

The Split Domination in Product Graphs

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Abstract— The paper concentrates on the theory of domination in graphs. The split domination in graphs was introduced by Kulli and Janakirm. In this paper; we have investigated some properties of the split domination number of some product graphs and obtained several interesting results.

Index Terms—Domination, Split domination set, Split domination number, Standard graphs, Product Graphs.

I. INTRODUCTION

Graph theory is one of the most flourishing branches of modern mathematics. The last 30 years have witnessed spectacular growth of Graph theory due to its wide applications to discrete optimization problems, combinatorial problems and classical algebraic problems. It has a very wide range of applications in many fields like engineering, physical, social and biological sciences, linguistics etc., The theory of domination has been the nucleus of research activity in graph theory in recent times. This is largely due to a variety of new parameters, that can be developed from the basic definition of domination. The NP-completeness of the basic domination problems and its close relationship to other NP-completeness problems have contributed to the enormous growth of research activity in domination theory. It is clearly established from the exclusive coverage of the “Topics on domination in graph” in the 86th issue of the Journal of Discrete mathematics (1990), that the theory of domination is a very popular area of research activity in graph theory. Most of the terminology of the paper considered from references [1] [3].

The rigorous study of dominating sets in graph theory began around 1960, even though the subject has historical roots dating back to 1862 when the Jaenisch studied the problems of determining the minimum number of queens which are necessary to cover or dominate a $n \times n$ chessboard. In 1958, Berge defined the concept of the domination number of a graph, calling this as “coefficient of External Stability”. In 1962, Ore used the name ‘dominating set’ and ‘domination number’ for the same concept. In 1977 Cockayne and Hedetniemi made an interesting and extensive survey of the results know at that time about dominating sets in

graphs. They have used the notation $\gamma(G)$ for the domination number of a graph, which has become very popular since then.

The survey paper of Cockayne and Hedetniemi has generated lots of interest in the study of domination in graphs. In a span of about twenty years after the survey, more than 1,200 research papers have been published on this topic, and the number of papers continued to be on the increase. Since then a number of graph theorists Konig, Ore, Bauer, Harary, Lasker, Berge, Cockayne, Hedetniemi, Alavi, Allan, Chartrand, Kulli, Sampthkumar, Walikar, Armugam, Acharya, Neeralgi, Nagaraja Rao, Vangipuram many others have done very interesting and significant work in the domination numbers and the other related topics [5] [6]. A recent book on domination [2], has stimulated sufficient inspiration leading to the expansive growth of this field of study. It has also put some order into this huge collection of research papers, and organized the study of dominating sets in graphs into meaningful sub areas, placing the study of dominating sets in even broader mathematical and algorithmic contexts.

The split domination in graphs was introduced by Kulli & Janakiram [4]. They defined the split dominating set and the split domination number and obtained several interesting results regarding the split domination number of some standard graphs. They have also obtained relations of split domination number with the other parameters such as domination number, connected domination number, vertex covering number etc., Sampathkumar [7] obtained some interesting results on tensor products of graphs. Vasumathi & Vangipuram [8] and Vijayasradhi & Vangipuram [9] obtained domination parameters of an arithmetic Graph and also they have obtained an elegant method for the construction of an arithmetic graph with the given domination parameter.

Motivated by the study of domination and split domination we have investigated some properties of a split domination number of the product graphs.

Important definitions:

1.1: Dominating set:

A subset D of V is said to be a dominating set of G if every vertex in $V \setminus D$ is adjacent to a vertex x in D .

1.2: Dominating number:

The dominating number $\gamma(G)$ of G is the minimum cardinality of a dominating set.

1.3: Split dominating set:

A dominating set D of a graph G is called a split dominating set, if the induced subgraph $\langle V-D \rangle$ is disconnected.

1.4: Split domination number:

The split dominating number $\gamma_s(G)$ of G is the minimum cardinality of the split dominating set.

