

# Introduction to Neutrosophic Topological Spatial Region, Possible Application to GIS Topological Rules

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**Abstract**—Neutrosophic set is a power general formal framework, which generalizes the concept of the classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and interval intuitionistic fuzzy set from philosophical point of view. In Geographical Information Systems (GIS) there is a need to model spatial regions with indeterminate boundary and under indeterminacy. In this paper, we first gives fundamental concepts and properties of a neutrosophic spatial region.

**Index Terms**—Neutrosophic Sets, Neutrosophic Topology, Geographical Information Systems, Neutrosophic Spatial Region.

## I. INTRODUCTION

Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The paper will discuss several possible contributions to the GIS field including remote sensing, object reconstruction from airborne laser scanner, real time tracking, routing applications and modeling cognitive agents. In this paper, a simple neutrosophic region and fundamental concepts for uncertainty and indeterminacy modeling of spatial relationships are analyzed from the viewpoint of neutrosophic (NS) logic. This paper gives fundamental concepts and properties of a neutrosophic spatial region.

## II. RELATED WORK

Algebraic topological models for spatial objects were introduced in (White 1979). Thirteen topological relations between two temporal intervals were identified by J.F.Allen [6]. After the 4-intersection model proposed by M.J Egenhofer [16], M.J. Egenhofer and R.D.Franzosa [17], the 9-intersection approach introduced by M.J Egenhofer and J. R. Herring [18] was proposed as a formalism for topological relations. This approach is based on point-set topological concepts. In the 9-intersection method, a spatial object A is

decomposed into three parts: an interior denoted by  $Ao$ , an exterior denoted by  $AE$ , and a boundary denoted by  $\partial A$ . There are nine intersections between six parts of two objects. The other significant approach known as RCC (Region-Connection Calculus) has been provided by Cohn et al. D.A. Randell, Z. Cui [25]; M.N.Gotts, J.M.Gooday et al. [21]; A.G.Cohn, B.Bennet et al. [12]. During recent years, the topological relations have been extended into fuzzy domains. An example of a fuzzy object was provided by P. Fisher [19]. A number of papers of M. Schneider [27, 28, 29, 30] was presented to model fuzzy set in GIS community and to design a system of fuzzy spatial data types including operations and predicates. M. Molenaar [24] extended the formal model into fuzzy domain and based on this model T. Cheng [8] proposed a process-oriented spatio-temporal data model. The intersection model is extended to vague regions by three main approaches: the work of E. Clementini and Di Felice [9, 11] on regions with "broad boundary", the work of F. B Zhan [34] who developed a method for approximately analyzing binary topological relations between geographic regions with indeterminate boundaries based on fuzzy sets, and X. Tang and W. Kainz [32] that provided a 3\*3, a 4\*4, and a 5\*5 intersection matrix based on different topological parts of two fuzzy regions. The extension of the RCC schemes to accommodate vague region has been addressed by F.Lehmann and A.G Cohn [23], and by A.G.Cohn and M.N .Gotts [13]. In this direction J.G. Stell and M.F Worboys [31] have used Heyting structures. The notion of intuitionistic ets (IFS) was introduced by K. Atanassov. [1, 2, 3] as generalization of fuzzy sets. The notion of neutrosophic sets (NS) was introduced by F. Smarandache [45, 46, 47] as generalization of intuitionistic fuzzy sets. In other research, A. A Salama [36] introduced the concept of neutrosophic topology. A. A. Salama, et,al [42] proposed a new mathematical model called "Neutrosophic crisp Sets and Neutrosophic crisp Topological Spaces.

## III. PRELIMINARIES

First we present the fundamental concepts and definitions given by Salama and Smarandache. We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [7, 8], Atanassov in [1, 2, 3] and Salama [9]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where  $]0^-, 1^+[$  is nonstandard unit interval.

