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Application of Weighted Additive Fuzzy Goal Programming Approach to Quality Control System Design

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Abstract— The problem of decision-making in designing a quality control system (QCS), is one of the most difficult problems decisions facing the manager in the industrial firms , this problem of decision requires of fixing the levels of inputs and variables that meet the required output specifications. in the context of the problem a QCS, the parameters can be imprecise and expressed through intervals or fuzzy. The aim of this study is to presents the formulation for designing a QCS based on Weighted fuzzy goal programming (WAFGP) developed by Yaghoobi and Tamiz [12] and Yaghoobi et al [13], the advantage of the proposed formulation as a linear , use all types of membership functions and integrate explicitly the decision-maker's preference . Finally, we compare the results of our model with the major important mathematical models used in the QCS It has been shown that the best model.

Index Terms— Fuzzy Goal Programming, Additive Approach, Quality Control System

I. Introduction

Even though some real-world problems can be reduced to a matter of a single objective very often it is hard to define all the aspects in terms of a single objective. Defining multiple objectives often gives a better idea of the task. Multi objective optimization has been available for about two decades, and its application in real-world problems is continuously increasing. In contrast to the plethora of techniques available for single-objective optimization, relatively few techniques have been developed for multi objective optimization , Goal programming(GP) is one of the most important methods of Multi objective optimization ,it is an extension to linear programming . the basic idea is to establish a specific numeric goal for each of the objectives, formulate an objective function for each objective, then seek a solution that minimizes the significance of GP lies in its perspective of sharing goals with their priorities and providing an optimal solution ,keeping in line the goals and their priorities. Where linear programming usually deals with a one-dimensional objective such as profit maximization, goal programming solves multiple and frequently conflicting objectives, such as profitability, liquidity, and solvency. Some of the many recent applications of GP in management have been considered. In this paper we introduce this approach, describe its underlying philosophy for QCS in the presence of certain features which is a complex decision making process.

Sengupta [11] proposed a lexicographic GP model for QCS design in paper industry, he determined the levels of inputs and process variables in order to meet a required specification of output which is common for QCS design. Schniederjans and Karuppan [10] developed a new formulation based on GP for QCS design in service organizations by using a zero-one GP model to help in select the "best" set of quality control instruments for customer data collection purposes. Badri [1] proposed an extension of Schniederjans and Karuppan's model by combining the Analytic Hierarchy Process method and GP model for designing QCS in service organizations. Lee and Wen [7] proposed an application of fuzzy goal programming (FGP) which has been developed by Hannan [4] for Water Quality Management in a River Basin. Sadok et al [3] proposed two formulations for designing QCS based on the imprecise GP model , first based on Hannan[4] approach (Minmax approach) and second based on GP with satisfaction functions which was later developed by Martel and Aouni [8] , they applied his formulations of paper industry.

This study presents two formulations of QCS design based on additive FGP, the first was developed by Hannan(1981) it minimized an additive summation of deviations , and the second was developed by Yaghoobi and Tamiz [12] and Yaghoobi et al [13] and its application of paper industry is the same example that had been developed by Sengupta [11].

II. GP Approach for Designing to QCS in the Paper Factory

2.1 Sengupta's Approach

Sengupta(1981) described a process control problem in the paper industry in which levels of inputs variables ($X_1; \ldots; X_l$) and process variables ($R_1; \ldots; R_k$) were to be fixed in order to meet required specifications of several output characteristics ($Y_1; \ldots; Y_r$). The permissible range of values for inputs and process variables were predetermined. The output characteristics to be achieved are either specified as a permissible range of values or are of the 'close to' type. The problem, as stated, is to find a solution in which the input levels and process variables meet all the specifications on output characteristics subject to their constraints and if no such solution exists, then to find the best compromise solution.

The relationship between the output quality characteristics with the inputs and the process variables established through multiple linear regression analysis. These relationships are then used in a GP formulation with a pre-emptive priority structure to solve the problem.

The details of the input, process variables and output variables in the paper industry are illustrated in Table 1.

Table 1: Target specification for input characteristic, process variables, and output characteristics

Specification/permissible limit		
Input characteristic	(X_1) Hardwood (%)	$[20, 40]$
Process variables	(R_1) Upper cooking zone temperature (℃)	$[140, 175]$
	(R_2) Lower cooking zone temperature (℃)	$[140, 173]$
	(R_3) LP steam pressure (kg/cm^2)	$[2,0, 4,4]$
	(R_4) HP steam pressure (kg/cm^2)	$[8,0, 20,5]$
	(R_5) Active alkali as NaOH (%)	$[20, 35]$
	(R_6) Sulphidity of white liquor (%)	$[13, 25]$
	(R_7) Alkali index (no)	$[12,5, 18,7]$
Output characteristics	(Y_1) K-number	$[16, 18]$
	(Y_2) Burst factor	Close to 35
	(Y_3) Breaking length	Close to 5000

The problem was to fix the levels of the input and the process variables so that specification is met. A follow-up study was undertaken linking the input with the output through the process variables. 46 sets of such data were collected over a period of 13 days. Multiple linear regression analysis was undertaken and the following relationships were obtained.

