

# Comparative Performance Evaluation of Entropic Thresholding Algorithms Based on Shannon, Renyi and Tsallis Entropy Definitions for Electrical Capacitance Tomography Measurement Systems

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**Abstract**—The concept of entropy as a measure of information has been extensively applied in information theory and related fields. The complex nature of information has resulted in some proposed entropy definitions. In image processing, the entropy concept has been used in developing thresholding techniques based on maximum entropy principles for image segmentation, enhancement and object detection purposes. In this article, entropy definitions are analysed to establish their relationship and after that evaluate their performance in image thresholding. Static simulated data from Electrical Capacitance Tomography measurement system for annular and stratified flows in multiphase hydrocarbons production has been used. Performance evaluation results of thresholding algorithms using Renyi entropy has shown to improve the measurements, particularly for stratified flow regimes. The improvement is solely based on the entropy definition, and it has been observed the introduced controlling parameters do not affect its performance. Renyi entropic thresholding algorithm is relatively robust as it is independent of the controlling parameter  $q$  and the grey level resolution. Therefore, there is the potential possibility of using Renyi entropic thresholding to improve measurements in hydrocarbons flow measurement using Electrical Capacitance Tomography measurement system.

**Index Terms**—Image segmentation, Thresholding, Entropy, Electric capacitance tomography, multiphase flows.

## I. INTRODUCTION

The Electrical Capacitance Tomography (ECT) is one of the imaging system developed for monitoring and measurement of industrial processes [1-6]. The system has been extensively evaluated for applications in hydrocarbons multi-phase flow measurement and

monitoring [7-15]. In this application, one of the requirements for ECT system is to be able to generate at least 100 frames per second. As a result, the most commonly used reconstruction algorithm is the Linear Back Projection (LBP), a crude but simple and fast algorithm suitable for fast flowing industrial processes. However, the problem with the LBP algorithm is that it generates poor quality images, which are mostly used for qualitative analysis only. Improving the quality of the reconstructed images from these systems has attracted the attention of researchers. Some reconstructions algorithms have been proposed to address the problem [16-24]. Others have improved images from LBP by further preprocessing the images using different approaches such as filtering [25], segmentation [26] and curve fitting [27].

One of the investigated image pre-processing segmentation techniques is thresholding [28-31], the simplest method of image segmentation, enhancement and object detection. Entropy-based thresholding techniques have been applied to images generated from ECT system for multiphase flow imaging applications in hydrocarbons industry [26, 32]. However, in their work, only two definitions of entropies as proposed by Shannon [33] and Pal [34] were used with Shannon entropy emerging as relatively better entropy for the application. The problem with the Shannon entropy definition is that it does not guarantee the best generalisation of results, and assume implicit control of trade-offs in the grey level distributions under considerations. A more generalised definition of entropy with the potential of revealing additional unique information and explicit control of the trade-offs between the grey level distributions was desirable to accommodate the existing complexity of the concept of information [35].

Ref.[36], proposed a generalised entropy definition that is an extensive quantity for statistically independent systems. Ref. [37], extended the generalisation of Renyi entropy by adding an extra controlling parameter.

Independently, Ref.[38], proposed another generalised entropy definition that is a nonextensive quantity for statistical mechanics systems. In the case of images, Tsallis entropy assumes the presences of non-additive information in some class of images, not like Renyi. However, Tsallis definition has been shown by ref. [39] to be the un-normalized definition of a previous definition proposed by Havrda and Charvat [40] which was extending the generalisation of Renyi entropy definition.

In this contribution, the evaluation work on entropic thresholding in ECT applications is extended to include other definitions of entropy; the Renyi entropy that generalises the Shannon entropy, the Kapur entropy, which further generalises the Renyi entropy and the Havrda-Charvat-Tsallis entropy for online measurement of the component fraction in the hydrocarbon industry. Results show that both Renyi and Havrda-Charvat-Tsallis entropies produce the same results. The extra controlling parameters introduced in Renyi and Kapur entropies do not influence the performance of the thresholding algorithms. On comparing with the Shannon entropy thresholding algorithms, Renyi entropy thresholding algorithms are flow regime dependent, improving performance for stratified flow regime only. Renyi entropy algorithms are relatively robust as they are grey level resolution independent, a feature that can reduce computation cost.

