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Comparative Analysis for the Equilibrium Bidding Strategies of the Standard Auction Based on the Revenue Equivalence Principle

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Abstract

It has analyzed the transition processes for the content of the revenue equivalence principle. It used the revenue equivalence principle to derive the equilibrium bidding strategies in the first, second and third price auction and the all-pay auction from the expected payment of the bidders. Based on this, it compared the equilibrium bidding strategies in the four auction forms and got some useful conclusions.

Index Terms: revenue equivalence principle; standard auction; equilibrium bidding strategy; incentive compatibility

1. Introduction

The Revenue Equivalence Principle is the most important research result in the auction theory, which constitutes a benchmark of the theory of private value auctions. For the research contents, Vickery (1961)put forward the private value model of the auction and based on this, he proposed the Revenue Equivalence Principle for the common auction[1]. Through the Revelation Principle, Myerson(1981)expanded the Revenue Equivalence Principle in the much more common way and for the method of proof[2]. Milgrom (1989) gave the description and proof with much more general to the Revenue Equivalence Principle[3]. Moreover, Klemperer (1999)used a very concise way to prove the Revenue Equivalence Principle by using the equilibrium[4]. One of the importance roles of Revenue Equivalence Principle is to provide a number of easy ways for solving equilibrium bidding strategies of the auction model. This paper uses the Revenue Equivalence Principle to derive the equilibrium bidding strategies in the sealed first, second and third price auction and the all-pay

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auction; moreover, it compared and analyzed the equilibrium bidding strategies in the four auction ways and got some useful conclusions.

2. The Contents Analysis for The Revenue Equivalence Principle

The so-called revenue equivalence means that from the view of the sellers, the expected revenue of the seller for the same single and inseparable goods are sold by the said four auctions is the same with what they get. In the case of certain bidders, the equal expected revenue of the sellers means the equal expected payment of the buyers and the expected payment of the buyers is paid in advance. The traditional revenue equivalence principle assumed that the evaluation of the bidders is independently and identically distributed and the risk for the bidders is neutral; moreover, for the sellers, in the case that the evaluation of the bidders for the auction goods is zero and then the expected payment is zero, the symmetric increasing equilibrium auctioned by arbitrary standards produces the equal expected revenue [5]. Obviously, traditional Revenue Equivalence Principle required the evaluation x_i of the bidders is independently and identically distributed, which is only suitable for the private value model. Assumed the bidders are i and their evaluation is x_i , then their expected payment is

$$m_i(x_i) = \int_0^{x_i} yg(y)dy \tag{1}$$

The extended Revenue Equivalence Principle means if the direct mechanism (Q, M) is incentive compatibility, then for all the bidders i , their evaluation are x_i and their expected payment is

$$m_i(x_i) = m_i(0) + q_i(x_i)x_i - \int_0^{x_i} q_i(t_i)dt_i \tag{2}$$

As long as the evaluation of the bidders can be independently and differently distributed and conditions is relatively loosening compared with traditional Revenue Equivalence Principle, then it will be suitable for the more general hybrid value model. Wherein, $m_i(0)$ at the right of the equation is determined by the payment rule M and $q_i(x_i)x_i - \int_0^{x_i} q_i(t_i)dt_i$ is determined by the distributive rule Q [5]. In the traditional Revenue Equivalence Principle, $m_i(0) = 0$; therefore, $m_i(x_i)$ is completely determined by the distribution rule; moreover, the standard auction[1] means the auction which distributes the auction goods to the bidders who give the highest valuation, that is to say, the distribution rules are the same. Thereby, it gets the conclusion that all the expected payment of the bidders is the same. In this sense, traditional Revenue Equivalence Principle is the special case in the expended Revenue Equivalence Principle. Thus, in any two mechanisms which have the same distribution rule and incentive compatibility, except for a constant, the expected payment of them is equal.

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[1] The standard auction is assumed to satisfy the given conditions: Bidders are risk neutral; Independent private values holds; Bidders are symmetric; Payment is a function of bids alone.