**Definition 3.1**[47]

Let T, I, F be real standard or nonstandard subsets of  $]0^-, 1^+[$ , with

$$\begin{aligned} \text{Sup}_T &= t_{\text{sup}}, \text{inf}_T = t_{\text{inf}} \\ \text{Sup}_I &= i_{\text{sup}}, \text{inf}_I = i_{\text{inf}} \\ \text{Sup}_F &= f_{\text{sup}}, \text{inf}_F = f_{\text{inf}} \\ n\text{-sup} &= t_{\text{sup}} + i_{\text{sup}} + f_{\text{sup}} \\ n\text{-inf} &= t_{\text{inf}} + i_{\text{inf}} + f_{\text{inf}}, \end{aligned}$$

T, I, F are called neutrosophic components

We shall now consider some possible definitions for basic concepts of the neutrosophic set and its operations due to Salama et al [ ].

**Definition 3.2**

Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  where  $\mu_A(x), \sigma_A(x)$  and  $\gamma_A(x)$  which represent the degree of membership function (namely  $\mu_A(x)$ ), the degree of indeterminacy (namely  $\sigma_A(x)$ ), and the degree of non-membership (namely  $\gamma_A(x)$ ) respectively of each element  $x \in X$  to the set A.

**Remark 3.1**

A neutrosophic  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  can be identified to an ordered triple  $\langle \mu_A, \sigma_A, \gamma_A \rangle$  in  $]0^-, 1^+[$  on X.

**Remark 3.2**

For the sake of simplicity, we shall use the symbol  $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$  for the NS  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$

**Example 3.1**

Every IFS A a non-empty set X is obviously on NS having the form  $A = \{ \langle x, \mu_A(x), 1 - (\mu_A(x) + \gamma_A(x)), \gamma_A(x) \rangle : x \in X \}$

Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the NSS  $0_N$  and  $1_N$  in X as follows:

$0_N$  may be defined as four types:

- (0<sub>1</sub>) **Type 1.**  $0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$
- (0<sub>2</sub>) **Type 2.**  $0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$

- (0<sub>3</sub>) **Type 3.**  $0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$
- (0<sub>4</sub>) **Type 4.**  $0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$

$1_N$  may be defined as four types:

- (1<sub>1</sub>) **Type 1.**  $1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$
- (1<sub>2</sub>) **Type 2.**  $1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$
- (1<sub>3</sub>) **Type 3**  $1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$
- (1<sub>4</sub>) **Type 4.**  $1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$

**Definition 3.3**

Let  $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$  a NS on X, then the complement of the set A ( $C(A)$ , for short) maybe defined as three kinds of complements

- (C<sub>1</sub>)  $C(A) = \{ \langle x, 1 - \mu_A(x), 1 - \gamma_A(x) \rangle : x \in X \}$ ,
- (C<sub>2</sub>)  $C(A) = \{ \langle x, \gamma_A, \sigma_A(x), \mu_A(x) \rangle : x \in X \}$
- (C<sub>3</sub>)  $C(A) = \{ \langle x, \gamma_A, 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}$

One can define several relations and operations between NSS follows:

**Definition 3.4**

Let X be a non-empty set, and NSS A and B in the form  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ ,  $B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$ , then we may consider two possible definitions for subsets ( $A \subseteq B$ )

( $A \subseteq B$ ) may be defined as two types:

- (1) Type 1:  $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \gamma_A(x) \geq \gamma_B(x)$  and  $\sigma_A(x) \leq \sigma_B(x) \quad \forall x \in X$
- (2) Type 2:  $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \gamma_A(x) \geq \gamma_B(x)$  and  $\sigma_A(x) \geq \sigma_B(x)$

**Proposition 3.1**

For any neutrosophic set A the following are holds

- (1)  $0_N \subseteq A, 0_N \subseteq 0_N$
- (2)  $A \subseteq 1_N, 1_N \subseteq 1_N$

**Definition 3.5**

Let X be a non-empty set, and

$$\begin{aligned} A &= \langle x, \mu_A(x), \gamma_A(x), \sigma_A(x) \rangle, \\ B &= \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle \text{ are NSS. Then} \end{aligned}$$

- (1)  $A \cap B$  may be defined as three types:

- ( $I_1$ ) **Type1:**  $A \cap B = \langle x, \mu_A(x), \mu_B(x), \sigma_A(x), \sigma_B(x), \gamma_A(x), \gamma_B(x) \rangle$   
 $\gamma_A(x), \gamma_B(x) \rangle$   
 ( $I_2$ ) **Type 2:**  $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$   
 $\gamma_A(x) \vee \gamma_B(x) \rangle$   
 ( $I_3$ ) **Type 3:**  $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$   
 $\gamma_A(x) \vee \gamma_B(x) \rangle$