1.5: Kronecker Product of two graphs

If G_1, G_2 are two simple graphs with their vertex sets $V_1: \{u_1, u_2, \dots\}$ and $V_2: \{v_1, v_2, \dots\}$ respectively, then the Kronecker product of these two graphs is defined to be a graph with its vertex set as $V_1 \times V_2$, where $V_1 \times V_2$ is the Cartesian product of the sets V_1 and V_2 and two vertices $(u_i, v_j), (u_k, v_l)$ are adjacent if and only if $u_i u_k$ and $v_j v_l$ are edges in G_1 and G_2 respectively.

This product graph is denoted by $G_1(k)G_2$.

1.6: Cartesian product of two graphs

If G_1, G_2 are two simple graphs with their vertex sets $V_1: \{u_1, u_2, \dots\}$ and $V_2: \{v_1, v_2, \dots\}$ respectively, then the Cartesian product of these two graphs is defined to be a graph with its vertex set as $V_1 \times V_2: \{w_1, w_2, \dots\}$ and two vertices $w_1 = (u_1, v_1)$ and $w_2 = (u_2, v_2)$ are adjacent if and only if either (i) $u_1 = u_2$ and $v_1 v_2 \in E(G_2)$ or (ii) $u_1 u_2 \in E(G_1)$ and $v_1 = v_2$.

This product graph is denoted by $G_1(C)G_2$.

1.7: Lexicograph product of two graphs

If G_1, G_2 are two simple graphs with their vertex sets $V_1: \{u_1, u_2, \dots\}$ and $V_2: \{v_1, v_2, \dots\}$ respectively, then the Lexicograph product of these two graphs is defined to be a graph with its vertex set as $V_1 \times V_2: \{w_1, w_2, \dots\}$ and two vertices $w_1 = (u_1, v_1)$ and $w_2 = (u_2, v_2)$ are adjacent if and only if either (i) $u_1 u_2 \in E(G_1)$ or (ii) $u_1 = u_2$ and $v_1 v_2 \in E(G_2)$.

This product graph is denoted by $G_1(L)G_2$.

In this paper, we have obtained several interesting results on split domination of some product graphs. The organization of the paper is as follows: Section II describes several interesting relationships of $\gamma_s(G)$ with the other known parameters. Some results on Split dominating sets and Split domination numbers of certain types of Product Graphs are discussed in Section III and finally the conclusion of the paper is given in Section IV.

II. SOME RESULTS ON SPLIT DOMINATION OF STANDARD GRAPHS

In this paper, we assume that the graph contains a split dominating set. Kulli and Janakiram have obtained several interesting relationships of $\gamma_s(G)$ with the other known parameters. The following are some of the results of Kulli and Janakiram [4]. They have proved that

- i. $\gamma_s(G) \leq \alpha_o(G)$, where $\alpha_o(G)$ is a vertex covering number
- ii. $\gamma(G) \leq \gamma_s(G)$
- iii. $k(G) \leq \gamma_s(G)$
- iv. $\gamma_s(G) \leq P \cdot \Delta(G)$, where $\Delta(G)$ is the maximum degree of G . $\Delta(G)+1$
- v. $\gamma_s(G) \leq \delta(G)$; If $\text{Diam}(G) = 2$, where $\delta(G)$ is the minimum degree of G .
- vi. They have obtained the split domination number of some standard graphs as follows:
- vii. $\gamma_s(C_p) = \lceil p/3 \rceil$, where $\lceil x \rceil$ is the least +ve integer not less than x and C_p is a cycle with $p \geq 4$ vertices
- viii. $\gamma_s(W_p) = 3$, Where W_p is a wheel with $p \geq 5$ vertices
- ix. $\gamma_s(K_{m,n}) = m$, Where $2 \leq m \leq n$ and $K_{m,n}$ is a complete bipartite graph.

