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Control and Synchronization of Hyperchaotic System based on SDRE method

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Abstract—In this paper, stabilization and synchronization problems of the hyperchaotic system is investigated. For this reason, state dependent Riccati equation (SDRE) is used. First, stabilizer is designed by SDRE method. Then, robust controller is designed that it can stabilize hyperchaotic system with uncertainly. Finally, synchronization problem between two hyperchaotic systems is considered. The optimal controller is designed that it synchronizes two hyperchaotic systems. Numerical simulation results are presented to show the effectiveness of the proposed controllers.

Index Terms—Hyperchaotic system, state dependent Riccati equation (SDRE), optimal control, robust control, stabilization, synchronization.

I. Introduction

An interesting phenomenon of nonlinear systems is chaos. In recent years, studies of chaos and hyperchaos generation, control and synchronization have attracted. Therefore, various effective methods have been proposed one the past decades to achieve the control and synchronization of chaotic system, such as Robust Control [1], the sliding method control [2], linear and nonlinear feedback control [3], adaptive control [4], active control [5], backstepping control [6] and generalized backsteppig method control [7-9], ect. The purpose of the present work lies in the design of a robust optimal control system for the control and synchronization of new hyperchaotic system using the state-dependent Riccati equation (SDRE) method. The State-Dependent Riccati Equation (SDRE) techniques are general design methods that control problems involving nonlinear systems [10-14]. For a nonlinear system a form of linearization is required which is not an approximation but simply a rewriting of the mathematical model in a different form. This form, which is not unique, is then possible to obtain feedback control laws.

The rest of the paper is organized as follows: In section 2, a new hyperchaotic system is described. In section 3, stability conditions in new hyperchaotic system are derived by SDRE method. In section 4, the stability conditions in new hyperchaotic system with uncertainly are derived by robust optimal method. In section 5, synchronization between two new hyperchaotic systems are achieved by SDRE method. Finally, section 6 isprovided conclusion of this work.

II. Hyperchaotic System

Recently, Dadras and Momeni proposed the new hyperchaotic system [15]. The system is described by:

$$\begin{aligned}\dot{x}_1 &= ax_1 - x_2x_3 \\ \dot{x}_2 &= x_1x_3 - bx_2 \\ \dot{x}_3 &= cx_1x_2 - dx_3 + gx_1x_4 \\ \dot{x}_4 &= fx_4 - hx_2\end{aligned} \quad (1)$$

Here x, y, z, w are the state variables and a, b, c, d, f, g, h are the posotive constant parameters. System (1) is hyperchaotic when $a = 8, b = 40, c = 2, d = 14, f = 0.05, g = 5, h = 0.2$. The corresponding phase portraits are depicted in Fig 1 and the state trajectory of the system (1) is displayed in Fig 2.

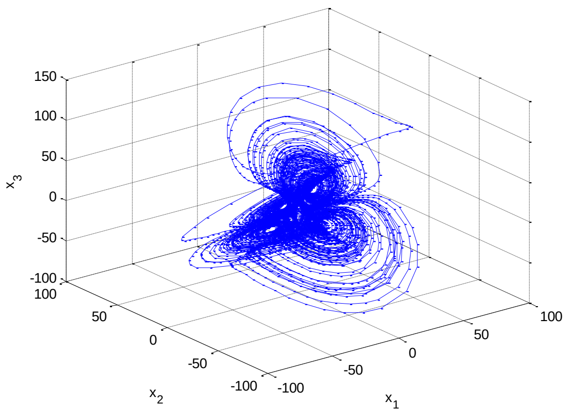


Fig. 1.phase portraits of the hyperchaotic (7).