## II. ECT SYSTEMS

ECT is one of the non-invasive measurement systems for imaging industrial processes. Its goal is to generate instantaneous images of dielectric material component permittivity distributions inside a vessel using external capacitance measurement [41, 42]. The imaging technique is capable of distinguishing between different components or phases in a multiphase mixture based on the permittivity distribution in the mixture.

The ECT system is mainly composed of three sub-systems; the sensor, sensor electronics and a computer where image processing take place (see Fig. 1). The focus of this work is on the image processing sub-system,

which has three basic functions; to control the sensor electronics, to process capacitance data and to provide the user interface to the system. Processing of capacitance data, in its basic form, involves capacitance data normalisation, image reconstruction from projection and filtering. Thresholding was added to data processing for this work.

### A. Reconstruction Algorithm.

The reconstruction algorithm has a role of converting measured capacitance data into an image showing the permittivity distribution in a vessel. The commonly implemented reconstruction algorithm is the Linear Back Projection (LBP), which owes its success because it is direct, fast and reasonably simple to implement. A brief theoretical consideration is as follows;

Capacitance and permittivity distribution are related using the equation (1)

$$C = \frac{Q}{V} = -\frac{1}{V} \iint_{\Omega} \varepsilon(x, y) \nabla \phi(x, y) d\Omega \quad (1)$$

where  $\varepsilon$  is the permittivity distribution in the sensing field,  $V$  the potential difference between two electrodes forming the capacitance,  $\phi$  the potential difference distribution, and  $\Omega$  the electrode's surface. In the linearized and discrete state, the LBP reconstructed image grey levels can be represented by the following equation [16].

$$g = \sum_{k=1}^N \frac{S_k^T \times C_k}{S_k^T} \quad (2)$$

where  $\mathbf{S}$  is the sensitivity map and  $\mathbf{C}$  the normalised capacitance vectors.

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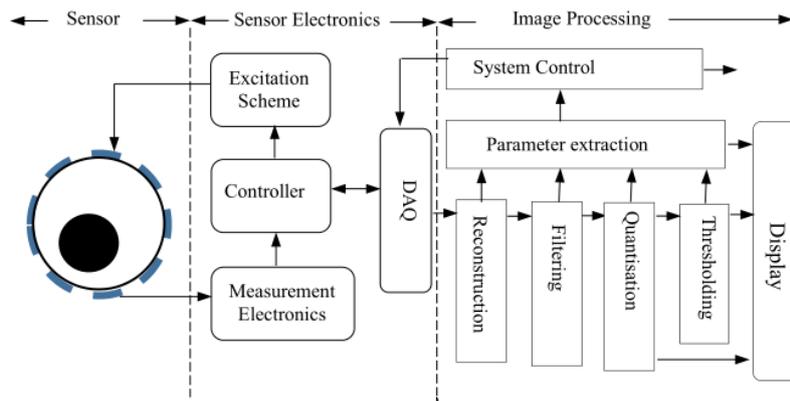


Fig.1. Basic building blocks for an ECT system

### B. Filtering.

Images generated by LBP reconstruction algorithm, however, are severely blurred. To avoid excessive smoothing and suppress aliasing, filtering is employed in the data after reconstruction [25].

$$g_k = \begin{cases} 0 & g_k < \beta \\ g_k & \beta \leq g_k < 1 \\ 1 & g_k \geq 1 \end{cases} \quad (3)$$

where

$$\beta = (1 - \gamma)\alpha \quad (4)$$

The thresholding value

$$\gamma = \frac{1}{M} \sum_{m=1}^M C_m \quad (5)$$

Average values of the normalised capacitance

$$\alpha = \frac{1}{N} \sum_{n=1}^N g_n \quad (6)$$

Average values of the reconstructed permittivity

### III. ENTROPIC THRESHOLDING.

The goal of thresholding is to select a set of optimal thresholding grey levels that can be used to discriminate objects from the background pixels in an image. The selection techniques can be classified into two groups such as bi-level and multi-level. In bi-level thresholding, only one thresholding value is used in thresholding process. The challenge is to develop cost functions that can be derived directly from data without further assumptions to capture as much information as possible within the data probability density function, upon which the selection will be based.

