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A Note on Determinant of Square Fuzzy Matrix

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Abstract—In this article, we would like to study the determinant theory of fuzzy matrices. The purpose of this article is to present a new way of expanding the determinant of fuzzy matrices and thereafter some properties of determinant are considered. Most of the properties are found to be analogous to the properties of determinant of matrices in crisp cases.

Index Terms — reference function**,** membership value, convergence of powers of fuzzy matrices, complement of a fuzzy set.

I. INTRODUCTION

The theory of fuzzy sets was first introduced by Zadeh [1], as an appropriate mathematical instrument for description of uncertainty observed in nature. Since the inception it has got intensive acceptability in various fields.

Fuzzy matrix has been proposed to represent fuzzy relation in a system based on fuzzy set theory, Ovehinnikov [2]. Fuzzy matrices were introduced first time by Thomson [3], who discussed the convergence of powers of fuzzy matrices. Several authors had presented a number of results on the covergence of power sequences of fuzzy matrices. Several authors have presented a number of results on the convergence of power sequence of fuzzy matrices ([4], [5], [6]). Fuzzy matrices play an important role in science and technology. It plays an important role in fuzzy set theory. It is well known that the matrix formulation of a mathematical formula gives extra advantage to handle/study the problem. When some problems are not solved by classical matrices, then the concepts of fuzzy matrices are used. Kim ([7], [8], [9]) represented some important result on the determinant of a square matrix. He defined the determinant of a square fuzzy matrix and contributed with very research works ([7], [8], [9], [10]) a lot to the study of determinant theory of square fuzzy matrices. The properties of the determinant of a square fuzzy matrix are analogous to the properties of determinant of the determinant of matrices in general.

It is important to mention here that the properties are studied taking into consideration of the complementation of fuzzy matrices in our way because we are not in a position to accept the existing definition of complementation of fuzzy sets due to lack of logical backgrounds. If it is so then same should be followed in case of fuzzy matrices also because it has already been mentioned that matrix has been proposed to represent fuzzy set theory. Just as classical relation can be viewed as a set, fuzzy relation can be viewed as a fuzzy subset.

In this article, our main intention is to introduce a new definition of determinant of square fuzzy matrices. Furthermore, efforts have been made to establish some properties with the help of the new introduced definition of determinant of square fuzzy matrices. But before proceeding further it is necessary to mention here that the fuzzy matrices are at first represented on the basis of reference function which can be found in Dhar ([11], [12]) and then the new procedure for finding the determinant is put forward.

The rest of the paper has been organized as follows: Section II deals with the definition of square fuzzy matrix and the new definition of determinant of fuzzy matrices. The process of finding determinant of a square fuzzy matrix is presented with the help of a numerical example. Section III provides some properties of determinant of square fuzzy matrices which are presented with the help of some numerical examples. Section IV gives our conclusions.

II. DETERMINANT OF A SQUARE FUZZY MATRIX

In this section, we would like to provide a new method of finding the determinant of a fuzzy square matrix. For this purpose let us have a look at the definition of fuzzy square matrix

2.1 Definition: Square Fuzzy Matrix

A fuzzy matrix is a matrix which has its elements from the interval [0, 1], called the unit fuzzy interval. An $m \times n$ fuzzy matrix for which m=n (i.e the number of rows is equal to the number of columns) and whose elements belong to the unit interval [0, 1] is called a fuzzy square matrix of order n.

A fuzzy square matrix of order two is expressed in the following way

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad (1)$$

where the entries a,b,c,d all belongs to the interval [0,1].

It is important to mention here the fact that since in case of fuzzy sets we prefer to represent it with the help of reference function and so the use of reference function in fuzzy matrices cannot be overlooked. But in case of usual matrices there would not be much difference because the membership value and membership functions are of course equal but in case of complementation it makes sense. It is for this reason we would like to deal with matrix complementation to show how the new representation also satisfy the properties which are seen in case of existing definition of fuzzy matrices.