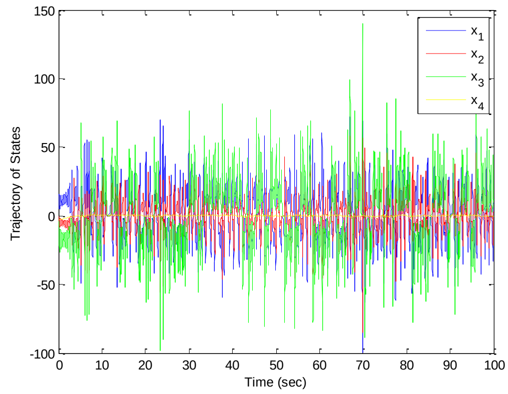


Fig. 2.state trajectory of the hyperchaotic (7).

Now, system (1) is described by state dependent Riccati equation (SDRE).

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}a & -x_3 & 0 & 0\\ x_3 & -b & 0 & 0\\ cx_2 & 0 & -dgx_1 & \\ 0 & -h & 0 & f\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \qquad (2)$$

III. Stabilization of Hyperchaotic System

In this section, the SDRE method is applied to stabilize hyperchaotic system (2) with now parameters.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}a & -x_3 & 0 & 0\\ x_3 & -b & 0 & 0\\ cx_2 & 0 & -dgx_1 & \\ 0 & -h & 0 & f\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \qquad (3)$$

Where $u_1, u_2$are control function to be determined for achieving minimize cost function (4).

$$J=\int_0^{\infty}\left(x^T(t)Qx(t)+u^T(t)Ru(t)\right)dt \qquad (4)$$

Where

$$Q=I_{4\times 4}, R=I_{2\times 2} \qquad (5)$$

A state dependent Riccati equation (SDRE) is the solved at each point X along the trajectory to obtain a nonlinear feedback controller of the form (6), where P(t) is the solution of the SDRE.

$$u(t)=-k(t)x(t) \qquad (6)$$

$$k(t)=R^{-1}B^T(t)P(t) \qquad (7)$$

$$\dot{P}(t)+P(t)\left(A\left(x(t)\right)+\alpha I\right)+\left(A^T\left(x(t)\right)+\alpha I\right)P(t)+ Q-P(t)B(t)R^{-1}B^T(t)P(t)=0 \qquad (8)$$

Where scalerα is a design parameter [17]. We choose $\alpha=20$. As can be seen, the SDRE method produces a stabilizing solution. The time response of x_1, x_2, x_3, x_4 states for system (3) is shown in Fig 3. The time response of control inputs u_1, u_2 is shown in Fig 4.