3. Specific Applications of The Revenue Equivalence Principle

First, make some explanations for the related symbols in the derivation process. The evaluation of the bidders i is x_i ; the bidding function is the monotonically increasing function of x_i; and the bidding price is $b_i = \beta(x_i)$; moreover, make the order statistics $Y_1 = Y_1^{(n-1)} = \max_{j\neq i} b_j$, $Y_2 = Y_2^{(n-1)}$, the condition for the bidders i to win the bid is $b_i > \max_{j\neq i} b_j$, that is to say

$$b_i = \beta(x_i) > \max_{j\neq i} \beta(x_j) \Rightarrow b_i > \max_{j\neq i} \beta(x_j) = \beta(\max_{j\neq i} x_j) = \beta(Y_1)$$

that is

$$\beta(Y_1) < b_i \Rightarrow Y_1 < \beta^{-1}(b_i) \tag{3}$$

Here, $\max$ is the convex function, $\beta(x)$ is monotonically increasing, so, $\max_{j\neq i} \beta(x_j) = \beta(\max_{j\neq i} x_j)$. $F(x_i)$ and $f(x_i)$ are respectively the distribution function and density function for the evaluation x_i ,Wherein,

$$G(x_i) = p(Y_1 \le x_i) = p(\max_{j\neq i} b_j \le x_i) = \prod_{j\neq i} p(b_j \le x_i) = \prod_{j\neq i} F(x_i) \tag{4}$$

the Revenue Equivalence Principle requires the evaluation x_i of the bidders to be independently and identically distributed, then $G(x_i) = (F(x_i))^{n-1}$ .We can see from the symmetry that we make $i = 1$, and at the symmetric equilibrium, the bidding strategies of all the bidders are the same, that is to say, $\beta(x_1) = \beta(x_j)$, $j \neq 1$ .Wherein, A is any one of the standard auctions and then their expected payment is as following

$m^A(x) =$ prob (win the bid) $\times$ the expected payment in case of winning the bid +prob (lose the bid) $\times$ the expected payment in case of losing the bid (5)

3.1. Solving for the Equilibrium Bidding Strategies of the First-sealed Price Auction

In the first-price sealed-bid auction, the bidder 1 submits a sealed valuation b_1 and its evaluation is x ; and given the valuation b_j to other bidders, then the payment of the bidder 1 is

$$m_1 = \begin{cases} b_1 & b_1 > \max_{j\neq 1} b_j \\ 0 & b_1 < \max_{j\neq 1} b_j \end{cases}$$

We can get to know from the Revenue Equivalence Principle that the expected payment of bidder 1 is

$$G(x)\beta^1(x) = \int_0^x yg(y)dx$$

$$\Rightarrow \beta^{\mathrm{I}}(x) = \frac{1}{G(x)}\int_0^x yg(y)dy$$

$$\Rightarrow \beta^{\mathrm{I}}(x) = \frac{\int_0^x yg(y)dy}{\int_0^x g(y)dy}$$

$$\Rightarrow \beta^{\mathrm{I}}(x) = x - \int_0^x \left[\frac{F(y)}{F(x)}\right]^{n-1} dx \tag{6}$$

3.2. Solving for the Equilibrium Bidding Strategies of the Second-price Sealed-bid Auction

In the second-price sealed-bid auction, the bidder 1 submits a sealed valuation b_1 and its evaluation is x ; and given the valuation b_j to other bidders, then the payment of the bidder 1 is

$$m_1 = \begin{cases} Y_1 = \max_{j\neq 1} b_j & b_1 > \max_{j\neq 1} b_j \\ 0 \quad b_1 < \max_{j\neq 1} b_j \end{cases}$$

We can get to know from the Revenue Equivalence Principle that the expected payment of bidder is

$$G(x)E(\beta^{\mathrm{II}}(Y_1)|Y_1 < x) = \int_0^x yg(y)dy$$

$$\Rightarrow G(x)\frac{\int_0^x \beta^{\mathrm{II}}(y)d(F(y))^{n-1}}{G(x)} = \int_0^x yg(y)dy$$

$$\Rightarrow \int_0^x \beta^{\mathrm{II}}(y)d(F(y))^{n-1} = \int_0^x yg(y)dy$$

$$\Rightarrow \beta^{\mathrm{II}}(x)(F(x))^{n-1} - \int_0^x (F(y))^{n-1}d\beta^{\mathrm{II}}(y) = xG(X) - \int_0^x G(y)dy \Rightarrow (n-1)(F(x))^{n-2} f(x)\beta^{\mathrm{II}}(x) = xg(x)$$