Shannon, [43] proposed such cost functions based on an information-theoretic concept of entropy. The principle of entropy is to use uncertainty as a measure to describe the information contained in the image. The maximum information is achieved when no *a priori* knowledge is available, in which case, it results in maximum uncertainty.

Entropy-based thresholding technique considers an image grey levels histograms as the probability distribution, analyse the profile characteristics of the histograms, and then select optimal thresholding grey level values that yield Maximum Entropy (ME). Thresholding techniques using ME as optimal criteria for image thresholding was first introduced by Pun [44], whose theoretical work was corrected and improved by Kapur et al. [45]. Pal and Pal [34] addressed the inadequacy of Shannon's entropy when it comes to image data by proposing an alternative entropy definition.

Generated interest in the work of Kapur et al. [45] has resulted in some thresholding techniques addressing different issues and limitations such as spatial information, entropy correlation, cross entropy and fuzzy logic applications [46]. A comprehensive review can be found somewhere else in the literature [47-49].

The concept of information is so rich that perhaps, there is no single definition that will be able to quantify. Further, estimating entropy from data is in not a trivial matter. Therefore, the information-theoretic approach based on Shannon's entropy concept has received considerable interest resulting in other alternative measures of information or entropy. One of the problems with Shannon's entropy is that it does not guarantee the best generalisation of results, and assume implicit control of trade-offs in the distributions under considerations. Renyi [36] introduced parametric families of entropies as a mathematical generalisation of Shannon's entropy. Tsallis [38] also did the same albeit from a statistical physics point of view. Both introduced a parameter  $q$  in the definition of entropy which offers explicit control of some tradeoffs involved when ME thresholding and help to make the definitions of entropy less sensitive to the shape of the probability distribution. In both cases, it has been shown that when the parameter  $q \rightarrow 1$ , they reduce to Shannon's entropy.

#### A. Shannon's Entropy Based Thresholding.

According to Shannon [43], the entropy of an image is defined as

$$H_N^S = - \sum_{i=1}^n p_i \ln p_i \quad (7)$$

Where  $p_i$  is the probability of occurrence of the  $i^{th}$  grey level and  $i \in [1, n]$  the total number of grey-levels. In Kapur et al. [45], also synonymously referred to as Kapur, implementation of the ME thresholding based on Shannon's entropy, the object and background are considered as separate signal sources and the thresholding grey level that maximises their additive combined entropy is the most optimal one.

$$H_n^S(T) = H_o^S(T) + H_b^S(T) \quad (8)$$

$$H_o^S(T) = - \sum_{i=1}^T \frac{p_i}{p_T} \ln \frac{p_i}{p_T} \quad (9)$$

$$H_b^S(T) = - \sum_{i=1+T}^n \frac{p_i}{(1-p_T)} \ln \frac{p_i}{(1-p_T)} \quad (10)$$

where  $p_T = \sum_{i=1}^T p_i$  and  $(1 - p_T) = \sum_{i=T+1}^n p_i$  .the cumulative probability

$$T_{opt} = \arg \max [H_o^S(T) + H_b^S(T)] \quad (11)$$

### B. Renyi's Entropy Based Thresholding.

According to Renyi [36], a more generalised definition of entropy of an image is defined as

$$H_n^R = \frac{1}{(1-q)} \ln \left[ \sum_{i=1}^n (p_i)^q \right] \quad (12)$$

where  $q \neq 1$  and  $q > 0$ . The definition is an extensive quantity for statistically independent sub-systems and has a concave distribution only for  $0 < q > 1$ . It has been shown that Renyi entropy reduces to Shannon entropy as a limiting case when  $q \rightarrow 1$ .