$$Y_1 = 22.84 + 0.06X_1 - 0.05R_1 + 0.004R_2 - 0.67R_3 + 0.24R_4 - 0.13R_5 + 0.19R_6 - 0.18R_7$$
$$(\textit{Multiple correlation coefficient} = 0.74)$$

$$Y_2 = 38.94 + 0.05X_1 - 0.02R_1 + 0.002R_2 + 1.67R_3 + 0.21R_4 + 0.06R_5 + 0.02R_6 - 0.69R_7$$
$$(\textit{Multiple correlation coefficient} = 0.72)$$

$$Y_3 = 3273.4 - 24.37X_1 + 9.997R_1 + 8.48R_2 - 268.68R_3 + 120.92R_4 + 67.27R_5 + 27.89R_6 - 138.46R_7$$
$$(\textit{Multiple correlation coefficient} = 0.66)$$

To formulate this problem as a GP problem, the first setup required to be transformed to obtain one sided specification only, and these transformed variables are used in the GP formulation described. For example, the input-hardwood percentage (X_1) should be between 20 and 40. This is transformed as

$$X_1' = X_1 - 20 \le 20 ,$$

and in other variables as

$$R_3' = R_3 - 2 \le 2.4 , Y_3' = Y_3 \approx 5000 ,$$

and next setup modified regression equation for example:

$$Y_1' = -0.334 + 0.06X_1' - 0.05R_1' + 0.004R_2' - 0.67R_3' + 0.24R_4' - 0.13R_5' + 0.19R_6' - 0.18R_7'$$

The Pre-emptive Priority factor is the K-number most important characteristic to be fulfilled gets the top priority. Priorities for others which in the fixed by the management after giving due consideration to the quality aspect as well as the ease of adjusting and modifying the levels of those variables. Sengupta [11] has formulated the GP problem as follows:

$$Min\ Z = P_1\delta^-_{Y_1'} + P_2(\delta^-_{Y_2'} + \delta^+_{Y_2'}) + P_3(\delta^-_{Y_3'} + \delta^+_{Y_3'})$$
$$+ P_4(\delta^-_{R_4'} + \delta^-_{R_7'}) + P_5(\delta^-_{X_1'} + \delta^-_{R_1'} + \delta^-_{R_2'} + \delta^-_{R_3'} + \delta^-_{R_5'} + \delta^-_{R_6'})$$

Subject to:

Output constraints:

$$Y_1' + \delta^-_{Y_1'} = 2 \ \ i.e$$
$$0.06X_1' - 0.05R_1' + 0.004R_2' - 0.67R_3' +$$
$$0.24R_4' - 0.13R_5' + 0.19R_6' - 0.18R_7' + \delta^-_{Y_1'} = 2.334$$

$$Y_2' + \delta^-_{Y_2'} - \delta^+_{Y_2'} = 35 \ \ i.e$$
$$0.05X_1' - 0.02R_1' + 0.002R_2' + 1.67R_3' + 0.21R_4'$$
$$+ 0.06R_5' + 0.02R_6' - 0.69R_7' + \delta^-_{Y_2'} - \delta^+_{Y_2'} = 0.6085$$
$$Y_3' + \delta^-_{Y_3'} - \delta^+_{Y_3'} = 5000 \ i.e$$
$$24.37X_1' + 9.997R_1' + 8.48R_2' - 268.68R_3' + 120.92R'$$
$$_4 + 67.89R_6' - 27.89R_6' - 138.46R_7' + \delta^-_{Y_3'} - \delta^+_{Y_3'} = 726.139$$

Input constraint

$$X_1' + \delta^-_{X_1'} = 20$$

Process constraints

$$R_1' + \delta^-_{R_1'} = 35; \quad R_2' + \delta^-_{R_2'} = 33;$$
$$R_3' + \delta^-_{R_3'} = 2.4; \quad R_4' + \delta^-_{R_4'} = 12.5; \quad R_5' + \delta^-_{R_5'} = 15;$$
$$R_6' + \delta^-_{R_6'} = 12; \quad R_7' + \delta^-_{R_7'} = 6.2;$$

With: $X_1' \geq 0; \ R_t' \geq 0, \quad t = 1,2,..,7$

The optimal solution is:

$$X_1 = 44; \ R_1 = 160; \ R_2 = 176; \ R_3 = 3;$$
$$R_4 = 11.5; \ R_5 = 28; \ R_6 = 23 \ and \ R_7 = 18.$$

This solution result in

$$Y_1 = 16.42; \ Y_2 = 35.43 \ and \ Y_3 = 5910.$$

2.2 GP with Satisfaction Functions Approach for Designing a QCS

Sadok et al [3] used a GP model with satisfaction function proposed by Martel and Aouni [8] for designing QCS in the paper industry . The general shape of the satisfaction function is shown in (fig 1).

