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Inverse Kinematics of Redundant Manipulator using Interval Newton Method

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Abstract

The paper presents an application of Interval Newton method to solve the inverse kinematics and redundancy resolution of a serial redundant manipulator. Such inverse problems are often encountered when the manipulator link lengths, joint angles and end-effector uncertainty bounds are given, which occurs due to because of inaccuracies in joint angle measurements, manufacturing tolerances, link geometries approximations, etc. The inverse kinematics of three degree of freedom planar redundant positioning manipulator without end-effector has been evaluated using the manipulability of Jacobian matrix as performance metric. To solve the nonlinear equation of inverse kinematics, the multidimensional Newton method is used. The inverse kinematics is intended to produce solutions for joint variables in interval of tolerances for specified end effector accuracy range. As exemplar problem solving, a planar 3-degrees-of-freedom serial link redundant manipulators is considered.

Index Terms: Interval Newton Method, Redundant Manipulator, Inverse Kinematics, Manipulability.

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1. Introduction

In robotics, a manipulator is called kinematically redundant if the number of degree of freedom (DOF) of the joint space is higher than the number of degree of freedom of task space coordinates. Inverse kinematics redundancy resolution for redundant serial manipulators remains a key topic in many robotics problems, where, presence of additional degrees of freedoms enhances manipulator's ability, and task versatility [1-4]. Secondary task of a redundant manipulator is defined by performance indices. These indices include, among many others, isotropic velocity behaviour, manipulability, dynamic manipulability, etc. Manipulability of the manipulator jacobian matrix is used by Yoshikawa [1] as performance metric for manipulating ability of robotic

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mechanisms in positioning and orienting end-effectors. Similarly condition number of the manipulator jacobian matrix is used by Salisbury [3] as an optimization performance criterion to evaluate the dimensions for the fingers of the Stanford/JPL articulated hand. To design a manipulator, Klein [4] used the condition number of jacobian matrix of manipulator for isotropy at a working point for a fixed total arm length. One problem of using condition number of a Jacobian matrix is the mismatch of units of rectilinear and rotational velocities. To overcome this discrepancy a characteristic length of manipulator is proposed by Angeles [5] to normalize the manipulator jacobian.

This article addresses the problem of inverse kinematics evaluation of redundant manipulator using an Interval Arithmetic based technique. This interval based technique is not much explored for redundant serial manipulator problem, can lead to design optimization of manipulators with dimensional/length uncertainties due to manufacturing tolerance, assembly variations and uncertainty inherent in behaviour of procured components, given that the uncertainty ranges are known. Some literatures are available, where, the researchers attempted to solve the inverse kinematics of non-redundant serial manipulator by the interval methods [6-8]. Castellet et al. [7] solved the inverse kinematics by using the n-bar mechanism and form a closed single-loop mechanism and apply the interval Newton cut approach. Roa et. al. [6] applied the interval Krawczyk method to solve the inverse kinematics equation of industrial robots. Pac et. al. [8] used the SIVIA algorithm (Set Inversion via Interval Analysis algorithm) to solved the solution to the inverse kinematics problem of the two-link and three link manipulator. This article attempts to solve the redundancy resolution and inverse kinematics of redundant manipulator by formulating the optimization problem as manipulability of Jacobian should be maximum. The maximization equality problem is converted in the form of General Fritz-John formulation and solved it using Interval Newton method.

The organization of the paper contains the problem formulation for inverse kinematics of redundant manipulator and some basics of interval arithmetic in Section 2. Section 3 describing the procedure for inverse kinematics solution with 3-DOF manipulator example, and conclusions are delineated in Section 4.

2. Problem Formulation and Interval Method

2.1 Inverse Kinematics Problem

The forward kinematic equation of an *n*-DOF manipulator can be expressed as

$$x = f(q) \tag{1}$$

where, $x \in R^m$ is the end effector pose in *m*-dimensional task space, $q \in R^n$ is the joint space variable vector of dimension n and f is a nonlinear vector function obtained from the particular kinematic structure of the manipulator. Inverse kinematics requires to find f^{-1} in order to find one or more joint angle vectors for a given end-effector position and orientation, such that

