N.m^2, $\xi = 0.07$ m, $l = 1.0$ m,

$$X_i(x) = \cosh(\beta_i x) - \cos(\beta_i x) - \frac{\cosh(\beta_i) + \cos(\beta_i)}{\sinh(\beta_i) + \sin(\beta_i)}(\sinh(\beta_i x) - \sin(\beta_i x)),$$

β_i is determined by $1 + \cos(\beta_i L)\cosh(\beta_i L) = 0$, $n = 4$.

The identification results of identification algorithms based on different subspace tracking algorithms are showed in Fig. 3 and 4. The output data incorporate stochastic white noise of signal-noise ratio SNR=43db.

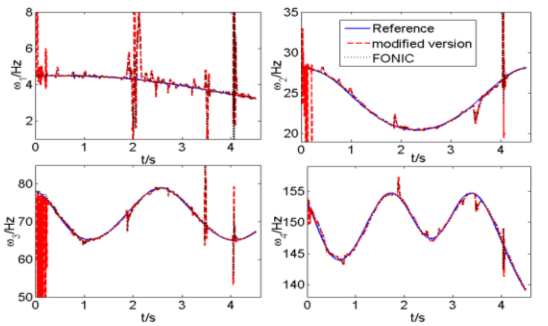

Fig. 3 Identification results with no noise disturbing

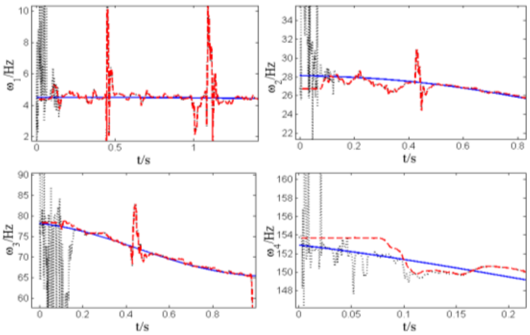

Fig. 4 Identification results with noise disturbing

As is shown in Fig. 3, the modified algorithm has an equivalent performance in signal subspace tracking with algorithms based on NIC and FONIC and all of them have a tolerance of noise. In theoretical analysis, NIC has a fast convergence speed and both of FONIC and the modified version have a faster convergence speed in the initial period showed in Fig.3 and 4, and after a period all of the three almost have the same performance.

5. Conclusion

The key of fast identification of time-varying modal parameters is to find efficient and fast signal subspace tracking algorithms. This paper adopts two-layer linear neural network learning for subspace tracking in fast identification algorithm of time-varying modal parameters. For the modified algorithm based on NIC, using in two-layer linear neural network learning for subspace tracking, we have no need to set a important factor in advance and has a faster convergence speed in the initial period demonstrated by simulation experiment results.

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