Weichsel [10] has proved that if G_1, G_2 are connected graphs, then $G_1(k)G_2$ is connected if and only if either G_1 (or) G_2 contains an odd cycle. It was further proved that if G_1, G_2 are connected graphs with no odd cycle, then $G_1(k)G_2$ is a disconnected graph. Sampath Kumar [7] has proved that if G is a connected graph with no odd cycles, then $G(k)K_2 = 2G$.

It can be easily seen that;

$$\deg_{G_1(k)G_2}(u_i, v_j) = \deg_{G_1}(u_i) \cdot \deg_{G_2}(v_j)$$

We recall some of the results on these product graphs.

Theorem 2.1:

In $G_1(k)G_2$, $\deg(u_i, v_j) = \deg(u_i) \cdot \deg(v_j)$

Theorem 2.2:

If G_1, G_2 are finite graphs without isolated vertices, then $G_1(k)G_2$ is a finite graph without isolated vertices.

Theorem 2.3:

If G_1, G_2 are regular graphs, then $G_1(k)G_2$ is also a regular graph.

Theorem 2.4:

If G_1, G_2 are bipartite graphs, then $G_1(k)G_2$ is a bipartite graph.

Remark 2.5:

However, it is to be noted that If G_1, G_2 are simple graphs, then $G_1(k)G_2$ can never be a complete graph, for (u_i, v_j) is not adjacent with (u_i, v_k) for any $j \neq k$ (By definition 1.5)

We have obtained the following results.

III. SPLIT DOMINATION OF PRODUCT GRAPHS

In this section, we have obtained some results on Split dominating sets and Split domination numbers of certain types of Product Graphs.

Theorem 3.1:

If G_1 is any graph on p -vertices and S_n is a star on n -vertices.

$$\text{Then } \gamma_s(G_1(k)S_n) = p = |V(G_1)|$$

Proof: Let $V(G_1) = \{u_1, u_2, \dots, u_p\}$ and $V(S_n) = \{v_1, v_2, \dots, v_n\}$
 Let $D_s(G_1) = \{u_1, u_2, \dots, u_p\}$ and $D_s(S_n) = \{v_1\}$, where v_1 is the only vertex in S_n whose degree is >1 .
 Now the set $D_s(G_1) \times D_s(S_n)$ is a vertex cut of $G_1(k)S_n$ as illustrated in Fig 1.
 However this set is not a dominating set.
 It can be easily seen that the set of vertices

$$D = \{(u_1, v_1), (u_2, v_1), \dots, (u_p, v_1)\}$$

is a dominating set.
 Further this is also a split dominating set.
 For, the vertex of the form (u_i, v_j) , for $j \neq 1$ is not adjacent with any vertex in $\langle V - D \rangle$.
 Now D is a split dominating set of minimum cardinality.
 Suppose, if we delete a vertex (u_i, v_1) from the set D , then this vertex (u_i, v_1) is not adjacent to any vertex in the set $D_s(G_1) \times D_s(S_n)$. Thus D is not a dominating set and hence not a split dominating set.
 Hence this is a split dominating set of minimum cardinality.

$$\text{Therefore } \gamma_s(G(k)S_n) = p = |V(G_1)|$$

Illustration:

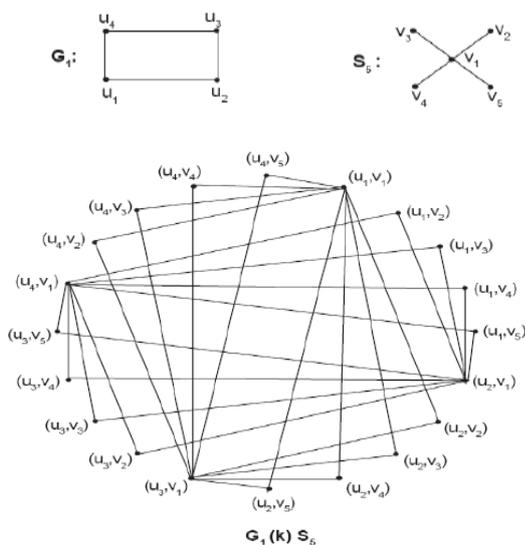


Figure 1: Graph of $G_1(k)S_6$

Here $D = \{(u_1, v_1), (u_2, v_1), (u_3, v_1), (u_4, v_1)\}$
 The removal of D vertices from $G_1(k)S_6$ will make the induced sub graph $\langle V - D \rangle$ disconnected.
 Thus D is a split dominating set. The cardinality of this set is 4.
 Hence $\gamma_s[G_1(k)S_n] = 4 = |V(G_1)|$

We have also obtained the following result regarding the split domination of the Kronecker product graph $G_1(k)G_2$, where G_1, G_2 are any two graphs.

Theorem 3.2:

If G_1, G_2 are any two graphs, then $\gamma_s[G_1(k)G_2] \leq \min[\gamma_s(G_1) \cdot |V_2|, |V_1| \cdot \gamma_s(G_2)]$

Proof: Let G_1 be a graph with p_1 vertices with

$$V(G_1) = \{u_1, u_2, \dots, u_{p_1}\} = V_1 \text{ say}$$

and G_2 is a graph with p_2 vertices with

$$V(G_2) = \{v_1, v_2, \dots, v_{p_2}\} = V_2 \text{ say}$$

$$\text{Let } D_1 = \{u_{d_1}, u_{d_2}, \dots, u_{d_r}\},$$

$D_2 = \{v_{d_1}, v_{d_2}, \dots, v_{d_s}\}$ be the split dominating sets of minimum cardinality of G_1, G_2 respectively.

The removal of the D_1 vertices in G_1 will make the induced sub graph $\langle V_1 - D_1 \rangle$ to be disconnected graph.
 Consider the set of vertices

$$D_s = \{(u_{d_1}, v_1), (u_{d_1}, v_2), \dots, (u_{d_1}, v_{p_2}), \\ (u_{d_2}, v_1), (u_{d_2}, v_2), \dots, (u_{d_2}, v_{p_2}), \\ \dots, (u_{d_r}, v_1), (u_{d_r}, v_2), \dots, (u_{d_r}, v_{p_2})\}.$$

This is dominating set of $G_1(k)G_2$ as shown in Fig 2.
 For, if (u, v) is any vertex of $\langle V - D_s \rangle$ in $G_1(k)G_2$, u is adjacent with at least one vertex in $\{u_{d_1}, u_{d_2}, \dots, u_{d_r}\}$, as this is the dominating set of G_1 being its split dominating set.

Now suppose v is adjacent with some vertex v_k in G_2 (certainly there exists at least one vertex v_k in G_2 , since G_2 has no isolated vertices).

Thus (u, v) is adjacent with (u_{d_i}, v_k) in D_s .
 Therefore D_s is a dominating set.

Further this is also a split dominating set.
 It can be established as follows:

We know that D_1 is a split dominating set of G_1 . The induced sub graph $\langle V_1 - D_1 \rangle$ is disconnected graph.

Let u_i, u_j be any two vertices in two different components of $\langle V_1 - D_1 \rangle$

If v_k, v_l are any two non-adjacent vertices in G_2 , then the vertices (u_i, v_k) & (u_j, v_l) will be in two different components in the induced sub graph $\langle V - D_s \rangle$ in $G_1(k)G_2$.

Thus D_s is a split dominating set of $G_1(k)G_2$ as illustrated in Fig 3.