$$q = f^{-1}(x) \tag{2}$$

Solution of (1), for redundant manipulator, gives an infinite number of configurations. Although in principle (2) may exist, in practice it is not easy to obtain a closed form inverse kinematic function of (1) for spatial manipulators having more than three degrees of freedom in particular. In such a case, iterative and algorithmic techniques are generally employed. One such technique makes use of jacobian of the forward kinematic function in (1), this is defined in the differential motion relations

$$\dot{x} = J(q)\dot{q} \text{ or } \Delta x = J(q)\Delta q \tag{3}$$

where, $J = \partial f / \partial q_i, i = 1,..,n.$ Inverse kinematics is solved either using first of (3) and integrating the velocities for position with respect to an initial configuration, or, by an iterative method using increment in second of (3) again with knowledge of an initial configuration. A specific inverse kinematic solution with minimum norm can be obtained by using the Moor-Penrose pseudo inverse [9] of J(q) but it does not guarantee avoidance of all occurrences of singularity. A more general solution of second of (3) is given by

$$\Delta q = J^{\#}\Delta x + (I_{nxn} - J^{\#}J)\Delta q_0 \tag{4}$$

which is a non-minimum norm solution, where a homogeneous term is added to the minimum norm solution. $I_{n\,x\,n}$ is an identity matrix of dimension *n* and Δq_0 is an arbitrary vector, denoting null-space motion, which is added with the minimum-norm-term through a projection operator which is added with the minimum-norm-term through a projection operator $(I_{nxn} - J^{\#}J)$. This non-minimum norm solution allows reaching a secondary objective and achieves Manipulability. In this article, Manipulability of Jacobian is taken as the performance metric. The redundancy resolution and inverse kinematics is solved by prioritizing the tasks – the primary task being the reachability to the goal point in the workspace and the secondary task is maximize the manipulability of Jacobin matrix. Based on the primary and secondary task of manipulator formulate the optimization problems and solve them using interval Newton method. In this article, the inverse kinematic solution is attempted using optimization problem that is based on manipulability of Jacobian matrix, by incorporating interval technique to handle uncertainties in link dimensions and goal position.

2.2 Interval Arithmetic

Interval analysis is a relatively new mathematical branch of computational mathematics where computations are carried out on intervals instead of real numbers. Interval analysis is used to design interval algorithms for solving systems of linear and nonlinear equations and optimization problems [10-12]. Moore [13] among few firsts made important discussions with the basic interval operations.

An interval X is defined as the closed bounded set of real numbers x denoted by $X = [\underline{X}, \overline{X}]$, such that, $\underline{x} \le x \le \overline{x}$ where $\underline{x}, \overline{x}$, represent lower and upper bounds respectively. Four important elementary definitions based on unary operations are stated as:

a Midpoint: $m(X) = (\underline{x} + \overline{x})/2$

b Width: $w(X) = (\overline{x} - \underline{x})$

c Radius: $rad(X) = (\overline{x} - \underline{x})/2$

d Absolute value: $|X| = \max\{|\overline{x}|, |\underline{x}|\}$

(Note that $|x| \le |X|$ for every $x \in X$)

The basic interval arithmetic operations are defined on interval vector such that the interval result includes all possible real values. For a given interval $X = [\underline{X}, \overline{X}]$ and $Y = [\underline{Y}, \overline{Y}]$ the elementary operations are defined as $XopY = \{xopy : x \in X, y \in Y\}$ where *op* denotes one of $\{+, -, \times, \div\}$. In case of division, $0 \notin Y$.

(a) Addition: $X + Y = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$

(b) Subtraction: $X - Y = [\underline{x} - \overline{y}, \overline{x} - \underline{y}]$

(c) Multiplication: $X \times Y = [\min\{\underline{x}\times\underline{y}, \underline{x}\times\overline{y}, \overline{x}\times\underline{y}, \overline{x}\times\overline{y}\}, \max\{\underline{x}\times\underline{y}, \underline{x}\times\overline{y}, \overline{x}\times\underline{y}, \overline{x}\times\overline{y}\}]$

(d) Division: $X / Y = [\underline{x}, \overline{x}] \times [1/\overline{y}, 1/\underline{y}]$ if $0 \notin Y$.