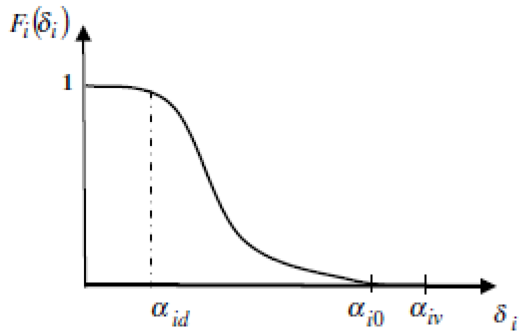


Fig. 1: General shape of the satisfaction function

Were $F_i(\delta_i)$: satisfaction function associated with deviations δ_i , α_{id} : indifference threshold; α_{i0} : dissatisfaction threshold; α_{iv} :veto threshold.

The GP model with satisfaction function proposed by sadok et al [2] can be formulated as follows :

$$Maximize\ Z = \sum_{i=1}^{r}(w^+_{Y_i} F^+_{Y_i}(\delta^+_{Y_i}) + w^-_{Y_i} F^-_{Y_i}(\delta^-_{Y_i})) +$$
$$\sum_{j=1}^{l}(w^+_{X_j} F^+_{X_j}(\delta^+_{X_j}) + w^-_{X_j} F^-_{X_j}(\delta^-_{X_j}))$$
$$+ \sum_{t=1}^{k}(w^+_{R_t} F^+_{R_t}(\delta^+_{R_{ti}}) + w^-_{R_t} F^-_{R_t}(\delta^-_{R_t}))$$

$$subject \quad to:$$

$$Y_i + \delta^-_{Y_i} - \delta^+_{Y_i} = g_{Y_i} \qquad (for\ i = 1,2,...,r)$$
$$X_j + \delta^-_{X_j} - \delta^+_{X_j} = g_{X_j} \ (for \ \ j = 1,2,...,l)$$
$$R_t + \delta^-_{R_t} - \delta^+_{R_t} = g_{R_t} \qquad (for \ \ t = 1,2,...,k)$$

$$with \ \ \delta^-_{Y_i} \ and \ \delta^+_{Y_i} \leq \alpha_{iv}$$
$$\delta^-_{X_j} \ and \ \delta^+_{R_t} \leq \alpha_{iv}$$
$$\delta^-_{R_t} \ and \ \delta^+_{R_t} \leq \alpha_{iv}$$
$$\delta^-_{Y_i}, \delta^+_{Y_i}, \delta^-_{X_j}, \delta^+_{X_j}, \delta^-_{R_i}, \delta^+_{R_t}, Y_i, X_j \ and \ R_t \geq 0$$

Where w_i express the relative importance of the objectives.

Sadok et al [3] have used this model in the papers industry , the optimal solution is:

$$X_1 = 40; \ R_1 = 158; \ R_2 = 145; \ R_3 = 3.387;$$
$$R_4 = 9.321; \ R_5 = 23; \ R_6 = 23 \ and \ R_7 = 18.152$$

This solution results in

$$Y_1 = 16 \ ; \ Y_2 = 35 \ and \ Y_3 = 5052{,}356$$

Despite the good results obtained by Sadok et al but that the formulation in problem of a QCS designing the use of GP with satisfaction function proposed by Martel and Aouni [8] we will get to the formulation of non-linear programming (LP), to be converted to the LP this is what makes the model's contains a many constraints, as it would be very difficult to be applied in the firms that produce some products which contain many inputs and process variables.

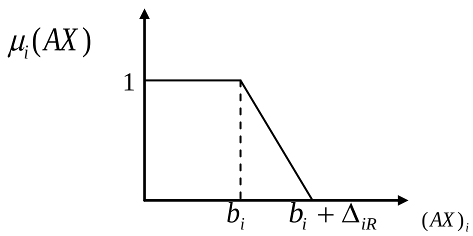
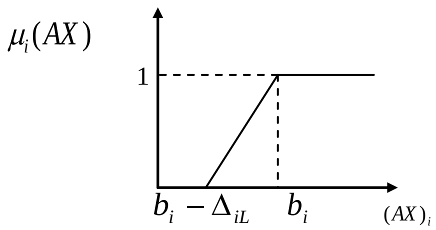
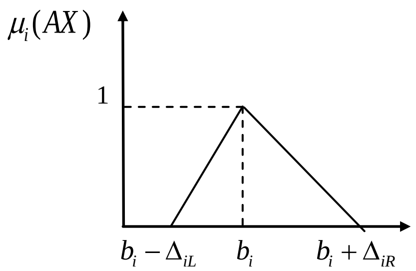
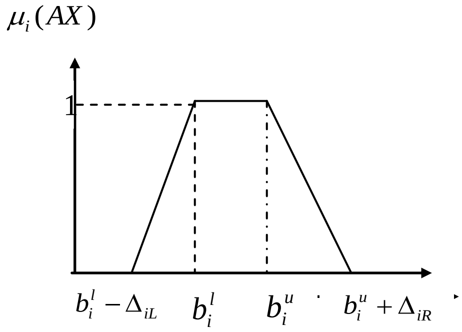
Membership function	**Analytical definition**
	$\mu_i(AX)_i = \begin{cases} 1 & if \ (AX)_i \le b_i \\ 1-\frac{(AX)_i - b_i}{\Delta_{iR}} & if \ b_i \le (AX)_i \le b_i + \Delta_{iR} \quad i=1,..,i_0 \\ 0 & if ..(AX)_i \ge b_i + \Delta_{iR} \end{cases}$ (1)
Type 1	
	$\mu_i(AX)_i = \begin{cases} 1 & if \ (AX)_i \ge b_i \\ 1-\frac{b_i-(AX)_i}{\Delta_{iL}} & if \ b_i - \Delta_{iL} \le (AX)_i \le b_i \quad i=i_0+1,..., j_0 \\ 0 & if \ (AX)_i \le b_i - \Delta_{iL} \end{cases}$ (2)
Type 2	
	$\mu_i(AX)_i = \begin{cases} 0 & if \ (AX)_i \le b_i - \Delta_{iL} \\ 1-\frac{(AX)_i-b_i}{\Delta_{iR}} & if \ b_i-\Delta_{iL} \le (AX)_i \le b_i \quad i=j_0+1,...,k_0 \\ 1-\frac{b_i-(AX)_i}{\Delta_{iL}} & if \ b_i \le (AX)_i \le b_i + \Delta_{iL} \\ 0 & if \ (AX)_i \ge b_i + \Delta_{iR} \end{cases}$ (3)
Type 3	
	$\mu_i(AX)_i = \begin{cases} 0 & if \ (AX)_i \le b_i^l - \Delta_{iL} \\ 1-\frac{b_i^l-(AX)_i}{\Delta_{iL}} & if \ b_i^l-\Delta_{iL} \le (AX)_i \le b_i^l \\ 1 & if \ b_i^l \le (AX)_i \le b_i^u \quad i=k_0+1,..., K \\ 1-\frac{(AX)_i-b_i^u}{\Delta_{iR}} & if \ b_i^u \le (AX)_i \le b_i^u+\Delta_{iR}. \\ 0 & if \ (AX)_i \ge b_i^u + \Delta_{iR} \end{cases}$ (4)
Type 4	