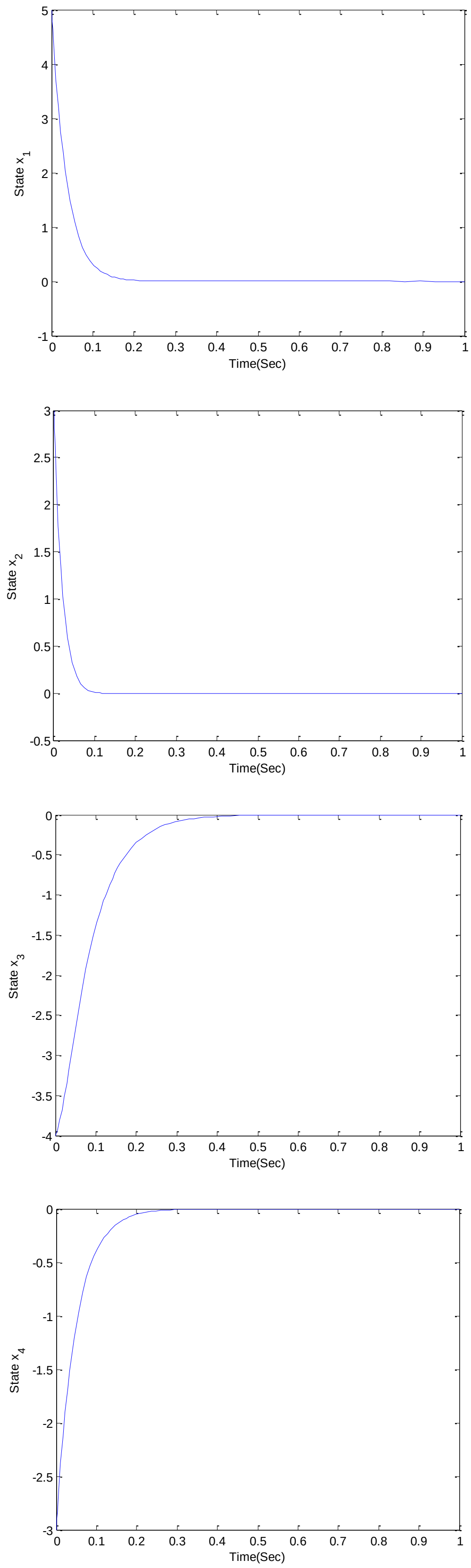


Fig. 3.the time response of signals (x_1, x_2, x_3, x_4) for system (3).

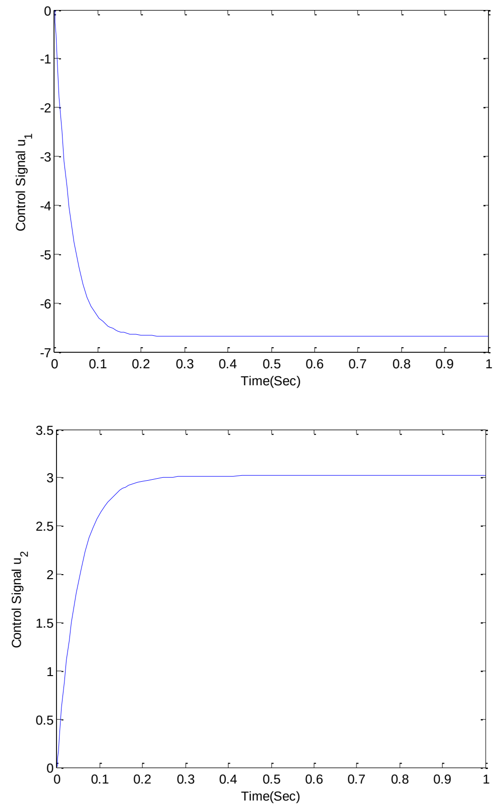


Fig. 4.the time response of the control inputs (u_1, u_2).

IV. STABILIZATION OF HYPERCHAOTIC SYSTEM WITH UNCERTAINLY

In practical situations, some or all of system parameter can not be exactly know in advance. We assume the system (3) with uncertainly.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} a+\theta_1 & -x_3 & 0 & 0\\ x_3 & -b & 0 & 0\\ cx_2 & 0 & -d & gx_1\\ 0 & -h & 0 & f+\theta_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \quad (9)$$

Where θ_1, θ_2 are uncertainly in system and $\theta_i \in [0,30], i = 1,2$. Now, robust controller is designed by presented method in [16]. For this reason, states matrix is analysed as follows:

$$\begin{bmatrix} a+\theta_1 & -x_3 & 0 & 0\\ x_3 & -b & 0 & 0\\ cx_2 & 0 & -d & gx_1\\ 0 & -h & 0 & f+\theta_2\end{bmatrix} - \begin{bmatrix} a & -x_3 & 0 & 0\\ x_3 & -b & 0 & 0\\ cx_2 & 0 & -d & gx_1\\ 0 & -h & 0 & f\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\phi(\theta) = B\phi(\theta) \quad (10)$$

Where

$$\phi(\theta) = \begin{bmatrix}\theta_1 & 0 & 0 & 0\\ 0 & 0 & 0 & \theta_2\end{bmatrix} \quad (11)$$

System (9) is rewrote as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} a & -x_3 & 0 & 0\\ x_3 & -b & 0 & 0\\ cx_2 & 0 & -d & gx_1\\ 0 & -h & 0 & f\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} + B\phi(\theta)\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \quad (12)$$

We must find the control signals (6) that it stabilizes the system (12) wwithuncertainly $\theta_i, i = 1,2$. This problem is solved by optimal control of system (3) that it minimizes cost function (13) [16].