(2)  $A \cup B$  may be defined as two types :

- ( $U_1$ ) **Type1:**  $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle$   
 $\gamma_A(x) \wedge \gamma_B(x) \rangle$   
 ( $U_2$ ) **Type 2:**  
 $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle$   
 $\gamma_A(x) \wedge \gamma_B(x) \rangle$

- (3)  $[ ]A = \langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x) \rangle$   
 $\langle \rangle A = \langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x) \rangle$

### Definition 3.6

Let  $\{A_j : j \in J\}$  be a arbitrary family of NSS in  $X$ , then

- (1)  $\cap A_j$  may be defined as two types :
- (i) **Type 1:**  $\cap A_j = \langle x, \bigwedge_{j \in J} \mu_{A_j}(x), \bigwedge_{j \in J} \sigma_{A_j}(x), \bigvee_{j \in J} \gamma_{A_j}(x) \rangle$   
 (ii) **Type 2:**  $\cap A_j = \langle x, \bigwedge_{j \in J} \mu_{A_j}(x), \bigvee_{j \in J} \sigma_{A_j}(x), \bigvee_{j \in J} \gamma_{A_j}(x) \rangle$
- (2)  $\cup A_j$  maybe defined as two types:
- (i) **Type1:**  $\cup A_j = \langle x, \bigvee, \bigwedge, \bigwedge \rangle$   
 (ii) **Type2:**  $\cup A_j = \langle x, \bigvee, \bigwedge, \bigwedge \rangle$

### Definition 3.7

Let  $A$  and  $B$  are neutrosophic sets then  $A|B$  may be defined as

$$A|B = \langle x, \mu_A \wedge \mu_B, \sigma_A(x) \sigma_B(x), \gamma_A \vee \mu_B(x) \rangle$$

### Proposition 3.2

For all  $A, B$  two neutrosophic sets then the following are true

- (1)  $C(A \cap B) = C(A) \cup C(B)$   
 (2)  $C(A \cup B) = C(A) \cap C(B)$ .

Salama et al. extended the concepts of fuzzy topological space [4], and intuitionistic fuzzy topological space [5, 7] to the case of neutrosophic sets.

### Definition 3.8

A neutrosophic topology ( $NT$  for short) and a non empty set  $X$  is a family  $\tau$  of neutrosophic subsets in  $X$  satisfying the following axioms

- ( $NT_1$ )  $0_N, 1_N \in \tau$ ,  
 ( $NT_2$ )  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$ ,  
 ( $NT_3$ )  $\cup G_i \in \tau \quad \forall \{G_i : i \in J\} \subseteq \tau$

In this case the pair  $(X, \tau)$  is called a neutrosophic topological space ( $NTS$  for short) and any neutrosophic set in  $\tau$  is known as neutrosophic open set ( $NOS$  for short) in  $X$ . The elements of  $\tau$  are called open neutrosophic sets, A neutrosophic set  $F$  is closed if and only if it  $C(F)$  is neutrosophic open.

### Example 3.2

Any fuzzy topological space  $(X, \tau_0)$  in the sense of Chang is obviously a NTS in the form  $\tau = \{A : \mu_A \in \tau_0\}$  wherever we identify a fuzzy set in  $X$  whose membership function is  $\mu_A$  with its counterpart.

### Remark 3.3

Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allow more general functions to be members of fuzzy topology.

### Example 3.3

Let  $x = \{x\}$  and

$$A = \{\langle x, 0.5, 0.5, 0.4 \rangle : x \in X\}$$

$$B = \{\langle x, 0.4, 0.6, 0.8 \rangle : x \in X\}$$

$$D = \{\langle x, 0.5, 0.6, 0.4 \rangle : x \in X\}$$

$$C = \{\langle x, 0.4, 0.5, 0.8 \rangle : x \in X\}$$

Then the family  $\tau = \{0_N, 1_N, A, B, C, D\}$  of NSSs in  $X$  is neutrosophic topology on  $X$ .