Kapur [37] extended the generalisation of Renyi entropy by adding another parameter  $\beta$ .

$$H_n^K = \frac{1}{(1-q)} \ln \left[ \frac{\sum_{i=1}^n (p_i)^{q+\beta-1}}{\sum_{i=1}^n (p_i)^\beta} \right] \quad (13)$$

where  $q \neq 1$  and  $q > 0$ ;  $\beta > 0$  and  $q + \beta - 1 > 0$ . The definition reduces to Renyi entropy when  $\beta = 1$ . Further, it reduces to Shannon entropy when  $\beta = 1$  and as a limiting case when  $q \rightarrow 1$ . In this case, the two entropy definitions yield the same results, hence treated as the same.

The first use of Renyi entropy in image thresholding was introduced by Sahoo et al. [50]. Their implementation follows a similar approach of maximum entropy as proposed by Kapur (see equation 11).

$$H_n^R(T) = H_o^R(T) + H_b^R(T) \quad (14)$$

$$H_o^R(T) = \frac{1}{(1-q)} \ln \left[ \sum_{i=1}^T \left( \frac{p_i}{P_T} \right)^q \right] \quad (15)$$

$$H_b^R(T) = \frac{1}{(1-q)} \ln \left[ \sum_{i=T+1}^n \left( \frac{p_i}{(1-P_T)} \right)^q \right] \quad (16)$$

$$T_{opt} = \arg \max [H_o^R(T) + H_b^R(T)] \quad (17)$$

As it turns out, this approach makes the entropy correlation approach suggested by Yen et al. [51] its special case when  $q = 2$ .

### C. Tsallis's Entropy Based Thresholding.

Independently, Tsallis (1988) proposed a more generalised Shannon's entropy of an image defined as

$$H_n^T = \frac{\left\{ 1 - \left[ \sum_{i=1}^n (p_i)^q \right] \right\}}{(q-1)} \quad (18)$$

where  $q \neq 1$  and  $q \geq 0$ . The definition is a non-extensive

quantity of the systems in statistical mechanics. It is used due to the presence of non-additive information in some classes of images, unlike Renyi entropy. Tsallis entropy reduces to Shannon entropy as a limiting case when  $q \rightarrow 1$ .

It turns out that this Tsallis entropy is the un-normalized case of a previously reported definition of Havrda and Charvat entropy [40] which was normalised to 1. Otherwise, they are essentially the same expressions [39].

$$H_n^H = \frac{\left\{ 1 - \left[ \sum_{i=1}^n (p_i)^q \right] \right\}}{(1-2^{1-q})} \quad (19)$$

In Albuquerque et al. [52], the first implementation of the ME thresholding based on Tsallis entropy, follow a similar approach as Kapur but replaces the pure addition expression (equation 11) by pseudo-additive expression.

$$H_n^T(T) = H_o^T(T) + H_b^T(T) + (1-\alpha)H_o^T(T)H_b^T(T) \quad (20)$$

$$H_o^T(T) = \frac{\left\{ 1 - \left[ \sum_{i=1}^T \left( \frac{p_i}{P_T} \right)^q \right] \right\}}{(q-1)} \quad (21)$$

$$H_b^T(T) = \frac{\left\{ 1 - \left[ \sum_{i=1+T}^n \left( \frac{p_i}{1-P_T} \right)^q \right] \right\}}{(q-1)} \quad (22)$$

$$T_{opt} = \arg \max \left[ \frac{H_o^T(T) + H_b^T(T) + (1-\alpha)H_o^T(T)H_b^T(T)}{(1-\alpha)H_o^T(T)H_b^T(T)} \right] \quad (23)$$

The use of Maximum Entropy principles in these algorithms yields the same results for both Renyi and Tsallis entropic thresholding. It can be shown that the two entropy definitions are monotonic functions of each other [53].

$$H_n^R = \ln[1 + (1-q)H_n^T](1-q)^{-1} \quad (24)$$

## IV. METHODS

The main objective of this work was to analyse and evaluate the performance of the suggested thresholding algorithms based on the new definitions of entropies in measuring component volume fraction in a two-components multiphase mixture of oil and gas. After that, compare the results with the previously evaluated Shannon based thresholding algorithms.