In accordance with the process of defining complementation of a fuzzy set as defined by Baruah ([13], [14]) a fuzzy set

$$A=\{x,\mu(x),x\in X\} \tag{2}$$

would be defined in this way as

$$A=\{x,\mu(x),0,x\in X\} \tag{3}$$

so that the complement would become

$$A^c=\{x,1,\mu(x),x\in X\} \tag{4}$$

Thus a square fuzzy matrix $A=[a_{ij}]_{n\times n}$ would be represented according to the new definition as $A=[(a_{ij},0)]_{n\times n}$ and similarly the complement matrix of the matrix A would be $A^c=[(1,a_{ij})]_{n\times n}$. The following example will make it clear.

The matrix A would be defined according to our way as

$$A=\begin{pmatrix}(a,0) & (b,0)\\ (c,0) & (d,0)\end{pmatrix} \tag{5}$$

because we are interested to define fuzzy sets with the help of reference function.

2.2 Determinant of square fuzzy matrices

Before proceeding further, let us define two operations which are mostly required in case of finding the determinant of fuzzy matrices.

(a ,b) + (c, d) = {max (a, c), min (b, d)} (6)

(a,b) (c ,d) = {min (a, c), max(b,d)} (7)

The determinant of the fuzzy matrix A would be denoted by

$$|A|=\begin{vmatrix}(a,0) & (b,0)\\ (c,0) & (d,0)\end{vmatrix} \tag{8}$$

And we would like to expand the above determinat in the following way

= [max{min(a,d),min(c,b),min{max(0,0),max(0,0)}] (9)

The complement of the above matrix A would be defined in our way as

$$A^c=\begin{pmatrix}(1,a) & (1,b)\\ (1,c) & (1,d)\end{pmatrix} \tag{10}$$

Now for a square fuzzy matrix of order 3

$$B=\begin{pmatrix}(a_1,0) & (b_1,0) & (c_1,0)\\ (a_2,0) & (b_2,0) & (c_2,0)\\ (a_3,0) & (b_3,0) & (c_3,0)\end{pmatrix} \tag{11}$$

The determinant would be denoted by

$$\begin{vmatrix}(a_1,0) & (b_1,0) & (c_1,0)\\ (a_2,0) & (b_2,0) & (c_2,0)\\ (a_3,0) & (b_3,0) & (c_3,0)\end{vmatrix}$$

and this would be defined as

$$=(a_1,0)\begin{vmatrix}(b_2,0) & (c_2,0)\\ (b_3,0) & (c_3,0)\end{vmatrix}+(b_1,0)\begin{vmatrix}(a_2,0) & (c_2,0)\\ (a_3,0) & (c_3,0)\end{vmatrix}$$
$$+(c_1,0)\begin{vmatrix}(a_2,0) & (b_2,0)\\ (a_3,0) & (b_3,0)\end{vmatrix}$$
$$=(a_1,0)[\max\{\min(0,0),\min(0,0)\},\min\{\max(b_2,c_3),\max(b_3,c_2)\}]+$$
$$(b_1,0)[\max\{\min(0,0),\min(0,0)\},\min\{\max(a_2,c_3),\max(a_3,c_2)\}]+$$
$$(c_1,0)[\max\{\min(0,0),\min(0,0)\},\min\{\max(b_3,a_2),\max(b_2,a_3)\}] \tag{12}$$

This is the way of defining the fuzzy matrices in the usual case. But in fuzzy areas there arises some cases when complement of fuzzy matrices is involved. In order to meet the need of the situation it is important to define the complement of fuzzy matrices in our way. We would like to define the complement of fuzzy matrices in accordance with the definition of complement of fuzzy sets as defined by Baruah ([13], [14]). This result has been used in the works of Dhar ([15], [16], [17], [18], [19] & [20]) to draw some conclusions. The new definition of complementation of fuzzy sets has been discussed in details in our previous works and so we would like to mention about in brief in this very article. The following properties of determinant can be easily verified if we represent fuzzy matrices on the basis of reference function.

2.2.1 Properties of determinant of square fuzzy matrices

The following are some of the properties of determinant of square fuzzy matrices which are observed to hold both for usual fuzzy matrices and the complement of fuzzy matrices when fuzzy matrices are represented in the way proposed by us.

Property1

If the rows and columns are interchanged then the value of the determinant remains the same.

Property2

If any two rows or any two columns are interchanged in their positions, the value of the determinant remains the same.

Property3

If the elements in a row (column) are all zero, the value of the determinant is also zero.