### Example 3.4

Let  $(X, \tau_0)$  be a fuzzy topological space in changes sense such that  $\tau_0$  is not indiscrete suppose now that  $\tau_0 = \{0_N, 1_N\} \cup \{V_j : j \in J\}$  then we can construct two NTSS on  $X$  as follows

- a)  $\tau_0 = \{0_N, 1_N\} \cup \{\langle x, V_j, \sigma(x), 0 \rangle : j \in J\}$ .  
 b)  $\tau_0 = \{0_N, 1_N\} \cup \{\langle x, V_j, 0, \sigma(x), 1 - V_j \rangle : j \in J\}$ .

### Proposition 3.3

Let  $(X, \tau)$  be an NTS on  $X$ , then we can also construct several NTSS on  $X$  in the following way:

- a)  $\tau_{o,1} = \{[ ]G : G \in \tau\}$ ,  
 b)  $\tau_{o,2} = \{\langle \rangle G : G \in \tau\}$ .

### Definition 3.9

Let  $(X, \tau_1), (X, \tau_2)$  be two neutrosophic topological spaces on  $X$ . Then  $\tau_1$  is said to be contained in  $\tau_2$  (in symbols  $\tau_1 \subseteq \tau_2$ ) if  $G \in \tau_2$  for each  $G \in \tau_1$ . In this case, we also say that  $\tau_1$  is coarser than  $\tau_2$ .

**Proposition 3.4**

Let  $\{\tau_j : j \in J\}$  be a family of NTSS on  $X$ . Then  $\bigcap \tau_j$  is a neutrosophic topology on  $X$ . Furthermore,  $\bigcap \tau_j$  is the coarsest NT on  $X$  containing all  $\tau_j$ , s

**Definition 3.10**

The complement of  $A$  ( $C(A)$  for short) of NOS.  $A$  is called a neutrosophic closed set (NCS for short) in  $X$ .

Now, we define neutrosophic closure and interior operations in neutrosophic topological spaces:

**Definition 3.11**

Let  $(X, \tau)$  be NTS and  $A = \langle x, \mu_A(x), \gamma_A(x), \sigma_A(x) \rangle$  be a NS in  $X$ .

Then the neutrosophic closure and neutrosophic interior of  $A$  are defined by

$$NCl(A) = \bigcap \{K : K \text{ is an NCS in } X \text{ and } A \subseteq K\}$$

$$NInt(A) = \bigcup \{G : G \text{ is an NOS in } X \text{ and } G \subseteq A\}.$$

It can be also shown that  $NCl(A)$  is NCS and  $NInt(A)$  is a NOS in  $X$

- $A$  is in  $X$  if and only if  $NCl(A)$ .
- $A$  is NCS in  $X$  if and only if  $NInt(A) = A$ .

**Proposition 3.5**

For any neutrosophic set  $A$  in  $(X, \tau)$  we have

- $NCl(A^c) = (NInt(A))^c$ ,
- $NInt(A^c) = (NCl(A))^c$ .

**Proposition 3.6**

Let  $(X, \tau)$  be a NTS and  $A, B$  be two neutrosophic sets in  $X$ . Then the following properties hold:

- $NInt(A) \subseteq A$ ,
- $A \subseteq NCl(A)$ ,
- $A \subseteq B \Rightarrow NInt(A) \subseteq NInt(B)$ ,
- $A \subseteq B \Rightarrow NCl(A) \subseteq NCl(B)$ ,
- $NInt(NInt(A)) = NInt(A)$ ,  
 $NCl(NCl(A)) = NCl(A)$
- $NCl(A \cup B) = NCl(A) \cup NCl(B)$ ,  
 $NInt(A \cap B) = NInt(A) \cap NInt(B)$

- $NInt(1_N) = 1_N$ ,
- $NCl(0_N) = 0_N$ ,

IV. SOME NEUTROSOPHIC TOPOLOGICAL NOTIONS OF NEUTROSOPHIC REGION

In this section, we add some further definitions and propositions for a neutrosophic topological region.