Fig 2 : A type of linear membership functions

III. Fuzzy Goal Programming

A useful tool for dealing with imprecision is fuzzy set theory [14] .An objective with an imprecise aspiration level can be treated as a fuzzy goal. Initially, Narasimhan [9] incorporated fuzzy set theory in GP and presented an FGP model . Hannan [4] simplified the Narasimhan method to an equivalent simple LP . These pioneering works led to extensive research in the use and application of FGP to real life problems.

To solve FGP problems various models based on different approaches have been proposed. A survey and classification of FGP models had been presented by Chanas and Kuchta [2].There are three types of fuzzy goals which are the most common. The following FGP model contains these fuzzy goals.

$$OPT\ (AX)_i \underset{\approx}{\leq} b_i \quad i=1,..,i_O$$
$$(AX)_i \underset{\approx}{\geq} b_i \quad i=i_O+1,...,j_0$$
$$(AX)_i \cong \quad i=j_O+1,...,K$$
$$X \in C_{S,}$$

Where OPT means finding an optimal decision X such that all fuzzy goals are satisfied, $(AX)_i = \sum_{j=1}^{n} a_{ij}x_j \quad i=1,...,k$, b_i is the aspiration level for $i.th$ goal.

3.1 Membership Functions

Narasimhan [9] and Hannan [4] ,were the first to give a FGP formulation by using the concept of the membership function. These functions are defined on the interval [0, 1]. So, the membership function for the *i-th* goal have a value of 1 when this goal is attained and the decision maker's is totally satisfied; otherwise the membership functions assume a value between 0 and 1.

Linear membership functions are used in theory and practice more than other types of membership functions. For the above four types of fuzzy goals linear membership functions are defined and depicted as follows (Fig. 2).

IV. FGP for Designing a QCS

4.1 FGP for Designing a QCS Using Hannan Approach

To deal with FGP problems some models use the concept of deviational variables in GP. These models try to minimize an additive summation of deviations from imprecise aspiration levels of fuzzy goals.

Hannan [4] introduced the first formulation in the FGP his model is only isosceles triangular linear membership function (Fig1-type3) which considered, ($\Delta_{iL} = \Delta_{iR}$) indicates both left and right admissible violations for the ith fuzzy goal. δ_i^- and δ_i^+, Hannan [4] proposed two approaches in the FGP (Minmax approach and Additive approach) , the first approach Maximizes the degree of membership functions and the seconds Minimizes an additive summation of deviations. The application of two objective functions to Hannan [4] for designing the QCS in the paper factory is as follows:

- **Minmax approach**: maximize degree of memberships functions μ_i its model is as follows:

$Max\ z = \mu$

subject to

$$-0{,}06X_1 + 0{,}05R_1 - 0{,}004R_2 + 0{,}67R_3 - 0{,}24R_4 + 0{,}13R_5 - 0{,}19R_6 + 0{,}18R_7 - \delta_{y_1}^- + \delta_{y_1}^+ = 5{,}84 \quad (1)$$

$$-0{,}025X_1 + 0{,}01R_1 - 0{,}001R_2 - 0{,}835R_3 - 0{,}105R_4 - 0{,}03R_5 - 0{,}01R_6 + 0{,}345R_7 - \delta_{y_2}^- + \delta_{y_2}^+ = 1{,}97 \quad (2)$$

$$-0{,}0812X_1 + 0{,}033R_1 + 0{,}028R_2 - 0{,}895R_3 + 0{,}403R_4 + 0{,}224R_5 + 0{,}092R_6 - 0{,}461R_7 - \delta_{y_3}^- + \delta_{y_3}^+ = 5{,}753 \quad (3)$$

$$0{,}1X_1 + \delta_{X_1}^- - \delta_{X_1}^+ = 3 \quad (4)$$
$$0{,}0571 + \delta_{R_1}^- - \delta_{R_1}^+ = 9 \quad (5)$$
$$0{,}0606R_2 + \delta_{R_2}^- - \delta_{R_2}^+ = 9{,}485 \quad (6)$$
$$0{,}833R_3 + \delta_{R_3}^- - \delta_{R_3}^+ = 2{,}666 \quad (7)$$
$$0{,}16R_4 + \delta_{R_4}^- - \delta_{R_4}^+ = 2{,}28 \quad (8)$$
$$0{,}133R_5 + \delta_{R_5}^- - \delta_{R_5}^+ = 3{,}666 \quad (9)$$
$$0{,}166R_6 + \delta_{R_6}^- - \delta_{R_6}^+ = 3{,}166 \quad (10)$$
$$0{,}322R_7 + \delta_{R_7}^- - \delta_{R_7}^+ = 5{,}032 \quad (11)$$
$$\mu + \delta_{y_i}^- + \delta_{y_i}^+ \leq 1 \quad (12)$$
$$\mu + \delta_{X_1}^- + \delta_{X_1}^+ \leq 1 \quad (13)$$
$$\mu + \delta_{R_t}^- + \delta_{R_t}^+ \leq 1 \quad (14)$$

$$\mu, \delta_{y_i}^-, \delta_{y_i}^+, \delta_{X_1}^-, \delta_{X_1}^+, \delta_{R_t}^-, \delta_{R_t}^+, Y_i, X_1\ and\ R_t \geq 0$$
$$(For\ i=1,2,3\ and\ t=1,2,..,7). \quad (15)$$

Using the LINGO package, the obtained optimal solution is as follows:

$X_1 = 35{,}886$,

$R_1 = 147{,}197$, $R_2 = 145$, $R_3 = 3{,}04$, $R_4 = 10{,}57$, $R_5 = 22{,}249$, $R_6 = 22{,}249$, $R_7 = 18{,}152$

This solution results in to

$Y_1 = 16{,}455$, $Y_2 = 36{,}177$, $Y_3 = 5229{,}926$.

- **Additive approach**: minimize a additive summation of deviations: the objective function and constraints in their model is as follows:

$$Min\ Z = \sum_{i=1}^{3}(\delta_{Y_i}^- + \delta_{Y_i}^+) + \sum_{j=1}^{1}(\delta_{X_i}^- + \delta_{X_i}^+) + \sum_{t=1}^{7}(\delta_{R_i}^- + \delta_{R_i}^+)$$

Subject to: Constraints (1)-(15).

The optimal solution is as follows: $X_1 = 36{,}288$, $R_1 = 139{,}99$, $R_2 = 156{,}502$, $R_3 = 3{,}2$, $R_4 = 11{,}605$

$R_5 = 20$, $R_6 = 18{,}99$, $R_7 = 18{,}302$ This solutions is yielding to $Y_1 = 17{,}04$, $Y_2 = 35$, $Y_3 = 5056{,}82$.

4.2 Weighted Additive Fuzzy Goal Programming (WAFGP) for Designing a QCS

4.2.1 WAFGP models

Yaghoobi and Tamiz [12] and Yaghoobi et al [13] who proposed other approaches for solving FGP problems with unequal weights can be formulated as a single LP problem with the concept of tolerance , The attempt to extend Kim and Whang [6] model by introducing an LP model that is able to use all types of memberships functions (type1-type4) their model can be formulated as follow:

$$Min\ z = \sum_{i=1}^{i_0} w_i \frac{\delta_i^+}{\Delta_{iR}} + \sum_{i=i_0+1}^{j_O} w_i \frac{\delta_i^-}{\Delta_{iL}} + \sum_{i=j_o+1}^{K} w_i (\frac{\delta_i^-}{\Delta_{iL}} + \frac{\delta_i^+}{\Delta_{iR}})$$

subject to:

$$(AX)_i - \delta_i^+ \le b_i \qquad i = 1,..,i_o$$

$$\mu_i + \frac{\delta_i^+}{\Delta_{iR}} = 1 \qquad i = 1,..,i_o$$

$$(AX)_i + \delta_i^- \ge b_i \qquad i = i_0 + 1,.., j_0$$

$$\mu_i + \frac{\delta_i^-}{\Delta_{iL}} = 1 \qquad i = i_0 + 1,.., j_0$$

$$(AX)_i + \delta_i^- - \delta_i^+ = b_i \qquad i = j_o + 1,.., k_0$$

$$\mu_i + \frac{\delta_i^-}{\Delta_{iL}} + \frac{\delta_i^+}{\Delta_{iR}} = 1 \qquad i = j_o + 1,.., K$$

$$(AX)_i + \delta_i^- - \delta_i^+ = b_i \qquad i = j_o + 1,.., K$$

$$(AX)_i - \delta_i^+ \le b_i^u \qquad i = k_0 + 1,.., K$$

$$(AX)_i + \delta_i^- \ge b_i^l \qquad i = k_0 + 1,.., K$$

$$\mu_i , \delta_i^- , \delta_i^+ \ge 0 \qquad i = 1,.., K$$

$$X \in C_s$$

Where C_s is an optional set of hard constraints as found in LP.