Property4

If A and B be two square fuzzy matrices then the following property will hold
det (AB)=detAdetB

Property5

If the elements of any row (or column) of a determinant are added to the corresponding elements of another row (or column) , the value of the determinant thus obtained is equal to the value of the original determinant.

2.3 Determinant of complement of square fuzzy matrices

Taking into consideration of the above mention procedure, the complement of the above fuzzy matrix can be written as

$$B^c = \begin{pmatrix} (1,a_1) & (1,b_1) & (1,c_1) \\ (1,a_2) & (1,b_2) & (1,c_2) \\ (1,a_3) & (1,b_3) & (1,c_3) \end{pmatrix} \tag{13}$$

Then the determinant of the above complement matrix when expanded along the first row would be evaluated as

$$= (1,a_1)\begin{vmatrix} (1,b_2) & (1,c_2) \\ (1,b_3) & (1,c_3) \end{vmatrix} + (1,b_1)\begin{vmatrix} (1,a_2) & (1,c_2) \\ (1,a_3) & (1,c_3) \end{vmatrix}$$
$$+ (1,c_1)\begin{vmatrix} (1,a_2) & (1,b_2) \\ (1,a_3) & (1,b_3) \end{vmatrix}$$
$$= (1,a_1)[\max\{\min(1,1),\min(1,1)\},\min\{\max(b_2,c_3),\max(b_3,c_2)\}]$$
$$+$$
$$(1,b_1)[\max\{\min(1,1),\min(1,1)\},\min\{\max(a_2,c_3),\max(a_3,c_2)\}]$$
$$+$$
$$(1,c_1)[\max\{\min(1,1),\min(1,1)\},\min\{\max(b_3,a_2),\max(b_2,a_3)\}] \tag{14}$$

It is to be noted here that the above determinant is expanded along the first row. But it can be easily observed that the value of the determinant remains unchanged if it is expanded along any rows or columns.

2.4 Numerical Examples

Here we shall put a numerical example to find the determinant of a fuzzy matrix in the way described above.

$$A^c = \begin{pmatrix} (1,0.5) & (1,0.3) & (1,0.8) \\ (1,0.6) & (1,0.2) & (1,0.9) \\ (1,0) & (1,0.7) & (1,0.4) \end{pmatrix}$$

Then the determinant of the above matrix when expanded along the first row would give us the following result

$$|A^c| = (1,0.5)\begin{vmatrix} (1,0.2) & (1,0.9) \\ (1,0.7) & (1,0.4) \end{vmatrix} + \tag{15}$$
$$((1,0.3)\begin{vmatrix} (1,0.6) & (1,0.9) \\ (1,0) & (1,0.4) \end{vmatrix} +$$
$$(1,0.8)\begin{vmatrix} (1,0.6) & (1,0.2) \\ (1,0) & (1,0.7) \end{vmatrix}$$

=(1,0.5)[max{min(1,1),min(1,1)},
min{max(0.2,0.4),min(0.7,0.9)}]+
(1,0.3)[max{min(1,1),min(1,1)},
min{max(0.6,0.4),max(0,0.9)}]+(1,0.8)[max{min(1,1),
min(1,1)},min{max(0.6,0.7),max(0,0.2)}]
=(1,0.5)[max(1,1), min(0.4,0.9)] + (1,0.3) [max(1,1),
min(0.6,0.9)] + (1,0.8)[max(1,1), min(0.7,0.2)]
=(1,0.5) (1,0.4)+ (1,0.3) (1,0.6)+ (1,0.8) (1,0.2)
={min(1,1), max(0.5, 0.4)} + {min(1,1), max(0.3, 0.6)}
+ {min(1,1), max(0.8, 0.2)}
= (1, 0.5) + (1, 0.6) + (1, 0.8)
= {max (1, 1), min (0.5, 0.6)} + (1, 0.8)
= (1, 0.5) + (1, 0.8)
= {max (1, 1), min (0.5, 0.8)}
= (1, 0.5)

Let us see what happens when the determinant is expanded along the 2nd row.