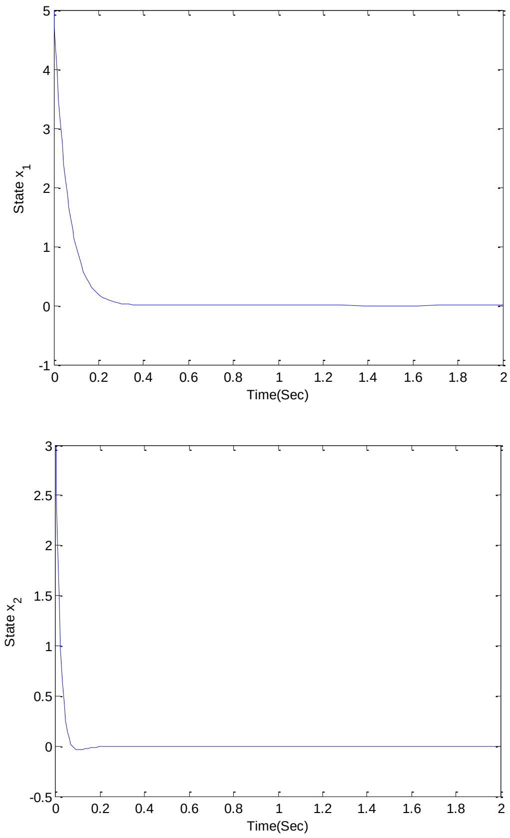
$$J = \int_0^\infty \left(x^T(t)Fx(t) + x^T(t)x(t) + u^T(t)Ru(t)\right)dt \quad (13)$$

Where F matrix is

$$\phi^T(\theta)\phi(\theta) \le F \quad (14)$$

$$F = \phi^T(\theta)\phi(\theta) = \begin{bmatrix}\theta_1^2 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \theta_2^2\end{bmatrix} = \begin{bmatrix}900 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 900\end{bmatrix} \quad (15)$$

Again, the Riccati equation is solved and the optimal feedback control (6) is obtained. The time response of x_1, x_2, x_3, x_4 states for system (9) is shown in Fig 5. The time response of control inputs u_1, u_2 is shown in Fig 6.



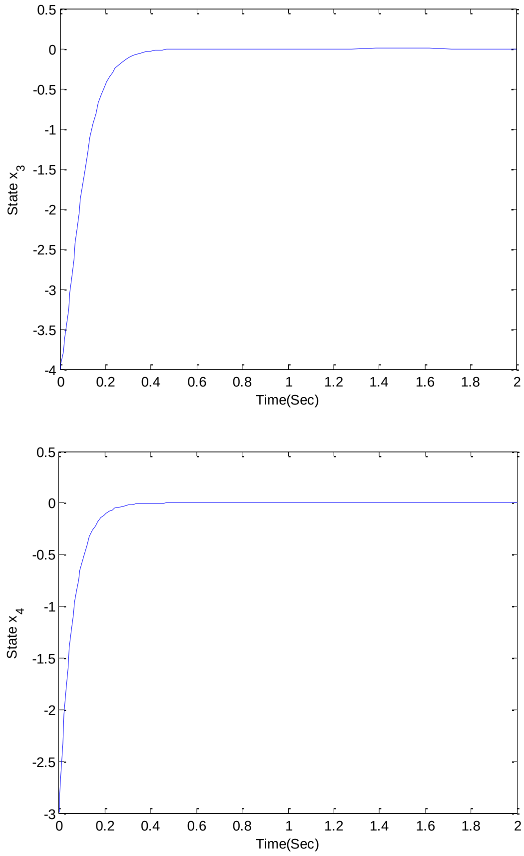


Fig. 5.the time response of signals (x_1, x_2, x_3, x_4) for system (9).