The advantages of the new model are :

- the WAFGP developed by Yaghoobi et al (2008) wich can be used for these types of membership functions .
- the new formulation determines the degree of membership function for every variable.
- the optimal solution of new model is equal to the degree of membership function for *ith* fuzzy goal.

Table 2: type and data of memberships function for every variables

Type of variables	Variables	Type of memberships functions	Data of membership functions	
Input characteristic	(X_1)	Type 3	$(\Delta_{iL}, b_i, \Delta_{iR})$	(15, 15 ,5)
Process variables	(R_1)	Type 4	$(b_i^l, \Delta_{iL}, b_i^u, \Delta_{iR})$	(158, 18 ,170, 5)
	(R_2)	Type 3	$(\Delta_{iL}, b_i, \Delta_{iR})$	(4 ,144 ,29)
	(R_3)	Type 3	$(\Delta_{iL}, b_i, \Delta_{iR})$	(1 , 3 ,1.4)
	(R_4)	Type 4	$(b_i^l, \Delta_{iL}, b_i^u, \Delta_{iR})$	(2 , 10.5 ,10)
	(R_5)	Type 3	$(\Delta_{iL}, b_i, \Delta_{iR})$	(7.5 , 27.5 ,7.5)
	(R_6)	Type 3	$(\Delta_{iL}, b_i, \Delta_{iR})$	(6 , 19 ,6)
	(R_7)	Type 4	$(b_i^l, \Delta_{iL}, b_i^u, \Delta_{iR})$	(3.1 , 15.6 ,3.1)
Output characteristics	(Y_1)	Type 4	$(b_i^l, \Delta_{iL}, b_i^u, \Delta_{iR})$	(16 ,0,5 ,18 ,0,5)
	(Y_2)	Type 3	$(\Delta_{iL}, b_i, \Delta_{iR})$	(2 , 35 , 2)
	(Y_3)	Type 3	$(\Delta_{iL}, b_i, \Delta_{iR})$	(100 ,5000 ,100)

4.2.2 Application of WAFGP for designing a QCS in the papers industry

The application of the previous model will be illustrated through the same example of the paper industry. First we will present the membership functions related to each specification (objective), and then we will define the type of membership functions. The details of the type of membership functions of input, process variables and output are shown in Table 2.

Based on the above information (Table 2) and using a methods developed by Yaghoobi and Tamiz [12], and

Yaghoobi et al [13] the by WAFGP formulation for QCS in the paper factory is as follows:

$$Min\ \ z=(\frac{\delta_{Y_1}^-+\delta_{Y_1}^+}{0,5})+(\frac{\delta_{Y_2}^-+\delta_{Y_2}^+}{2})+(\frac{\delta_{Y_3}^-+\delta_{Y_3}^+}{100})$$
$$+(\frac{\delta_{X_1}^-}{15}+\frac{\delta_{X_1}^+}{5})+(\frac{\delta_{R_1}^-}{18}+\frac{\delta_{R_1}^+}{5})+(\frac{\delta_{R_2}^-}{4}+\frac{\delta_{R_2}^+}{29})$$
$$+(\frac{\delta_{R_3}^-}{1}+\frac{\delta_{R_3}^+}{1,4})+(\frac{\delta_{R_4}^-}{2}+\frac{\delta_{R_4}^+}{10,5})+(\frac{\delta_{R_5}^-+\delta_{R_5}^+}{7,5})$$
$$+(\frac{\delta_{R_6}^-+\delta_{R_6}^+}{6})+(\frac{\delta_{R_7}^-+\delta_{R_7}^+}{3,1})$$
$$subject\quad to:$$
$$-0,06X_1+0,05R_1-0,004R_2+0,67R_3-0,24R_4$$
$$+0,13R_5-0,19R_6+0,18R_7-\delta_{y_1}^-\geq 5,84$$
$$-0,06X_1+0,05R_1-0,004R_2+0,67R_3-0,24R_4$$
$$+0,13R_5-0,19R_6+0,18R_7+\delta_{y_1}^+\leq 6,34$$
$$-0,05X_1+0,02R_1-0,002R_2-1,67R_3-0,21R_4$$
$$-0,06R_5-0,02R_6+0,69R_7-\delta_{y_2}^-+\delta_{y_2}^+=3,94$$
$$-24,37X_1+9,997R_1+8,48R_2-268,68R_3+120,92R_4$$
$$+67,27R_5+27,89R_6-138,46R_7-\delta_{y_3}^-+\delta_{y_3}^+=1726,6$$