The value of the determinant when expanded along the second row becomes

$$|A^c| = (1,0.6)\begin{vmatrix} (1,0.3) & (1,0.8) \\ (1,0.7) & (1,0.4) \end{vmatrix} +$$
$$((1,0.2)\begin{vmatrix} (1,0.5) & (1,0.8) \\ (1,0) & (1,0.4) \end{vmatrix} +$$
$$(1,0.9)\begin{vmatrix} (1,0.5) & (1,0.3) \\ (1,0) & (1,0.7) \end{vmatrix}$$

=(1,0.6)[max{min(1,1),min(1,1)},
min{max(0.3,0.4),max(0.7,0.8)+
(1,0.2)[max{min(1,1),min(1,1)},
min{max(0.5,0.4),max(0,0.8)}]+(1,0.9)
max{min(1,1),min(1,1)},min{max(0.5,0.7),
max (0,0.3)}]
=(1,0.6)[max(1,1), min(0.4,0.8)] + (1,0.2) [max(1,1),
min(0.5,0.8)] + (1,0.9)[max(1,1), min(0.7,0.3)]
=(1,0.6) (1,0.4)+ (1,0.2) (1,0.5)+ (1,0.9) (1,0.3)
={min(1,1), max(0.6, 0.4)} + {min(1,1), max(0.2, 0.5)}
+{min(1,1)},max(0.9, 0.3)}
= (1, 0.6) + (1, 0.5) + (1, 0.9)
= {max (1, 1), min (0.6, 0.5)} + (1, 0.9)
= (1, 0.5) + (1, 0.9)
= {max (1, 1), min (0.5, 0.9)}
= (1, 0.5)

The value of the determinant when expaned along the 3rd row would give us the following result

$$|A^c| = (1,0)\begin{vmatrix} (1,0.3) & (1,0.8) \\ (1,0.2) & (1,0.9) \end{vmatrix} + \tag{16}$$
$$((1,0.7)\begin{vmatrix} (1,0.5) & (1,0.8) \\ (1,0.6) & (1,0.9) \end{vmatrix} +$$
$$(1,0.4)\begin{vmatrix} (1,0.5) & (1,0.3) \\ (1,0.6) & (1,0.2) \end{vmatrix}$$

=(1,0)[max{min(1,1),min(1,1)},
min{max(0.3,0.9),min(0.2,0.8)}]+
(1,0.7)[max{min(1,1),min(1,1)},min{max(0.5,0.9),
max(0.6,0.8)}]+(1,0.4)[max{min(1,1),min(1,1)},
min {max(0.5,0.2),max(0.6,0.3)}]
=(1,0)[max(1,1), min(0.9,0.8)] + (1,0.7) [max(1,1),
min(0.9,0.8)] + (1,0.4)[max(1,1), min(0.5,0.6)]
=(1,0) (1,0.8)+ (1,0.7) (1,0.8)+ (1,0.4) (1,0.5)
={min(1,1), max(0, 0.8)} + {min(1,1), max(0.7, 0.8)}
+{min(1,1)},max(0.4, 0.5)}
= (1, 0.8) + (1, 0.8) + (1, 0.5)
= {max (1, 1), min (0.8, 0.8)} + (1, 0.5)
= (1, 0.8) + (1, 0.5)
= {max (1, 1), min (0.8, 0.5)}
= (1, 0.5)

The value of the determinant when expanded along the 1st column becomes

$$\left|A^c\right| = (1,0.5)\begin{vmatrix}(1,0.2) & (1,0.9)\\(1,0.7) & (1,0.4)\end{vmatrix} + ((1,0.6)\begin{vmatrix}(1,0.3) & (1,0.8)\\(1,0.7) & (1,0.4)\end{vmatrix} + (1,0)\begin{vmatrix}(1,0.3) & (1,0.8)\\(1,0.2) & (1,0.9)\end{vmatrix} \quad (17)$$

=(1,0.5)[max{min(1,1),min(1,1)},
min{max(0.2,0.4),min(0.7,0.9)}]+
(1,0.6)[max{min(1,1),min(1,1)},min{max(0.3,0.4),
max(0.7,0.8)}]+(1,0)[max{min(1,1),min(1,1)},
min {max(0.3,0.9),max(0.2,0.8)}]
=(1,0.5)[max(1,1), min(0.4,0.9)] + (1,0.6)
[max(1,1), min(0.4,0.8)] +
(1,0)[max(1,1), min(0.9,0.8)]
=(1,0.5) (1,0.4)+ (1,0.6) (1,0.4)+ (1,0.4) (1,0.8)
={min(1,1), max(0.5, 0.4)} + {min(1,1),
max(0.6, 0.4)} +{min(1,1)},max(0.4, 0.8)}
=(1,0.5) + (1,0.6) + (1,0.8)
={max(1,1), min(0.5, 0.6)} + (1,0.8)
=(1, 0.5) + (1,0.8)
={max(1,1), min(0.5,0.8)}
= (1, 0.5)

Thus it can be easily verified that the determinant of the matrix when expanded along any rows or columns would give us the same result. In the next section, we shall study some of the properties of fuzzy square matrices.