(e) Intersection: $X \cap Y = [\max\{\underline{x}, \underline{y}\}, \min\{, \overline{x}, \overline{y}\}]$

(f) Union: $X \cup Y = \begin{cases} [\min\{\underline{x}, \underline{y}\}, \max\{, \overline{x}, \overline{y}\}] & if \quad X \cap Y \neq \phi \\ undefined, & otherwise \end{cases}$

An interval function is an *interval valued* function of one or more variables. For interval $X = (X1, \ldots.., Xn)$, an interval function $F(X1, \ldots.., Xn)$ is said to be an interval extension of a real function $F(x1, \ldots.., xn)$ if

$$F(x1, \ldots.., xn) \in F(X1, \ldots.., Xn) \tag{5}$$

$x_i \in X_i$ for all $i = 1, \ldots., n$. An interval function F, is called inclusion monotonic if $X_i \subset Y_i$ implies $F(X1, \ldots.., Xn) \subset F(Y1, \ldots.., Yn)$.

Interval extension of a mathematical function can be derived by partitioning its domain into monotonic regions. For example, the interval extension of *sin(X)* for 0 to 2π can be formulated as:

$$\sin(X) = \begin{cases} [\sin(\underline{x}), \sin(\overline{x})] & \{0 \le \underline{x} \le \overline{x} \le \pi/2\} \cup \{3\pi/2 \le \underline{x} \le \overline{x} \le 2\pi\} \\ [\sin(\overline{x}), \sin(\underline{x})] & \pi/2 \le \underline{x} \le \overline{x} \le 3\pi/2 \\ [\min(\sin(\underline{x}), \sin(\overline{x})), 1] & \{0 \le \underline{x} \le \pi/2\} \cap \{\pi/2 \le \overline{x} \le 3\pi/2\} \\ [-1, \max(\sin(\underline{x}), \sin(\overline{x}))] & \{\pi/2 \le \underline{x} \le 3\pi/2\} \cap \{3\pi/2 \le \overline{x} \le 2\pi\} \\ [-1, 1] & \{0 \le \underline{x} \le \pi/2\} \cap \{3\pi/2 \le \overline{x} \le 2\pi\} \end{cases}$$

Uniform subdivision of an interval vector $X = (X1, \ldots.., Xn)$ can be defined as below [13]:
Let *N* be a positive integer and define

$$X_{i,j} = [\underline{X}_i + (j-1)w(X_i)/N, \underline{X}_i + jw(X_i)/N], \quad j = 1, 2, \ldots.., N. \tag{6}$$

Equation (6) is used for generating uniform subdivision of the angle vector for the given angle range in the inverse kinematics procedure.

Intervals methods may be used in many ways to solve the nonlinear equations. Mostly these methods can be described in terms of contraction operators, or contractors [13]. Function of the contractors is either reducing the size of, or completely eliminates, the region in which solutions to the equation system of interest are being sought. The contraction strategies based methods are Krawczyk and interval-Newton methods that have been widely used in the solution of nonlinear equation systems. Here we next described the interval-Newton methods only which is used in this paper to solve the nonlinear equations.

2.2.1. Newton Method

Let us consider the nonlinear equations $f(x) = 0$, such that we need a solution in vector x. Suppose that $f(x)$ has a continuous derivative in the region of interest. By the mean value theorem:

$$f(x) = f(y) + f^{'}(\varepsilon)(x - y) \qquad (7)$$

where ε lies between x and y. Now assume $f(x) = 0$, and now (7) became

$$x = y - f(y) / f^{'}(\varepsilon) \qquad (8)$$

Suppose $X = [a, b]$, be an interval containing x and y, and $F^{'}(X)$ be an inclusion monotonic interval extension of $f^{'}(X)$ hence interval Newton algorithm [13] can be written as

$$X^{(k+1)} = X^{(k)} \cap N(X^{(k)}) \qquad k = 0,1,2,\ldots\ldots \qquad (9)$$

where

$$N(X) = m(X) - f(m(X)) / F^{'}(X) \qquad (10)$$

and

$$m(X) = (a + b) / 2$$

The above solution is for one dimensional variable. For multivariable case (7) can be written as

$$f(y) - f(x) = A(y - x) \qquad (11)$$

Where A is a matrix whose ith row is given by

$$A_i = \nabla^T f_i(c_i) = (\frac{\partial f_i}{\partial x_1}(c_i), \ldots\ldots\ldots\ldots\ldots, \frac{\partial f_i}{\partial x_n}(c_i)) \qquad (12)$$

To solve the multidimensional interval Newton method set the equation as assume $f(x) = 0$, then

$$x = y - A^{-1} f(y) \qquad (13)$$

Here A is replaced with $F^{'}(X)$. For simplicity it can be written as

$$x \in N(X) = y + V \qquad (14)$$

Where V bound the solution set to

$$F^{'}(X)v = -F(y) \qquad (15)$$

Using the Interval Gauss' elimination method solve (15). $N(X)$ in (14) is the multidimensional analog to (10).