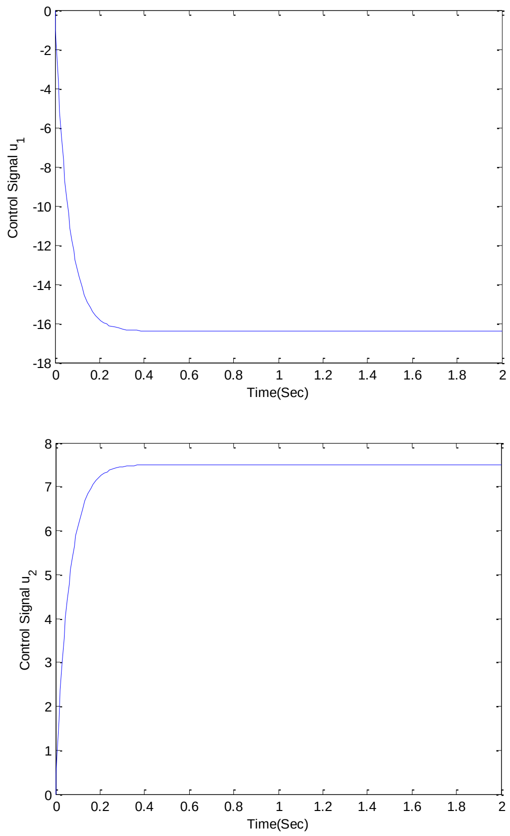


Fig. 6.the time response of the control inputs (u_1, u_2).

V. SYNCHRONIZATION OF HYPERCHAOTIC SYSTEM

In this section, the SDRE method is applied to synchronize two hyperchaoticsystem. Suppose the drive system takes the following form

$$\begin{aligned} \dot{x}_1 &= ax_1 - y_1 z_1 \\ \dot{y}_1 &= x_1 z_1 - by_1 \\ \dot{z}_1 &= cx_1 y_1 - dz_1 + gx_1 w_1 \\ \dot{w}_1 &= fw_1 - hy_1 \end{aligned} \tag{16}$$

And the response system is given as follows

$$\begin{aligned} \dot{x}_2 &= ax_2 - y_2 z_2 + u_1(t) \\ \dot{y}_2 &= x_2 z_2 - by_2 + u_2(t) \\ \dot{z}_2 &= cx_2 y_2 - dz_2 + gx_2 w_2 + u_3(t) \\ \dot{w}_2 &= fw_2 - hy_2 + u_4(t) \end{aligned} \tag{17}$$

Where u_1, u_2, u_3, u_4 are control inputs. Define state errors between system (16) and (17) as follows

$$\begin{aligned} e_x &= x_2 - x_1 \\ e_y &= y_2 - y_1 \\ e_z &= z_2 - z_1 \\ e_w &= w_2 - w_1 \end{aligned} \tag{18}$$

We obtain the following error dynamical system by subtracting the drive system (16) from the response system (17)

$$\begin{aligned} \dot{e}_x &= ae_x - e_y e_z + z_1 e_y + y_1 e_y + u_1(t) \\ \dot{e}_x &= e_x e_z - be_y - x_1 e_z - z_1 e_x + u_2(t) \\ \dot{e}_z &= ce_x e_y - de_z + ge_x e_w - cx_1 e_y - cy_1 e_x - gx_1 e_w \\ &\quad - gw_1 e_x + u_3(t) \\ \dot{e}_w &= fe_w - he_y + u_4(t) \end{aligned} \tag{19}$$

Error dynamical system (19) is rewrote as follows

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_z \\ \dot{e}_w \end{bmatrix} = A \begin{bmatrix} e_x \\ e_y \\ e_z \\ e_w \end{bmatrix} + \Delta A \begin{bmatrix} e_x \\ e_y \\ e_z \\ e_w \end{bmatrix} + B \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{20}$$