Similarly, by the same argument we can prove that

The set $D_s' = \{(u_1, v_{d_1}), (u_2, v_{d_1}), \dots, (u_{p_1}, v_{d_1}), (u_1, v_{d_2}), (u_2, v_{d_2}), \dots, (u_{p_1}, v_{d_2}), (u_1, v_{d_s}), (u_2, v_{d_s}), \dots, (u_{p_1}, v_{d_s})\}$ is also a split dominating set of $G_1(k)G_2$ as illustrated in Fig 4.

$$\text{Hence } \gamma_s[G_1(k)G_2] \leq \min[|D_s|, |D_s'|]$$

$$\text{Thus } \gamma_s[G_1(k)G_2] \leq \min[\gamma_s(G_1) \cdot |V_2|, |V_1| \cdot \gamma_s(G_2)]$$

Illustration:

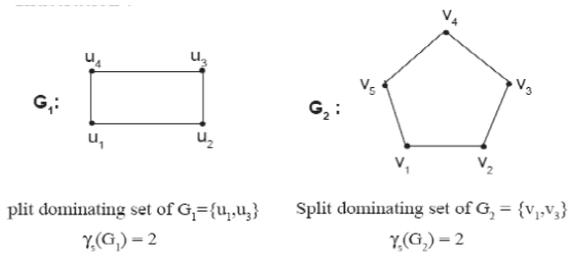


Figure 2: Graph of $G_1(k)G_2$

The split dominating set

$$D_s = \{u_1, u_3\} \times \{v_1, v_2, v_3, v_4, v_5\}$$

$$= \{(u_1, v_1), (u_1, v_2), (u_1, v_3), (u_1, v_4), (u_1, v_5), (u_3, v_1), (u_3, v_2), (u_3, v_3), (u_3, v_4), (u_3, v_5)\}$$

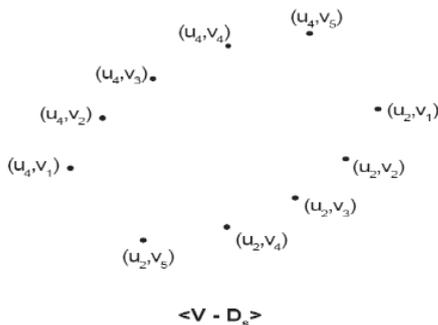


Figure 3: Graph of $\langle V(G_1(k)G_2) - D_s \rangle$

$\langle V - D_s \rangle$ is disconnected

that is $|D_s| = \gamma_s(G_1) \cdot |V(G_2)| = 2 \cdot 5 = 10$

$$D'_s = \{u_1, u_2, u_3, u_4\} \times \{v_1, v_3\}$$

$$= \{(u_1, v_1), (u_1, v_3), (u_2, v_1), (u_2, v_3), (u_3, v_1), (u_3, v_3), (u_4, v_1), (u_4, v_3)\}$$

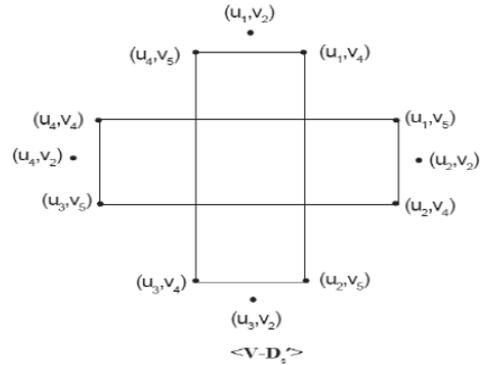


Figure 4: Graph of $\langle V(G_1(k)G_2) - D'_s \rangle$

Thus D'_s is another split dominating set.

$$|D'_s| = |V_1| \cdot \gamma_s(G_2) = 4 \cdot 2 = 8$$

$$\gamma_s[G_1(k)G_2] \leq \min[|D_s|, |D'_s|]$$

$$= \min[\gamma_s(G_1) \cdot |V_2|, |V_1| \cdot \gamma_s(G_2)]$$

$$= \min[10, 8]$$

$$= 8$$

Therefore $\gamma_s[G_1(k)G_2] \leq 8$

We recall some of the results already established by Sampath Kumar [7] for the Cartesian Product Graph.

Theorem 3.3:

If G_1, G_2 are bipartite graphs, then $G_1(C)G_2$ is also a bipartite graph.