III. SOME PROPERTIES OF THE DETERMINANT OF FUZZY SQUARE MATRIX

This section is contributed to deal with some of the properties of the determinant of square fuzzy matrices.

Following are some of the properties of fuzzy square matrices.

Property 1

The value of the determinant remains unchanged when any two rows or columns are interchanged. Let us consider the follwing matrix X which is obtained by interchanging the 1st and 2nd column of the fuzzy matrix written above.

$$X = \begin{pmatrix}(1,b_1) & (1,a_1) & (1,c_1)\\(1,b_2) & (1,a_2) & (1,c_2)\\(1,b_3) & (1,a_3) & (1,c_3)\end{pmatrix}$$

$$\begin{aligned}&= (1,b_1)[\max\{\min(1,1),\min(1,1)\},\\&\min\{\max(a_2,c_3),\max(a_3,c_2)\}]+\\&(1,a_1)[\max\{\min(1,1),\min(1,1)\},\\&\min\{\max(b_2,c_3),\max(b_3,c_2)\}]+\\&(1,c_1)[\max\{\min(1,1),\min(1,1)\},\\&\min\{\max(b_2,a_3),\max(b_3,a_2)\}]\end{aligned} \quad (18)$$

The determinant of this matrix when expanded along the first row would give us

$$\begin{aligned}&= (1,b_1)\begin{vmatrix}(1,a_2) & (1,c_2)\\(1,a_3) & (1,c_3)\end{vmatrix} + (1,a_1)\begin{vmatrix}(1,b_2) & (1,c_2)\\(1,b_3) & (1,c_3)\end{vmatrix} +\\&(1,c_1)\begin{vmatrix}(1,b_2) & (1,a_2)\\(1,b_3) & (1,a_3)\end{vmatrix}\\&= (1,b_1)[\max\{\min(1,1),\min(1,1)\},\min\{\max(a_2,c_3),\max(a_3,c_2)\}]+\\&(1,a_1)[\max\{\min(1,1),\min(1,1)\},\min\{\max(b_2,c_3),\max(b_3,c_2)\}]+\\&(1,c_1)[\max\{\min(1,1),\min(1,1)\},\min\{\max(b_2,a_3),\max(b_3,a_2)\}]\end{aligned} \quad (19)$$

In the following numerical example, we have tried to show how the above mentioned property holds.

Let us consider the determinant of the above mentioned matrix with 1st and 2nd columns interchanged.

$$\begin{vmatrix}(1,0.3) & (1,0.5) & (1,0.8)\\(1,0.2) & (1,0.6) & (1,0.9)\\(1,0.7) & (1,0) & (1,0.4)\end{vmatrix}$$

$$\left|A^c\right| = (1,0.3)\begin{vmatrix}(1,0.6) & (1,0.9)\\(1,0) & (1,0.4)\end{vmatrix} + ((1,0.5)\begin{vmatrix}(1,0.2) & (1,0.9)\\(1,0.7) & (1,0.4)\end{vmatrix} + (1,0.8)\begin{vmatrix}(1,0.2) & (1,0.6)\\(1,0.7) & (1,0)\end{vmatrix} \quad (20)$$

=(1,0.5)[max{min(1,1),min(1,1)},min{max(0.6,0.4),
max(0,0.9)}]
+(1,0.5)[max{min(1,1),min(1,1)},min{max(0.2,0.4),
max(0.7,0.9)}]
+(1,0.8)[max{min(1,1),min(1,1)},min{max(0.2,0),
max(0.6,0.7)}]
=(1,0.3) [max(1,1), min(0.6, 0.9)] + (1,0.5)
[max(1,1), min(0.4, 0.9)] + (1,0.8) [max(1,1),
min(0.2, 0.7)]
=(1,0.3) (1,0.6) + (1,0.5) (1,0.4)+(1,0.8) (1,0.2)
=[max{min(1,1)},min{max(0.3,0.6)}]+
[max{min(1,1)},min{max(0.5,0.4)}] +
[max{min(1,1)}, min{max(0.8, 0.2)}]
=(1,0.6)+(1,0.5)+(1,0.8)
= {max (1, 1), min (0.6, 0.5)}+(1,0.8)
= (1, 0.5) + (1, 0.8)
= {max (1, 1), min (0.5, 0.8)
= (1, 0.5)