Where

$$A = \begin{bmatrix} a & -e_z & 0 & 0 \\ e_z & -b & 0 & 0 \\ ce_y & 0 & -d & ge_x \\ 0 & -h & 0 & f \end{bmatrix}$$

$$\Delta A = \begin{bmatrix} 0 & z_1 & y_1 & 0 \\ -z_1 & 0 & -x_1 & 0 \\ -cy_1 - gw_1 & -cx_1 & 0 & -gx_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix} \tag{21}$$

We can rewrite ΔA matrix as follows

$$\Delta A = \begin{bmatrix} 0 & z_1 & y_1 & 0 \\ -z_1 & 0 & -x_1 & 0 \\ -cy_1 - gw_1 & -cx_1 & 0 & -gx_1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix}$$

$$\Delta A = B\phi(X) \tag{22}$$

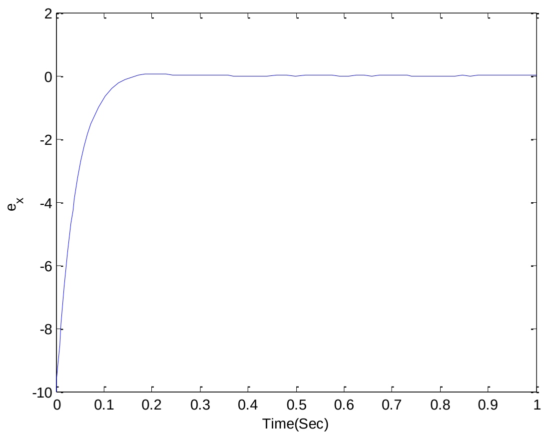
Where $X = [x, y, z, w]^T$. Substituting (22) into (20), the error dynamics is obtained.

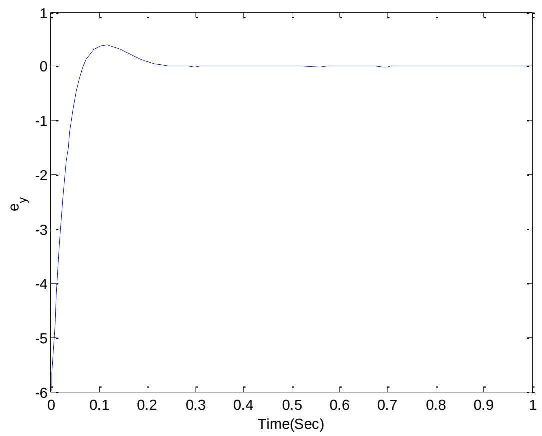
$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_z \\ \dot{e}_w \end{bmatrix} = A \begin{bmatrix} e_x \\ e_y \\ e_z \\ e_w \end{bmatrix} + B \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + B\phi(X) \begin{bmatrix} e_x \\ e_y \\ e_z \\ e_w \end{bmatrix} \tag{23}$$

We obtain optimal feedback control (6) that it minimizes the cost function (13) [16]. The F matrix is obtained.

$$F = \phi^T(X)\phi(X) = \begin{bmatrix} y_1^2 + z_1^2 & -x_1y_1 & -cx_1z_1 & 0 \\ -x_1y_1 & x_1^2 + z_1^2 & cy_1z_1 + gz_1w_1 & 0 \\ -cx_1z_1 & cy_1z_1 + gz_1w_1 & (cy_1 + gw_1)^2 + (cx_1)^2 + (gx_1)^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{24}$$

Again, the Riccati equation is solved and the optimal feedback control (6) is obtained. Synchronization errors e_x, e_y, e_z, e_w in hyperchaotic system are shown in Fig 7. The time response of x, y, z, w states for drive system (16) and response system (17) is shown in Fig 8. The time response of the control inputs u_1, u_2, u_3, u_4 for the synchronization of hyperchaotic system is shown in Fig 9.