Theorem 3.4:

If G_1, G_2 are simple finite graphs without isolated vertices, then $G_1(C)G_2$ is also a simple finite graph without isolated vertices.

Now we obtain a relationship between the split dominating number of $G_1(C)G_2$ and the split dominating numbers of G_1 and G_2 .

We obtain the following result.

Theorem 3.5:

If G_1, G_2 are any two graphs without isolated vertices, then $\gamma_s[G_1(C)G_2] \leq \gamma_s(G_1) \cdot |V_2|$

Proof : Let G_1 be a graph with p vertices with

$V(G_1) = \{u_1, u_2, \dots, u_p\} = V_1$ say

And G_2 be a graph with q vertices with

$V(G_2) = \{v_1, v_2, \dots, v_q\} = V_2$ say

Let $D_1 = \{u_{d_1}, u_{d_2}, \dots, u_{d_r}\}$ be the split dominating set of G_1 of minimum cardinality.

This means that the induced sub graph $\langle V_1 - D_1 \rangle$ of G_1 is a disconnected graph.

Let $D_2 = \{v_{d_1}, v_{d_2}, \dots, v_{d_s}\}$ be a split dominating set of G_2 of minimum cardinality.

The induced sub graph $\langle V_2 - D_2 \rangle$ is a disconnected graph.

Now consider the set of vertices

$$D_s = \{ (u_{d_1}, v_1), (u_{d_1}, v_1), \dots, (u_{d_r}, v_1),$$

$$(u_{d_1}, v_2), (u_{d_2}, v_2), \dots, (u_{d_r}, v_2),$$

$$(u_{d_1}, v_q), (u_{d_2}, v_q), \dots, (u_{d_r}, v_q) \}$$

This is a dominating set of $G_1(C)G_2$ as illustrated in Fig 5.

For, if (u, v) is any vertex of $\langle V - D_s \rangle$ in $G_1(C)G_2$, v will be adjacent with at least one vertex in D_2 , which is the split dominating set of G_2 say v_{d_i} , then (u, v) is adjacent to a vertex (u, v_{d_i}) in D .

Hence D_s is a dominating set of $G_1(C)G_2$ as illustrated in Fig.6.

Let v_i, v_j be two vertices in $\langle V_2 - D_2 \rangle$ in two different components.

Then it follows that (u, v_i) and (u, v_j) are in two different components of an induced sub graph

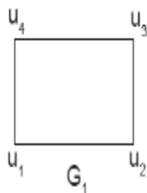
$$\langle V - D_s \rangle \text{ of } G_1(C)G_2.$$

Thus D_s is a split dominating set.

$$\text{Hence } \gamma_s [G_1(C)G_2] \leq |D_s|$$

$$\text{and so } \gamma_s [G_1(C)G_2] \leq \gamma_s (G_1) \cdot |V_2|$$

Illustration:



Split dominating set of $G_1 = \{u_1, u_3\}$

$$\gamma_s(G_1) = 2$$

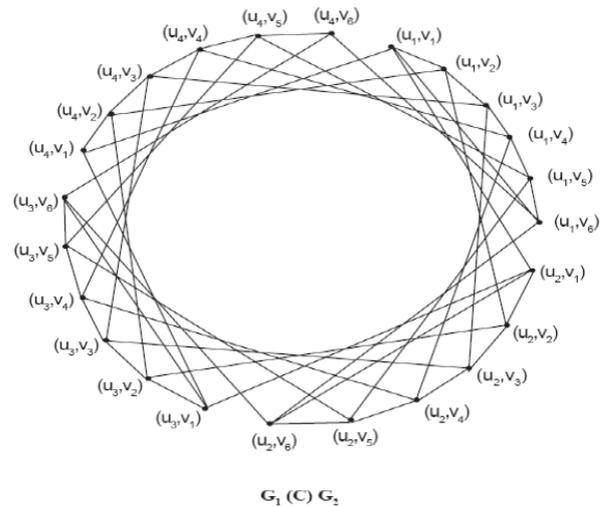
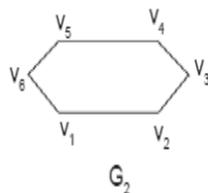