Thus we can say that the values of the determinant remain unchanged if any two rows or columns are interchanged.

Property2

The values of the determinant of fuzzy square matrix remain unchanged when rows and columns are interchanged.

Let us consider the following matrix which is obtained by interchanging the rows and columns of the above matrix

$$\begin{pmatrix}(1,a_1) & (1,a_2) & (1,a_3)\\(1,b_1) & (1,b_2) & (1,b_3)\\(1,c_1) & (1,c_2) & (1,c_3)\end{pmatrix}$$

Now expanding along the first column we get the same value of the determinant as

$$= (1,a_1)\begin{vmatrix}(1,b_2) & (1,b_3)\\(1,c_2) & (1,c_3)\end{vmatrix} + (1,b_1)\begin{vmatrix}(1,a_2) & (1,a_3)\\(1,c2) & (1,c_3)\end{vmatrix} + (1,c_1)\begin{vmatrix}(1,a_2) & (1,a_3)\\(1,b_2) & (1,b_3)\end{vmatrix}$$

$$= (1,a_1)[\max\{\min(1,1),\min(1,1)\},\min\{\max(b_2,c_3),\max(b_3,c_2)\}] + (1,b_1)[\max\{\min(1,1),\min(1,1)\},\min\{\max(a_2,c_3),\max(a_3,c_2)\}] + (1,c_1)[\max\{\min(1,1),\min(1,1)\},\min\{\max(b_2,a_3),\max(b_3,a_2)\}] \quad (21)$$

Thus we can see from the above that the value of the determinant remains unchanged when the rows and columns are interchanged.

Numerical Example

The citation of the following numerucal example will help to understand the process discussed above.

$$\begin{vmatrix}(1,0.5) & (1,0.6) & (1,0)\\(1,0.3) & (1,0.2) & (1,0.7)\\(1,0.8) & (1,0.9) & (1,0.4)\end{vmatrix} = (1,0.5)\begin{vmatrix}(1,0.2) & (1,0.7)\\(1,0.9) & (1,0.4)\end{vmatrix} + (1,0.6)\begin{vmatrix}(1,0.3) & (1,0.7)\\(1,0.8) & (1,0.4)\end{vmatrix} + (1,0)\begin{vmatrix}(1,0.3) & (1,0.2)\\(1,0.8) & (1,0.9)\end{vmatrix} \quad (22)$$

=(1,0.5)[max{min(1,1),min(1,1)},min{max(0.2,0.4), max(0.7,0.9)}]
+(1,0.6)[max{min(1,1),min(1,1)},min{max(0.3,0.4), max(0.7,0.8)}]
+(1,0.8)[max{min(1,1),min(1,1)},min{max(0.3,0.9), max (0.8,0.2)}]
=(1,0.5)[max(1,1), min(0.4,0.9)] + (1,0.6)[max(1,1), min(0.4,0.8)]+(1,0.8)[max(1,1), min(0.9, 0.8)]
=(1,0.5)(1,0.4)+(1,0.6)(1,0.4)+(1,0.8)(1,0.8)
=(1,0.5)+(1,0.6)+(1,0.8)
= (1, 0.5) + (1, 0.8)
= (1, 0.5)

Property3

If the elements of a row (column) of a determinant are all zero, then the value of the determinant is zero.

It is important to mention here the fact that the elements zero in our case indicate that membership value of the element of any row or column is zero.