Computer simulated data were used in the evaluation process to establish static properties of the algorithms under considerations over the full component volume fraction range. Static image and capacitances data were generated from an 8 - electrodes ECT sensor unit

simulator [54]. The simulator is capable of generating multi-component permittivity distributions of annular and stratified flows in a circular pipe, herein referred to as true or reference distribution. After that related simulated capacitance measurements were obtained from the capacitance forward problem solver implemented using 2D Finite Element Methods (FEM). These simulated capacitance measurements were used by the capacitance inverse problem solver, also referred to as reconstruction algorithm to try to generate the original true distribution.

In this work, the generated images from an ECT system is a result of a combination of signal processing techniques such as image reconstruction, filtering, quantization and thresholding. The LBP is the implemented reconstruction algorithms. Filtering technique as proposed by Xie et al. also referred as Xie has been implemented. Three quantizing resolutions, 16, 256 and 512 have been used for evaluation. Two thresholding techniques based on maximum entropy and maximum correlation thresholding have been used. Entropy definitions by Shannon, Renyi, and Tsallis have been implemented in the ME thresholding, with maximum correlation as a special case of Renyi thresholding implementation. As a result, some algorithms implemented for evaluation, are summarised in table 1.

Table 1. Algorithms evaluated and associated abbreviations. Q, X and G stands for quantization, Xie filtering and gas as an object to be searched, respectively.

Algorithms	Description
LBP_XQ	Conventional Quantized filtered LBP algorithm.
SHA_XG	Thresholding algorithm with Shannon entropy
REN_XG	Thresholding algorithm with Renyi entropy
TSA_XG	Thresholding algorithm with Tsallis entropy
YEN_XG	Maximum correlation thresholding algorithm

## V. PERFORMANCE EVALUATION CRITERIA.

The performance evaluation criteria for the proposed algorithms can be summarised based on accuracy, robustness under the various combination and operating environment and usability to the hydrocarbon industry applications.

### A. Accuracy.

Accuracy refers to the capability of the algorithm to produce reconstructed images close to the original images, both spatially and volumetrically. The similarity metrics used include qualitative visual inspection and quantitative Mean Square Error (MSE) and Gas Volume Fraction Error (GVFE).

$$MSE = \left( \frac{1}{N} \sum_{i=1}^N \| \mathbf{g}_i^{rec} - \mathbf{g}_i^{ref} \|^2 \right) / (\mathbf{g}_i^{ref}) \quad (25)$$

where N is the number of pixels and  $\mathbf{g}_i$  the  $i^{\text{th}}$  pixel grey level value

$$GVFE = \| \alpha_{rec} - \alpha_{ref} \| \quad (26)$$

where  $\alpha = \frac{1}{A_{pipe}} \sum_{i=1}^N A_i g_i$  and  $A_i$  the  $i^{\text{th}}$  pixel area and  $A_{pipe}$  the total area of the pipe.

### B. Robustness.

The performance of the thresholding algorithms is influenced by; the reconstruction algorithm used, filters applied, the quantizer resolution, the components distribution in the mixture, the object to be searched and flow regime nature. Therefore, robustness refers to the capability of the algorithm to consistently perform over full component volume fraction range and under various combination and conditions. A good algorithm should be independent of the factors mentioned above.

### C. Usability.

Usability refers to practical applicability of the algorithm in measuring component volume fraction of two components multiphase flow in hydrocarbons industry (see Table 2).