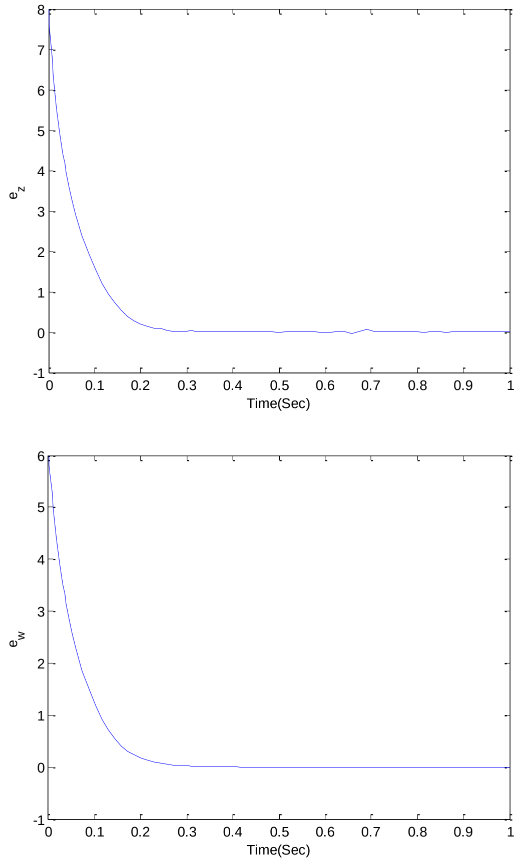
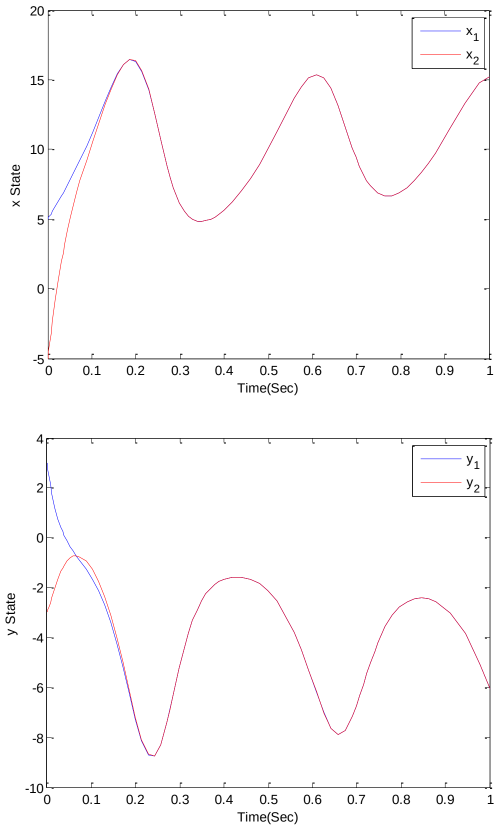


Fig. 7.synchronization errors (e_x, e_y, e_z, e_w) in drive system (16) and response system (17).