Let us consider the following matrix

$$C^c = \begin{pmatrix}(1,1) & (1,1) & (1,1)\\(1,1) & (1,0.9) & (1,0)\\(1,0.8) & (1,0.2) & (1,0.3)\end{pmatrix}$$

$$= (1,1)\begin{vmatrix}(1,0.9) & (1,0)\\(1,0.2) & (1,0.3)\end{vmatrix} + (1,1)\begin{vmatrix}(1,1) & (1,0)\\(1,0.8) & (1,0.3)\end{vmatrix} + (1,1)\begin{vmatrix}(1,1) & (1,0.9)\\(1,0.8) & (1,0.2)\end{vmatrix} \quad (23)$$

=(1,1)[max({min(1,1),min(1,1)},min{max(0.9,0.3), max(0,0.2)}]+
(1,1)[max(min(1,1),min(1,1)},min{max(1,0.3), max(0,0.8)}]+(1,1)[max{min(1,1),min(1,1)}, min{max(1,0.2),max(0.8,0.9)}]
=(1,1)[max(1,1), min(0.9,0.2)] + (1,1)[max(1,1), min(1,0.8)]+(1,1)[max(1,1), min(1, 0.9)]
=(1,1)(1,0.2) +(1,1)(1,0.8)+(1,1)(1,0.9)
=(1,1) +(1,1)+(1,1)
= (1, 1)

The membership value of this is zero and hence the result.

Property 4

If A^c and B^c be two square fuzzy matrices of same order then the following property will hold

$$\det(A^c B^c) = \det A^c \det B^c$$

Here for convenience we shall consider the above square matrix A^c as one and let us consider another fuzzy square matrix of order 3 as follows

$$B^c = \begin{pmatrix}(1,0.3) & (1,1) & (1,0.7)\\(1,1) & (1,0.9) & (1,0)\\(1,0.8) & (1,0.2) & (1,0.3)\end{pmatrix}$$

Proceeding in the similar manner we would get the value of the determinant as (1, 0.3).
Hence we get

$$\det A^c \det B^c = (1,0.5)(1,0.3) = (1,\ 0.5) \quad (24)$$

Now in order to find $\det(A^c B^c)$) we would have to define the product of the two fuzzy matrices and . So we should first define the multiplication of two fuzzy matrices when represented with the help of reference function.

Before proceeding further let us define the multiplication of two fuzzy matrices for illustration purposes.

3.1 Definition: Multiplication of matrices

The product of two fuzzy matrices under usual matrix multiplication is not a fuzzy matrix. It is due to this reason; a conformable operation analogus to the product which again happens to be a fuzzy matrix was introduced by many researchers which can be found in fuzzy literature. However, even for this operation if the product AB to be defined if the number of columns of the first fuzzy matrix is must be equal to the number of rows of the second fuzzy matrix. In the process of finding multiplication of fuzzy matrices, if this condition is satisfied then the multiplication of two fuzzy matrices A and B, will be defined in the following manner:

$$AB = \{\max\min(a_{ij}, b_{ji}), \min\max(r_{ij}, r'_{ji}\}$$

Where a_{ij} and r_{ij} , $1 \le i, j \le n$ stands for the membership function of the fuzzy matrix A and the corresponding reference function whereas b_{ij} stands for the membership function of the fuzzy matrix B with the corresponding reference function r'_{ij} where $1 \le i, j \le n$.

The multiplication of two fuzzy matrices which are conformable for multiplication are discussed below

Let

$$C = \begin{pmatrix} (1,a_{11}) & (1,a_{12}) & (1,a_{13}) \\ (1,a_{21}) & (1,a_{22}) & (1,a_{23}) \\ (1,a_{31}) & (1,a_{32}) & (1,a_{33}) \end{pmatrix}$$

and

$$D = \begin{pmatrix} (1,b_{11}) & (1,b_{12}) & (1,b_{13}) \\ (1,b_{21}) & (1,b_{22}) & (1,b_{23}) \\ (1,b_{31}) & (1,b_{32}) & (1,b_{33}) \end{pmatrix}$$

be two fuzzy matrices. Then the multiplication of these two matrices wolud be defined in our way as

$$CD = \begin{pmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{pmatrix}$$

Were

$$E_{11} = [\max\{\min(1,1), \min(1,1), \min(1,1)\},$$
$$\min\{\max(a_{11}, b_{11}), \max(a_{12}, b_{21}), \max(a_{13}, b_{31})\}]$$

$$E_{12} = [\max\{\min(1,1), \min(1,1), \min(1,1)\},$$
$$\min\{\max(a_{11}, b_{12}), \max(a_{12}, b_{22}), \max(a_{13}, b_{23})\}]$$