$$X_1+\delta_{X_1}^--\delta_{X_1}^+=35$$
$$R_1-\delta_{R_1}^+\leq 170$$
$$R_1+\delta_{R_1}^-\geq 158$$
$$R_2+\delta_{R_2}^--\delta_{R_2}^+=144$$
$$R_3+\delta_{R_3}^--\delta_{R_3}^+=3$$
$$R_4+\delta_{R_4}^--\delta_{R_4}^+=10$$
$$R_5+\delta_{R_5}^--\delta_{R_5}^+=27,5$$
$$R_6+\delta_{R_6}^--\delta_{R_6}^+=19$$
$$R_7+\delta_{R_7}^--\delta_{R_7}^+=15,6$$
$$\mu_1+\frac{(\delta_{Y_1}^-+\delta_{Y_1}^+)}{0,5}=1$$
$$\mu_2+\frac{(\delta_{Y_2}^-+\delta_{Y_2}^+)}{2}=1$$
$$\mu_3+\frac{(\delta_{Y_3}^-+\delta_{Y_3}^+)}{100}=1$$
$$\mu_4+(\frac{\delta_{X_1}^-}{15}+\frac{\delta_{X_1}^+}{5})=1$$
$$\mu_5+(\frac{\delta_{R_1}^-}{18}+\frac{\delta_{R_1}^+}{3})=1$$
$$\mu_6+(\frac{\delta_{R_2}^-}{4}+\frac{\delta_{R_2}^+}{29})=1$$
$$\mu_7+(\frac{\delta_{R_3}^-}{1}+\frac{\delta_{R_3}^+}{1,4})=1$$
$$\mu_8+(\frac{\delta_{R_2}^-}{2}+\frac{\delta_{R_2}^+}{10,5})=1$$
$$\mu_9+(\frac{\delta_{R_5}^-+\delta_{R_5}^+}{7,5})=1$$
$$\mu_{10}+(\frac{\delta_{R_6}^-+\delta_{R_6}^+}{6})=1$$
$$\mu_{11}+(\frac{\delta_{R_7}^-+\delta_{R_7}^+}{3,1})=1$$
$$\mu,\delta_{y_i}^-,\delta_{y_i}^+,\delta_{X_1}^-,\delta_{X_1}^+,\delta_{R_t}^-,\delta_{R_t}^+,Y_i,X_1\ and\ R_t\geq 0$$
$$(For\ \ i=1,2,3\ \ and\ t=1,2,...,7).\quad (15)$$

Using the LINGO package, the obtained optimal solution is as follows:

$X_1=35$, $R_1=146,152$, $R_2=144$, $R_3=3,04$, $R_4=10$, $R_5=20$, $R_6=29$, $R_7=17$

This solutions results to

$Y_1=16,54$, $Y_2=35$, $Y_3=5051,839$

The proposed model determines degree of membership functions for the ith goal:

$$(\mu_1,\mu_2,\mu_3,\mu_4,\mu_5,\mu_6,\mu_7,\mu_8,\mu_9,\mu_{10},\mu_{11})=$$
$$(1,\ \ 1, 1, 1, 0.341, 1, 0.971\ \ ,1,\ \ 0\ \ ,1\ \ ,0.509)$$

We notice all the solutions lie within the target intervals, and that the model is simple and flexible that and its adaptation to every new situation. can accommodate the simultaneous the changes that can occur in models parameters (specification levels, the coefficients of the importance of deviation variables and membership functions), as Decision makers preferences can also be introduced to use all types of membership functions. Our model uses linear formulation directly contrary to the formulation of GP witch satisfaction function Which is very complex as it uses non-LP.

The Comparison between WAFGP – QCS model presented in this study, sengupta approach [11], FGP of Hannan (ADDITIVE and MINMAX Approach) and GP with satisfaction function indicated in Table 3.

Use of model WAFGP-QCS will depend largely on the goodness of the regression model because If the relationship between input characteristics, output characteristics and process parameters is week then the solution may deviate from the optimum, and depend also on type of membership functions.

Appendix : Fig. 3 presents the block diagram of the WAFGP-QCS model Development.

V. Conclusions

The QCS design is concerned with fixing the levels of inputs and process variables in order to meet a required specification of output, this problem can be tackled by using an imprecise GP model.

In this study we proposed an two formulations for designing a QCS based on Additive FGP model. First developed bay Hannan[4] Which uses a triangular linear membership functions and second developed by Yaghoobi and Tamiz [12]and Yaghoobi et al [13] named weighted additive fuzzy goal programming (WAFGP) Which uses all types of membership functions the proposed models are solved by using LINGO programme and getting optimal levels of input and process variables is to meet a required specification of output.

The major limitations of the proposed model concern the good quality of the regression model. If the relationship between input characteristics, output characteristics and process parameters is poor then the solution may deviate from the optimum. For future research we will use the fuzzy regression model developed by H Hassanpour et al [5] which will possible for us to uses it estimate of the relation between inputs variables, process variables and output variables.