**Corollary 4.1**

Let  $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$  and  $B = \langle \mu_B(x), \sigma_B(x), \nu_B(x) \rangle$  are two neutrosophic sets on a neutrosophic topological space  $(X, \tau)$  then the following are holds

- $NInt(A) \cap NInt(B) = NInt(A \cap B)$ ,
- $NCl(A) \cup NCl(B) = NInt(A \cup B)$ ,
- $NInt(A) \subseteq A \subseteq NCl(A)$ ,
- $(NInt(A))^c = NCl(A^c)$ ,  $(NCl(A))^c = NInt(A^c)$ .

**Definition 4.1**

We define a neutrosophic boundary (NB) of a neutrosophic set  $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$  by:  
 $\partial A = NCl(A) \cap NCl(A^c)$ .

The following theorem shows the intersection methods no longer guarantees a unique solution.

**Corollary 4.2:**

$\partial A \cap NInt(A) = 0_N$  iff  $NInt(A)$  is crisp (i.e.,  $NInt(A) = 0_N$  or  $NInt(A) = 1_N$ ).

**Proof** obvious

**Definition 4.2**

Let  $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$  be a neutrosophic sets on a neutrosophic topological space  $(X, \tau)$ . Suppose that the family of neutrosophic open sets contained in  $A$  is indexed by the family  $\langle \mu_{G_i}(x), \sigma_{G_i}(x), \nu_{G_i}(x) \rangle : i \in I$  and the family of neutrosophic open subsets containing  $A$  are indexed the family  $\langle \mu_{K_j}(x), \sigma_{K_j}(x), \nu_{K_j}(x) \rangle : j \in J$ . Then two neutrosophic interior, closure and boundaries are defined as following

- $NInt(A)_{[ ]}$  may be defined as two types
  - Type 1.  $NInt(A)_{[ ]} = \langle \max(\mu_{G_i}(x)), \max(\sigma_{G_i}(x)), \min(1 - \mu_{G_i}(x)) \rangle$
  - Type 2.  $NInt(A)_{[ ]} = \langle \max(\mu_{G_i}(x)), \min(\sigma_{G_i}(x)), \min(1 - \mu_{G_i}(x)) \rangle$
- $NInt(A)_{< >}$  may be defined as two types
  - Type 1.  $NInt(A)_{< >} = \langle \max(1 - \nu_{G_i}(x)), \max(\sigma_{G_i}(x)), \min(\nu_{G_i}(x)) \rangle$

ii. Type 2.  $N\text{int}(A)_{< >} = \langle \max(1 - \nu_{G_i}(x)), \min(\sigma_{G_i}(x)), \min(\nu_{G_i}(x)) \rangle$

c)  $Ncl(A)_{[ ]}$  may be defined as two types

i) Type 1.  $Ncl(A)_{[ ]} =$

$\langle \max(\mu_{K_j}(x)), \min(\sigma_{K_j}(x)), \max(1 - \mu_{K_j}(x)) \rangle$

ii) Type 2.  $Ncl(A)_{[ ]} =$

$\langle \max(\mu_{K_j}(x)), \max(\sigma_{K_j}(x)), \max(1 - \mu_{K_j}(x)) \rangle$

d)  $Ncl(A)_{< >}$  may be defined as two types

i) Type 1.  $Ncl(A)_{< >} =$

$\langle \min(1 - \nu_{K_i}(x)), \min(\sigma_{G_i}(x)), \max(\nu_{G_i}(x)) \rangle$

ii) Type 2  $Ncl(A)_{< >} =$

$\langle \min(1 - \nu_{K_i}(x)), \max(\sigma_{G_i}(x)), \max(\nu_{G_i}(x)) \rangle$ .

e) Neutrosophic boundaries defined as

i)  $\partial A_{[ ]} = Ncl(A_{[ ]}) \cap Ncl(A^c_{[ ]})$

ii)  $\partial A_{< >} = Ncl(A_{< >}) \cap Ncl(A^c_{< >})$

#### Proposition 4.1

a)  $N\text{int}(A)_{[ ]} \subseteq N\text{int}(A) \subseteq N\text{int}(A)_{< >}$ ,

b)  $Ncl(A)_{[ ]} \subseteq Ncl(A) \subseteq Ncl(A)_{< >}$

c)  $N\text{int}(A_{\{[ ], < >\}}) = \{[ ], < >\} N\text{int}(A)$  and  
 $Ncl(A_{\{[ ], < >\}}) = \{[ ], < >\} Ncl(A)$

#### Proof

We shall only prove (c), and the others are obvious.