$$E_{21} = [\max\{\min(1,1), \min(1,1), \min(1,1)\},$$
$$\min\{\max(a_{21}, b_{11}), \max(a_{22}, b_{21}), \max(a_{23}, b_{31})\}]$$

$$E_{22} = [\max\{\min(1,1), \min(1,1), \min(1,1)\},$$
$$\min\{\max(a_{21}, b_{12}), \max(a_{22}, b_{22}), \max(a_{23}, b_{32}\}]$$

$$E_{23} = [\max\{\min(1,1), \min(1,1), \min(1,1)\},$$
$$\min\{\max(a_{21}, b_{13}), \max(a_{22}, b_{23}), \max(a_{23}, b_{33}\}]$$

and so on.

Numerical Example

$$A^c B^c = \begin{pmatrix} (1,0.5) & (1,0.3) & (1,0.8) \\ (1,0.6) & (1,0.2) & (1,0.9) \\ (1,0) & (1,0.7) & (1,0.4) \end{pmatrix} = \begin{pmatrix} (1,0.3) & (1,1) & (1,0.7) \\ (1,1) & (1,0.9) & (1,0) \\ (1,0.8) & (1,0.2) & (1,0.3) \end{pmatrix} \quad (25)$$

Which when calculated with our method of multiplication would give us the following result

$$= \begin{pmatrix} (1,0.5) & (1,0.8) & (1,0.3) \\ (1,0.6) & (1,0.9) & (1,0.2) \\ (1,0.3) & (1,0.4) & (1,0.4) \end{pmatrix}$$

Now the determinant of the above fuzzy square matrix is (1, 0.5) which states the fact that

$$\det(A^c B^c) = \det A^c \det B^c \quad (26)$$

Property5

If the elements of any row (or column) of a determinant are added to the corresponding elements of another row (or column), the value of the determinant thus obtained is equal to the value of the original determinant.

Let us consider the following example to make the point clear.

$$B^c = \begin{pmatrix} (1,0.3) & (1,1) & (1,0.7) \\ (1,1) & (1,0.9) & (1,0) \\ (1,0.8) & (1,0.2) & (1,0.3) \end{pmatrix} \quad (27)$$

If the elements of first columns are added to the corresponding elements of the second columns then the above matrix would take the form

$$B^c = \begin{pmatrix} (1,0.3) & (1,1) & (1,0.7) \\ (1,0.9) & (1,0.9) & (1,0) \\ (1,0.2) & (1,0.2) & (1,0.3) \end{pmatrix}$$

The determinant of this matrix when evaluated would give the following result

=(1,0.3)(1,0.2) + (1,1)(1,0.2)+ (1,0.7)(1,0.2)

=(1,0.3)+(1,1)+(1,0.7)

= (1, 0.3) + (1, 0.7)

= (1, 0.3)

If the elements of first columns are added to the corresponding elements of the second and third columns then the above matrix would take the form

$$B^c = \begin{pmatrix} (1,0.3) & (1,1) & (1,0.7) \\ (1,0) & (1,0.9) & (1,0) \\ (1,0.2) & (1,0.2) & (1,0.3) \end{pmatrix}$$

The determinant of this matrix when evaluated would give the following result

=(1,0.3)(1,0.2)+ (1,1)(1,0.2)+(1,0.7)((1,0.2)

=(1,0.3)+(1,1)+(1,0.7)

= (1, 0.3) + (1, 0.7)

= (1, 0.3)

Thus we can see from all the above cases, the value of the determinant remains unchanged if the elements of any row (or column) of a determinant are added to the corresponding elements of another row (or column).

Thus it is observed from the above that the properties which hold for usual matrices also do hold for the complement of fuzzy matrices even if we proceed in our way.

IV. CONCLUSIONS

In this article, the expansions of determinant of fuzzy matrices which are represented on the basis of reference function are discussed. Further some of the properties are studied and these are supported by some numerical examples. It is obseved that some of the properties of the determinant of square fuzzy matrix are analogus to the properties of the determinant of square matrices in crisp case. But there are some exceptions too which cannot be overlooked. For the sake of convenience we have studied the said properties by considering matrices of order 3. It is important to mention here that these properties would hold for all square matrices.

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