3. Inverse Kinematics for Redundant Manipulator

Here the inverse kinematic of the manipulator is obtained based on optimization method for the redundancy resolution. The inverse kinematics of the 3-DOF planar manipulator is being studied with maximize the Manipulability of Jacobian as objective function and reachability as constraints**.**

The considered manipulator configuration of a 3-DOF planar redundant manipulator is shown in Fig. 1, which consists of three links of lengths l_1 , l_2 and l_3. The inverse kinematics of the manipulator for a given task position (x_g , y_g) is solved by formulating optimization problem as maximize the manipulability.

The manipulability of a manipulator is defined as

$$w = \sqrt{\det(JJ^T)}$$

where J is Jacobian of the manipulator.

Thus we formulate the problem as to find the value of angles θ_1, θ_2 and θ_3 that minimizes

$$P(\theta) = 1/w \tag{16}$$

In the bounded region of angles ranges, subject to the *j* (*j = 1,2*) equality constraints that satisfied the goal position i.e. $g_j(\theta) = 0$, is given below

Subject to: $$x_g - (l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)) = 0 \tag{17}$$

$$x_g - (l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)) = 0 \tag{18}$$

where (x_g , y_g) is the goal position of the manipulator in the workspace.

The equality constraint in (17) and (18) satisfies the reachability criterion (primary task), such that the vector function $g(q) = x - f(q) = 0$ gets satisfied for given task position.

General Fritz-John method [11] is applied to solve above equality optimization problem using interval method. The normalized Fritz-John condition for above can be written as below

$$u_0 \nabla P(\theta) + \sum_{j=1}^{2} v_j \nabla g_j(\theta) = 0 \tag{19}$$

$$g_j(\theta) = 0 \tag{20}$$

$$u_0 + \sum_{j=1}^{2} v_j^{\,2} = 1 \tag{21}$$

The initial bounds for multipliers are $0 \le u_0 \le 1$ and $-1 \le v_i \le 1$.

The interval Newton method is applied to the Fritz John conditions. The algorithm developed here works on boxes, each box being a set of intervals of angles. We assume the initial box X(0) is given in which the solution is sought. The algorithm is described below as:

Algorithm:

To solve the (19), (20), and (21) simultaneously ($f(x)=0$) using interval Newton method the following steps are followed.

(i) Initial interval vector *X(0)* (i.e., range of angles based on requirement) is given which forms a box.
(ii) Angle ranges are divided into several sub-boxes using the uniform interval subdivision strategy by applying (6) and form Stack, L_1.
(iii) Take one by one a sub box from Stack L_1 and repeat through step (x) until the Stack is empty.
(iv) Take one box X to be processed, find the value of *F(X).*
(v) Check, if $0 \notin F(X)$, means it do not have root. go to step (x)
(vi) Using (14) multidimensional interval Newton method, find $N(X)$

$$\text{if } N(X) \cap X = \phi, \text{ then go to step (x)}$$

(vii) If $N(X) \subseteq X = X$, output X go to step (x)
(viii) Now bisect X such that $X = X^{(1)} \cup X^{(2)}$

$$X^{(1)} = [\underline{X}, (\underline{X} + \overline{X})/2]$$
$$X^{(2)} = [(\underline{X} + \overline{X})/2, \overline{X}]$$

(ix) Push $X^1, X^{(2)}$ into the Stack
(x) Set X = top of Stack

3.1 Example: 3-DOF Planar Manipulator

The considered revolute type 3 DOF planar manipulator configuration is shown in Fig. 1, which consists of three links of lengths l_1, l_2 and l_3. The inverse kinematics of the manipulator for a given task position is solved using the above interval mentioned method. Interval calculations are carried out by using MATLAB and a toolbox called INTLAB developed by S. M. Rump [14].