$$\Rightarrow \beta^{\mathrm{II}}(x) = x \tag{7}$$

Here,

$$G(x) = \int_0^x g(y)dy = (F(x))^{n-1}, \; g(x) = (n-1)(F(x))^{n-1} f(x)$$

3.3. Solving for the Equilibrium Bidding Strategies of the Third-price Sealed-bid Price Auction

In the third-price sealed-bid auction, the bidder 1 submits a sealed valuation b_1 and its evaluation is x ; and given the valuation b_j to other bidders, then the payment of the bidder 1 is

$$m_1 = \begin{cases} Y_2 & b_1 > \max_{j \neq 1} b_j \\ 0 & b_1 < \max_{j \neq 1} b_j \end{cases}$$

We can get to know from the Revenue Equivalence Principle that the expected payment of bidder 1 is

$$G(x)E(\beta^{\text{III}}(Y_2) \big| Y_1 < x) = \int_0^x yg(y)dy$$

$$\Rightarrow G(x)\frac{1}{(F(x))^{n-1}}\int_0^x \beta^{\text{III}}(y)(n-1)(F(x)-F(y))f_1^{(n-2)}(y)dy = \int_0^x yg(y)dy$$

$$\Rightarrow f(x)\int_0^x \beta^{\text{III}}(y)(n-1)f_1^{(n-2)}(y)dy + (n-1)F(x)\beta^{\text{III}}(x)f_1^{(n-2)}(y) \qquad -(n-1)\beta^{\text{III}}(x)F(x)f_1^{(n-2)}(x) = xg(x)$$

$$\Rightarrow \int_0^x \beta^{\text{III}}(y)f_1^{(n-2)}(y)dy = x(F(x))^{n-2}$$

$$\Rightarrow \beta^{\text{III}}(x)f_1^{(n-2)}(x) = (F(x))^{n-2} + (n-2)x(F(x))^{n-3}f(x)$$

$$\Rightarrow \beta^{\text{III}}(x) = \frac{(F(x))^{n-2} + (n-2)x(F(x))^{n-3}f(x)}{f_1^{(n-2)}(x)} \Rightarrow \beta^{\text{III}}(x) = x + \frac{(F(x))^{n-2}}{f_1^{(n-2)}(x)}$$

$$\Rightarrow \beta^{\text{III}}(x) = x + \frac{F(x)}{(n-2)f(x)} \tag{8}$$

Here,

$$f_1^{(n-2)}(x) = (F_1^{(n-2)}(x))' = ((F(x))^{n-2})' = (n-2)(F(x))^{n-3}f(x)$$

3.4. Solving for the Equilibrium Bidding Strategies of the All- pay Auction

In the all-pay auction, the bidder 1 submits a sealed valuation b_1 and its evaluation is x; and given the valuation b_j to other bidders, then the payment of the bidder 1 is

$$m_1 = \begin{cases} b_1 & b_1 > \max_{j \neq 1} b_j \\ -b_1 & b_1 < \max_{j \neq 1} b_j \end{cases}$$

We can get to know from the Revenue Equivalence Principle that the expected payment of bidder is

$$G(x)\beta^{AP}(x) + (1-G(x))\beta^{AP}(x) = \int_0^x yg(y)dy$$

$$\Rightarrow \beta^{AP}(x) = \int_0^x yg(y)dy$$

$$\Rightarrow \beta^{AP}(x) = (F(x))^{n-1}\left(x - \int_0^x \left[\frac{F(y)}{F(x)}\right]^{n-1} dy\right) \quad \Rightarrow \beta^{AP}(x) = (F(x))^{n-1}\beta^{\mathrm{I}}(x) \tag{9}$$

4. Comparative Analysis on The Equilibrium Bidding Strategies in The Four Auction Forms

A comparative analysis of those equilibrium bidding strategies in the four auction forms shown by (6),(7), (8), (9)is given as follow