Figure 5: Graph of $G_1(C)G_2$

$$D_s = \{u_1, u_3\} \times \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

$$= \{(u_1, v_1), (u_1, v_2), (u_1, v_3), (u_1, v_4), (u_1, v_5), (u_1, v_6), (u_3, v_1), (u_3, v_2), (u_3, v_3), (u_3, v_4), (u_3, v_5), (u_3, v_6)\}$$

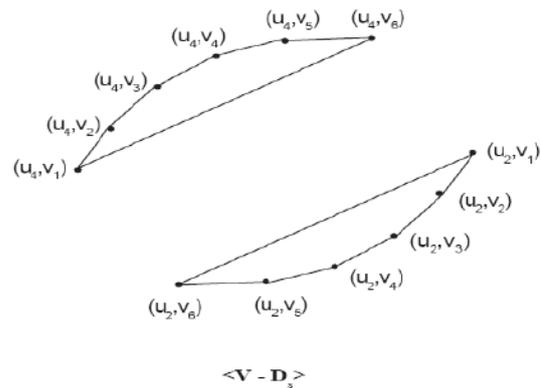


Figure 6: Graph of $\langle V - D_s \rangle$

D_s is a split dominating set.

$$|D_s| = \gamma_s(G_1) \cdot |V_2| = 2 \cdot 6 = 12$$

$$\gamma_s [G_1(C)G_2] \leq |D_s| = 12$$

$$\gamma_s [G_1(C)G_2] \leq 12.$$

It has already been established by Sampath Kumar [7] that

Theorem 3.6:

If G_1, G_2 are simple finite graphs without isolated vertices, then $G_1(L)G_2$ is a finite graph without isolated vertices.

Now we obtain an expression for the split domination number of $G_1(L)G_2$ in terms of the split domination numbers of G_1, G_2 .

Theorem 3.7:

If G_1, G_2 are any two graphs without isolated vertices, then $\gamma_s [G_1 (L) G_2] \leq \gamma_s (G_1) \cdot |V_2|$

Proof: Let G_1 be a graph with p vertices with

$$V(G_1) = \{u_1, u_2, \dots, u_p\} = V_1 \text{ say}$$

and G_2 be a graph with q vertices with

$$V(G_2) = \{v_1, v_2, \dots, v_q\} = V_2 \text{ say}$$

$$\text{Let } D_1 = \{u_{d_1}, u_{d_2}, \dots, u_{d_p}\} \text{ and}$$

$D_2 = \{vd_1, vd_2, \dots, vds\}$ be the split domination sets of minimum cardinality of G_1, G_2 respectively.

Consider the set

$$D_s = \{ (u_{d_1}, v_1), (u_{d_1}, v_2), \dots, (u_{d_1}, v_q),$$

$$(u_{d_2}, v_1), (u_{d_2}, v_2), \dots, (u_{d_2}, v_q),$$

$$(u_{d_p}, v_1), (u_{d_p}, v_2), \dots, (u_{d_p}, v_q) \}$$

This is the dominating set of $G_1(L) G_2$ as shown in Fig. 7.

For, if (u, v) is any vertex in $\langle V-D_s \rangle$, then we have the following two cases :

Either u is in D_1 or u is in $\langle V_1 - D_1 \rangle$.

If u is in D_1 , then $u = u_{di}$ for some i . The vertex $(u, v) = (u_{di}, v)$ and v will be adjacent to some vertex in $D_2 = \{vd_1, vd_2, \dots, vds\}$.

For the sake of definiteness, suppose v is adjacent with vd_j for some j ,

then $(u, v) = (u_{di}, v)$ is adjacent with (u_{di}, v_j)

Thus D_s is a dominating set.