Table 2. Accuracy requirements for a typical multiphase meter in hydrocarbons production over the full component volume fraction scale [55]

OIL INDUSTRY APPLICATION	DESIRED VOLUMETRIC ACCURACY
Reservoir management	$\sim \pm 10\%$ for all flow phases
Fiscal -Custody transfer	$\pm 2\text{-}5\%$ for all flow phases
Fiscal -Taxation/royalty	$\pm 0.25\%$ for oil
	$\pm 2\%$ for water
	$\pm 1\%$ for gas

## VI. RESULTS AND DISCUSSION

Qualitative evaluation through visual inspection shows clear differences between LBP reconstructed and thresholded images (Fig.2). Further, there is a notable difference between images generated by Shannon and the rest of the entropic thresholding algorithms. Shannon algorithm tends to overestimate the reference images whereas the rest of the entropic algorithm tends to underestimate the reference image, particularly for annular flows. There are no visible differences among Renyi, Tsallis and maximum correlation entropic thresholding unless the quantitative evaluation is considered.

Quantitative evaluation using MSE show a relatively poor average performance of the Shannon algorithm to the rest of the entropic thresholding algorithms (Table 3), particularly for annular flows tested. Also, it can be seen, that Renyi, Tsallis and maximum correlation entropic thresholding generates the same MSE results suggesting that they are the same. Maximum cross correlation is a special case of Renyi entropic thresholding ( $q = 2$ ). Under maximum entropy principle implemented in the thresholding process, Renyi and Tsallis will generate the same results always [53]. As a result, of this, the discussion that follows on algorithms performance evaluation will be focusing on Shannon and Renyi entropic thresholding algorithms only.

Quantitative evaluation has been extended to cover the whole component volume fraction range to see a broader picture of the performance of the thresholding algorithms for both annular and stratified flows. Plots of MSE and reference component fraction range have been used for these purposes.

Renyi introduced the parameter  $q$  in his entropy definition to generalise it. The performance of maximum

entropy thresholding technique using Renyi's entropy produces the same results irrespective of the value of  $q$  for both annular and stratified flows (Fig. 3 & 4). This indifference to  $q$  value suggests that the definition is solely responsible for the difference in performance when compared to their counterpart algorithms using Shannon's entropy.

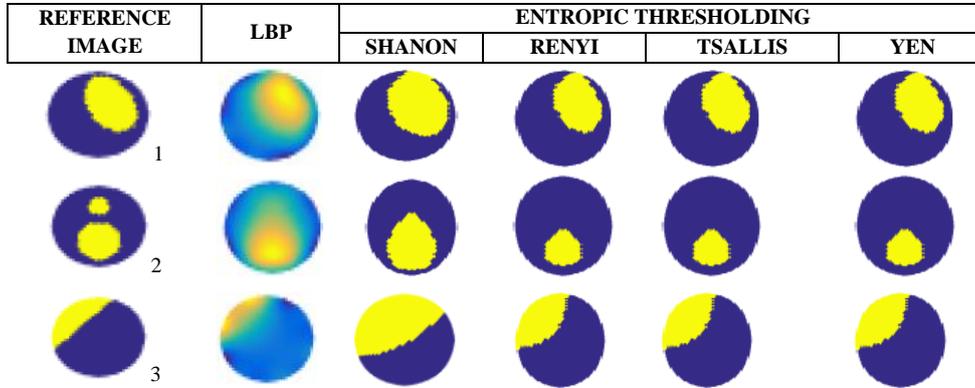


Fig.2. Qualitative visual inspection performance evaluation of algorithms for selected samples of annular and stratified flow

Table 3. Quantitative spatial accuracy performance evaluation results for algorithms for selected samples in Fig.2 using Minimum Square Error (MSE) criteria.