The considered 3-DOF planar manipulators has a reach of 1.2 m along with links length as $l_1 = 0.6m$, $l_2 = 0.3m$ and $l_3 = 0.3m$, with manufacturing tolerances 10^{-4} m and desired goal point is chosen to be $x_g = 0.8m$, $y_g = 0.8m$ with precision considered in the range of ±0.001m. The angle ranges of the joints considered are $q_1 = -\pi/2$ to π, $q_2 = -\pi/2$ to π and $q_3 = -\pi/2$ to $\pi/2$.

The link lengths can be represented in interval form as follows:

$$l_1 = [0.5999, 0.6001]$$
$$l_2 = [0.2999, 0.3001]$$
$$l_3 = [0.2999, 0.3001]$$

Similarly the goal point can be represented in interval form as follows

$$x_g = [0.799, 0.801], \; y_g = [0.799, 0.801]$$

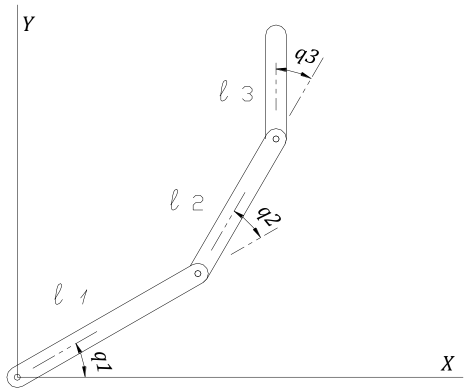

Fig.1. 3-DOF Planar Manipulator

The output angles are as follows

$$q1 = [0.4783, 0.4791]$$
$$q2 = [0.4187, 0.4191]$$
$$q3 = [0.3989, 0.3990]$$

The algorithm output is plotted in Fig. 2, in which the outer box represent the goal position and the inner box represent the manipulator end effector position (manipulator tip position).

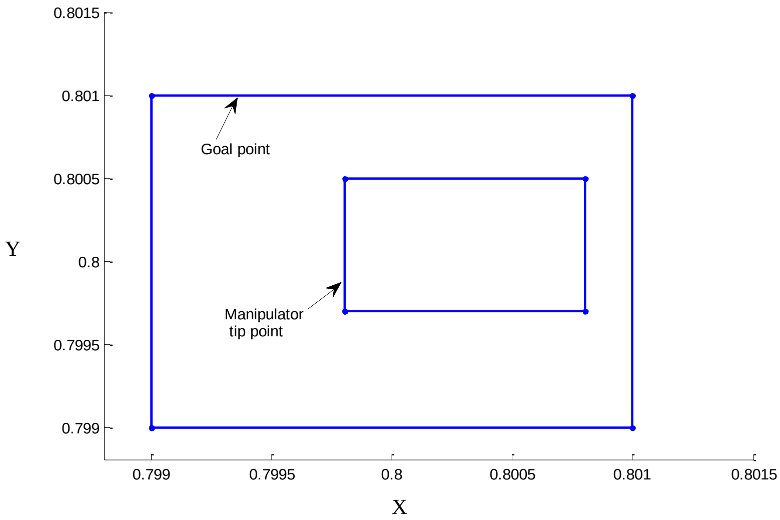

Fig.2. The Output Result – The End-Effector Position Lies Within The Goal Position Precision Range.

From Fig. 2, it is clear that the manipulator end effector (manipulator tip point) position which is plotted corresponding to the output angles is lie within the goal position.

4. Conclusion

Inverse kinematics is an important step in performance metric based design optimization of redundant manipulators. Handling manufacturing tolerances, complex geometry of links, etc. is a concern in design of a manipulator for desired performance accuracy. Interval arithmetic method is a viable technique in dealing with these tolerances, where, point solution is not sought, rather a solution interval is found. This paper attempts to evaluate the inverse kinematics of 3-DOF planar redundant manipulator arm using the interval Newton method. Given a tolerance range and acceptable goal point precision range, this paper develops a procedure to find inverse kinematics solution in intervals for redundant 3-DOF planar manipulator. The inverse kinematics solution simultaneously finds a posture, where, a performance metric is optimized. The performance criterion considered here is the so called manipulability of Jacobian of the manipulator. MATLAB© function from the INTLAB toolbox, developed by S.M. Rump, have been utilized in performing the interval numerical computations.

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