Suppose u_i, u_j be two vertices in two different components of the induced subgraph $\langle V_1 - D_1 \rangle$ of G_1 .

Then it follows from the definition of the split domination 1.3 and from the definition of the Lexicograph product graph 1.7, the vertices (u_i, v) ,

(u_j, v) will be in two different components of $\langle V-D_s \rangle$ of $G_1 (L) G_2$.

Thus D_s is a split dominating set of $G_1 (L) G_2$ as illustrated in Fig. 8.

Hence $\gamma_s [G_1(L) G_2] \leq |D_s|$

and so $\gamma_s [G_1(L) G_2] \leq \gamma_s (G_1) \cdot |V_2|$

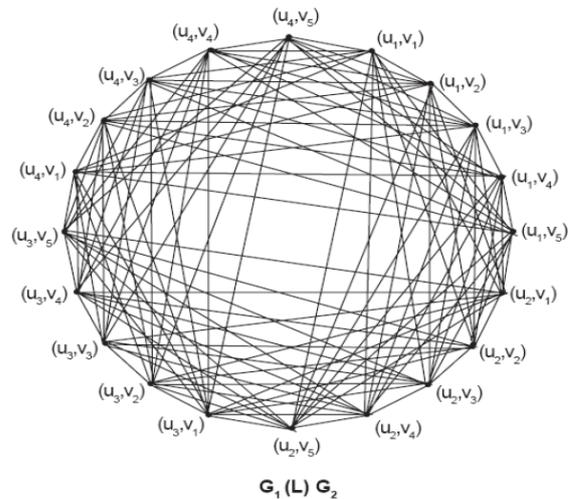


Figure 7: Graph of $G_1(L) G_2$

$$D_s = \{u_i, u_j\} \times \{v_1, v_2, v_3, v_4, v_5\}$$

$$= \{(u_1, v_1), (u_1, v_2), (u_1, v_3), (u_1, v_4), (u_1, v_5), (u_2, v_1), (u_2, v_2), (u_2, v_3), (u_2, v_4), (u_2, v_5)\}$$

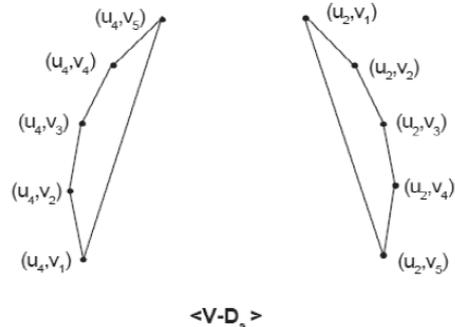


Figure 8: Graph of $\langle V-D_s \rangle$

D_s is a split dominating set.

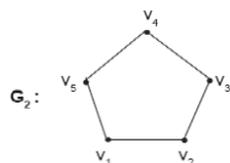
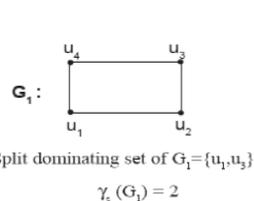
$$|D_s| = \gamma_s (G_1) \cdot |V_2| = 2 \cdot 5 = 10$$

$$\gamma_s [G_1 (L) G_2] \leq |D_s| = 10$$

$$\gamma_s [G_1 (L) G_2] \leq 10$$

Illustration:

Illustration :



IV. CONCLUSION

The tools of Graph theory enable us to develop a simple method of constructing a graph with a given cardinality of the split dominating set with amazing ease. It is also amazing to observe how such a graph with a given domination number can be enlarged to include more vertices and edges in a methodical, simple manner without affecting the domination number. We can apply this to many applications such as to eradicate pests in

Agriculture, to control viruses which produces diseases in an epidemic form, to maintain confidential in transferring the information, especially very useful for Defense sector. To some extent this may be due to the ever growing importance of computer science and its connection with graph theory.

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