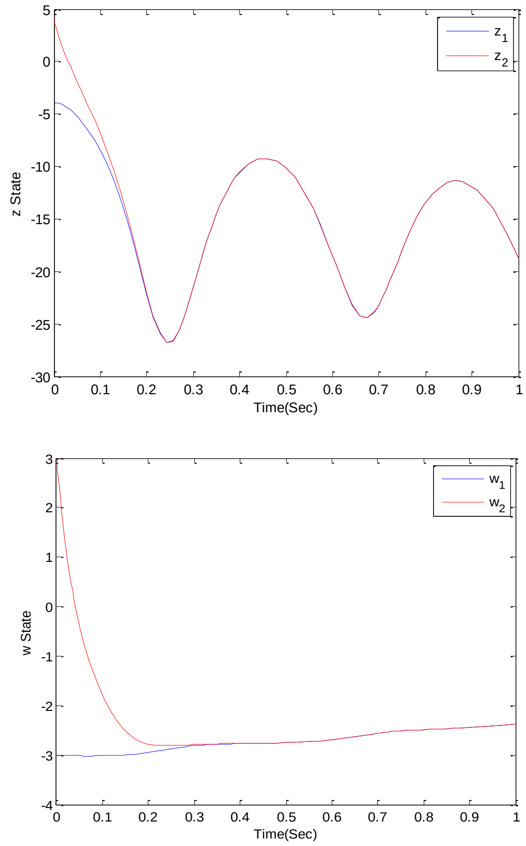
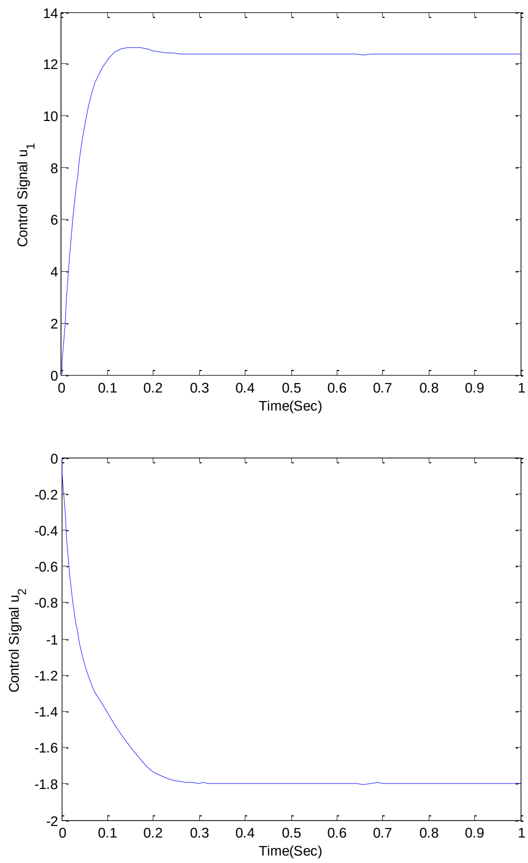


Fig. 8.the time response of signals (x, y, z, w) for drive system (16) and response system (17).



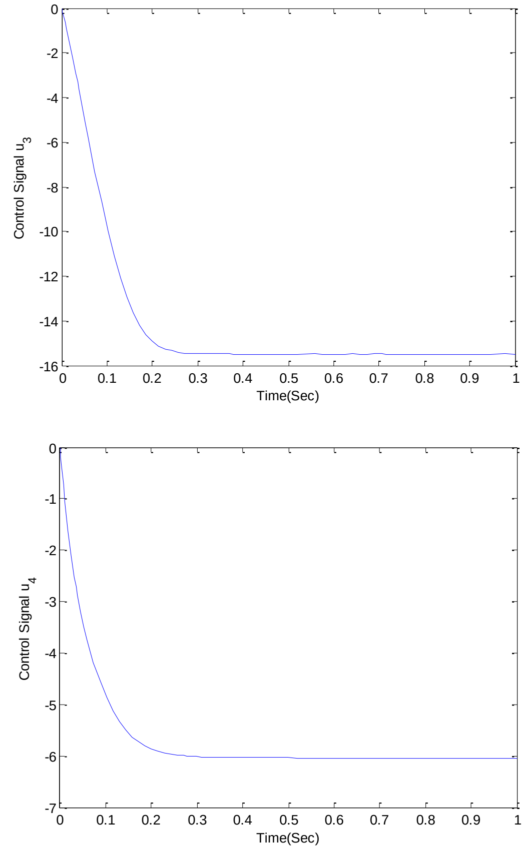


Fig. 9.the time response of the control inputs (u_1, u_2, u_3, u_4).

VI. Conclusion

In this paper, stabilization and synchronization problem of the new hyperchaotic system was investigated. For this reason, state dependent Riccati equation (SDRE) was used. This method was applied to new hyperchaotic system in three ways. Stabilized system with know parameters, stabilized system with unknow parameters, sysnchronized system. In every way, simulations proved abilities of method.

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