REF.	LBP	ENTROPIC THRESHOLDING			
		SHANON	RENYI	TSALLI	YEN
1	0.1882	0.2733	0.4289	0.4289	0.4289
2	0.1517	0.1167	0.0822	0.0822	0.0822
3	0.2379	0.1244	0.1078	0.1078	0.1078

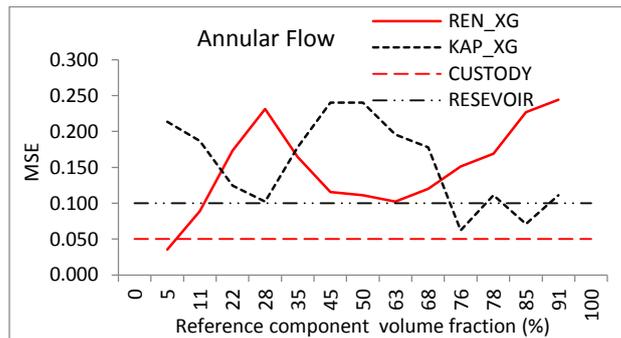


Fig.3. The performance of entropic thresholding algorithms based on Renyi entropy (REN\_XG) relative to Shannon entropy (KAP\_XG) definitions for annular flows

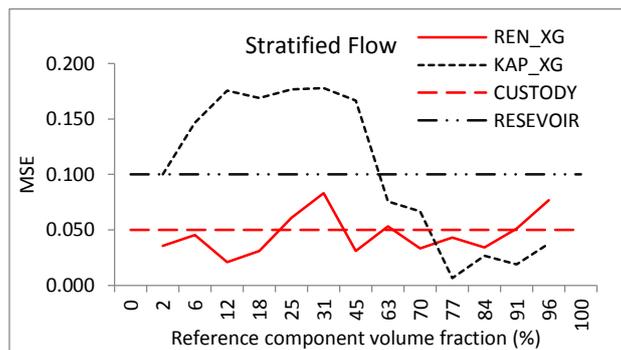


Fig.4. The performance of entropic thresholding algorithms based on Renyi entropy (REN\_XG) relative to Shannon entropy (KAP\_XG) definitions for stratified flows.

The algorithm is independent of the grey level resolution used. In this case, 16, 256 and 512 grey levels resolution were used for evaluation. No changes on the performance were observed for all the tested resolutions. This feature is attractive when it comes to speeding up the process as lower resolution can be used without any penalties. This resolution independence is another departure from what has been observed in the previous work using Shannon entropy definition.

Renyi entropic thresholding algorithms are dependent on the object being searched; in this case, the object is either oil or gas. Reasonable accurate results were only obtained when the gas was the object to be searched. When the object to be searched is oil the algorithm fails to distinguish the two-component distribution in the mixture. This failure is a departure from what has been observed using Shannon entropic thresholding algorithm.

The algorithm is flow regime dependent with the stratified flow being the best multiphase flow application to be used relative to annular flow. It can be seen from Fig.2 that REN\_XG measurements qualify for not only reservoir management but also fiscal-custody transfer purposes. Whereas in Fig 1, annular flow measurement does not qualify even for reservoir management purposes. However, the overall performance results are better for Renyi entropy than for Shannon entropy.

Renyi entropic thresholding seems to favour filtered data. Reasonably accurate measurements were obtained when filtered LBP algorithm is used. The results suggest that for better results this algorithm prefers filtered data.

The Renyi entropic thresholding algorithm like Shannon algorithm fails to perform over full component fraction range consistently. The problem is more visible or pronounced for annular flow regimes tested.

## VII. CONCLUSION

In this paper, comparative analysis and performance evaluation of the entropic thresholding algorithms based on maximum entropy principle has been presented. The comparative discussion presented was between the previously evaluated Shannon entropy definition and the new Renyi entropy definition. Performance evaluation of maximum entropy thresholding algorithms using Renyi entropy has shown to improve the measurements, particularly for stratified flow regimes. The improvement is solely based on the entropy definition, and it has been observed the introduced controlling parameters do not affect its performance. Renyi entropic thresholding algorithm is relatively robust as it is independent of the controlling parameter  $q$  and the grey level resolution. Therefore, there is the potential possibility of using Renyi entropic thresholding to improve measurements in hydrocarbons flow measurement using ECT measurement system.

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