$[ ] N\text{int}(A) = \langle \max(\mu_{G_i}(x)), \max(\sigma_{G_i}(x)), (1 - \max \mu_{G_i}(x)) \rangle$   
 or  $\langle \max(\mu_{G_i}(x)), \min(\sigma_{G_i}(x)), (1 - \max \mu_{G_i}(x)) \rangle$

Based on knowing that  $(1 - \max \mu_{G_i}(x)) = \min(1 - \mu_{G_i}(x))$  then

$[ ] N\text{int}(A) = \langle \max(\mu_{G_i}(x)), \max(\sigma_{G_i}(x)), \min(1 - \mu_{G_i}(x)) \rangle$   
 or  $\langle \max(\mu_{G_i}(x)), \min(\sigma_{G_i}(x)), \min(1 - \mu_{G_i}(x)) \rangle =$   
 $[ ] N\text{int}(A)$

In a similar way the others can prove.

#### Proposition 4.2

a)  $N\text{int}(A_{\{[ ], < >\}}) = (N\text{int}(A))_{\{[ ], < >\}}$

b)  $Ncl(A_{\{[ ], < >\}})_{\{[ ], < >\}} = (Ncl(A))_{\{[ ], < >\}}$

#### Proof

Obvious

#### Definition 4.3

Let  $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$  be a neutrosophic sets on a neutrosophic topological space  $(X, \tau)$ . We define neutrosophic exterior of A as follows:  $A^{NE} = 1_N \cap A^C$

#### Definition 4.4

Let  $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$  be a neutrosophic open sets and  $B = \langle \mu_B(x), \sigma_B(x), \nu_B(x) \rangle$  be a neutrosophic set on a neutrosophic topological space  $(X, \tau)$  then

a) A is called neutrosophic regular open iff  $A = N\text{int}(Ncl(A))$ .

b) If  $B \in NCS(X)$  then B is called neutrosophic regular closed iff  $A = Ncl(N\text{int}(A))$ .

Now, we shall obtain a formal model for simple spatial neutrosophic region based on neutrosophic connectedness.

#### Definition 4.5

Let  $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$  be a neutrosophic sets on a neutrosophic topological space  $(X, \tau)$ . Then A is called a simple neutrosophic region in connected NTS, such that

i)  $Ncl(A)$ ,  $Ncl(A)_{[ ]}$ , and  $Ncl(A)_{< >}$  are neutrosophic regular closed.

ii)  $N\text{int}(A)$ ,  $N\text{int}(A)_{[ ]}$ , and  $N\text{int}(A)_{< >}$  are neutrosophic regular open

iii)  $\partial(A)$ ,  $\partial(A)_{[ ]}$ , and  $\partial(A)_{< >}$  are neutrosophic connected.

Having  $Ncl(A)$ ,  $Ncl(A)_{[ ]}$ ,  $Ncl(A)_{< >}$ ,  $N\text{int}(A)$ ,  $N\text{int}(A)_{[ ]}$ ,  $N\text{int}(A)_{< >}$  are  $\partial(A)$ ,  $\partial(A)_{[ ]}$ , and  $\partial(A)_{< >}$  for two neutrosophic regions, we enable to find relationships between two neutrosophic regions

## V. CONCLUSION

Neutrosophic logic is well equipped to deal with missing data. By employing NSs in spatial data models, we can express a hesitation concerning the object of interest. This article has gone a step forward in developing methods that can be used to define neutrosophic spatial regions and their relationships. The main contributions of the paper can be described as the following: Possible applications have been listed after the definition of NS. Links to other models have been shown. We are defining some new operators to describe objects, describing a simple neutrosophic region. This paper has demonstrated that spatial object may profitably be addressed in terms of neutrosophic logic. Implementation of the named applications is necessary as a proof of concept.

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