Table 3: Comparisons between major QCS models

	variables	Target intervalles	Sengupta approach	Hannan Approach MINMAX Approach	Hannan Approach ADDITIVE Approach	Approach with satisfaction functions	WAFGP-QCS
Input characteristics	(X_1)	$[20, 40]$	**44**	**35,886**	**36,288**	**40**	**35**
Process variables	(R_1)	$[140, 175]$	**160**	**147,197**	**139,99**	**158**	**146,152**
	(R_2)	$[140, 173]$	**176**	**146,789**	**156,502**	**145**	**144**
	(R_3)	$[2{,}0, 4{,}4]$	3	3,634	3,2	3,387	3,040
	(R_4)	$[8{,}0, 20{,}5]$	11,5	10,570	11,605	9,321	10
	(R_5)	$[20, 35]$	28	23,085	20	23	20
	(R_6)	$[13, 25]$	23	22,249	18,99	23	19
	(R_7)	$[12{,}5, 18{,}7]$	18	17,429	18,302	18,152	17
Output characteristics	(Y_1)	$[16, 18]$	16,42	16,455	17,04	16	16,54
	(Y_2)	Close to 35	**35,43**	**36,177**	**35**	**35**	**35**
	(Y_3)	Close to 5000	**5910**	**5229,926**	**5056,86**	**5052,356**	**5051,83932**

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Appendix:

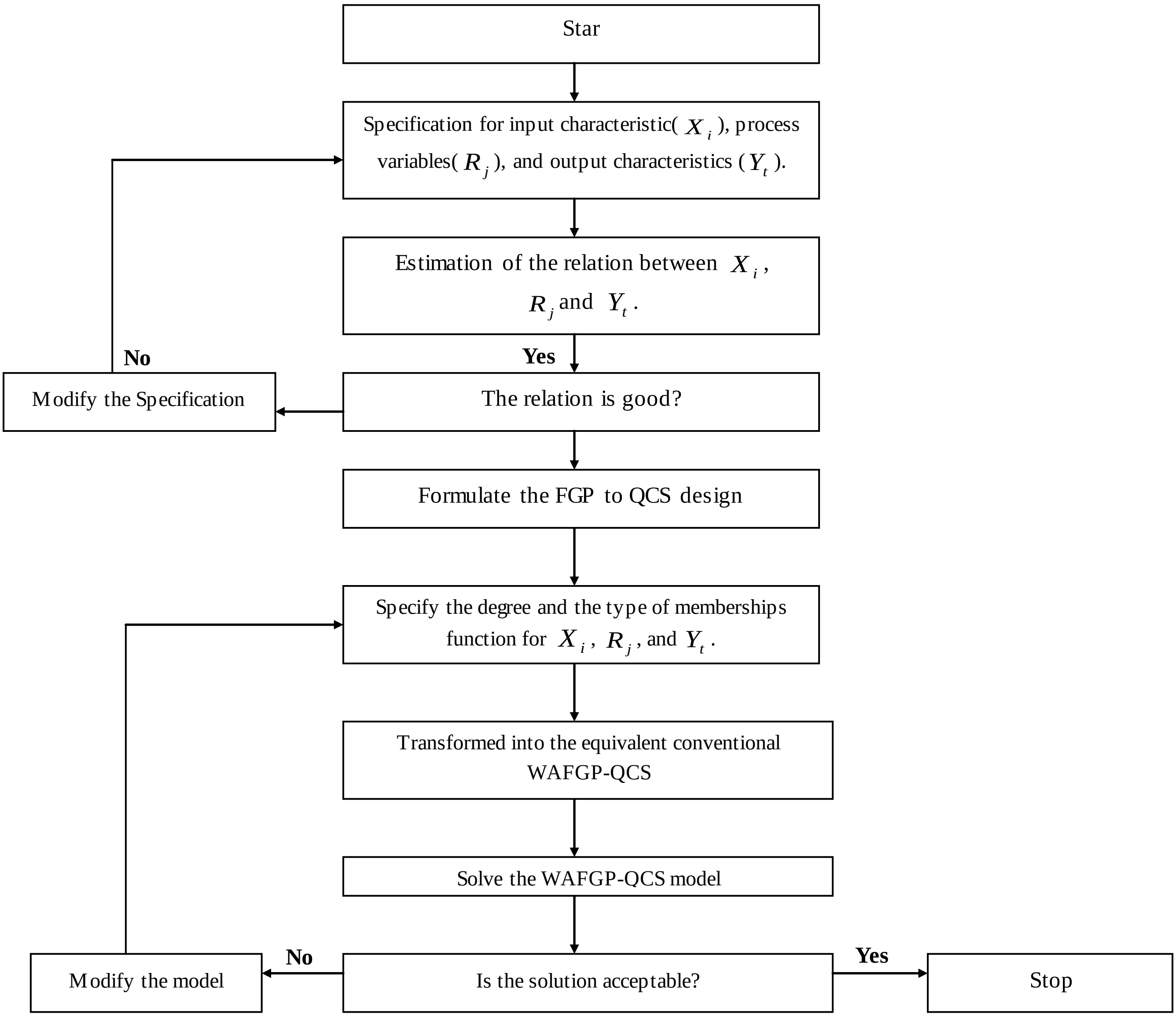

Fig. 3: The block diagram of WAFGP-QCS model development

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