(1) As they are shown in the (6),(7), (8), (9). the equilibrium bidding strategies order for the four auctions of the same bidder is $\beta^{\mathrm{AP}}(x) < \beta^{\mathrm{I}}(x) < \beta^{\mathrm{II}}(x) < \beta^{\mathrm{III}}(x)$ .In the all-pay auction, because the bidders need to pay for their bidding price no matter they win the bid or not, so the bidding strategy of the bidders is relatively cautious and their equilibrium bidding strategy is $\beta^{\mathrm{AP}}(x) = (F(x))^{n-1}\beta^{\mathrm{I}}(x) < \beta^{\mathrm{I}}(x)$, that is to say, it is smaller than the equilibrium bidding strategy in the case of winner-pay. Moreover, because the payment price is the ceiling price; so, the bidding strategy in the first price auction is relatively more cautious compared with the bidding strategy in the second price auction, that is to say

$$\beta^{\mathrm{I}}(x) = x - \int_0^x \left[\frac{F(y)}{F(x)}\right]^{n-1} dy < \beta^{\mathrm{II}}(x) = x \tag{10}$$

and the bidding price is lower than the evaluation x . Because the payment price is the second-price; so, the bidding strategy in the second price auction is relatively more cautious compared with the bidding strategy in the third price auction, that is to say

$$\beta^{\mathrm{II}}(x) = x < \beta^{\mathrm{III}}(x) = x + \frac{F(x)}{(n-2)f(x)} \tag{11}$$

and the bidding price is equal to the evaluation x , which truly discloses the type of the bidders. Besides, because the payment price is the third price; so, the bidding strategy in the third price auction is relatively fiercer compared with the bidding strategies in the mentioned three auctions, that is to say, its bidding price is greater than its evaluation x .

(2) In the all-pay auction, it gets $\beta^{AP} = m^{A}(x)$, which is in accordance with the definition of the all-pay auction[6]. That is to say, the bidding price of the bidders is equal to payment price no matter they win the bid or not, which is determined by the payment rule. When $n \to \infty$, and $\beta^{AP}(x) \to 0$, it means the number of the bidders is infinite and the probability of winning bid is small; what's more, the valuation of the bidders who is rational at the equilibrium is really small and close to zero. In the first price auction, because $\frac{F(y)}{F(x)} < 1$, $F(x)$ is monotonically increasing, $0 < y < x$ and $\left[\frac{F(y)}{F(x)}\right]^{n-1} < 1$; so, if n $n \to \infty$, and $\beta^{\mathrm{I}}(x) \to x$, it means when the

number of the bidders is infinite, the equilibrium bidding price of the bidders is close to their real evaluation. In the second price auction, the equilibrium bidding strategy of the bidders is $\beta^{\mathrm{II}}(x) = x$, which has no connection with the number of the bidders and the bidding price is the true evaluation; and the evaluation satisfies with the incentive compatibility. In the third price bidding, when $n \to \infty$, $\frac{F(x)}{(n-2)f(x)} \to 0$ and $\beta^{\mathrm{III}}(x) \to x$, with the increase number of the bidders, the bidding price is gradually returned to its true evaluation .

5. Conclusion

An auction mechanism is composed of distribution rules and payment rules. In the standard auctions, the distribution rules are the same which means the highest bidder gets the goods; moreover, the distribution rules are changeable and the changeable distribution rules will not affect the auctioneer's expected revenue or the expected payment of the bidders but it will affect the equilibrium bidding strategies of the bidders. According to the analysis on the mentioned four auction forms, we can know that if the auction wants to be the effective mechanism for the price found, it needs to encourage more bidders to join. Only when the participants are more enough can the true type of the bidders be disclosed and form the bidding price mechanism with incentive compatibility[7]. Thereby, it forms the incentive compatibility restriction in the most excellent auction mechanism; that is to say, the designed auction mechanism should encourage the bidders to truly report their types and the expected revenue obtained by truly reporting their types is not less than the expected revenue obtained by misreporting their types. And meanwhile, the precondition for encouraging entrance is to guarantee the expected revenue of the bidders for taking part in bidding no less than the expected revenue without bidding; therefore, it forms the individual rationality restriction in the most excellent auction mechanism[8]. All of these are the useful conclusions obtained through the analysis on the equilibrium bidding strategies in said four standard auctions.

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