I.J. Information Technology and Computer Science, **2014, 01, 1-12**
Published Online December 2013 in MECS (http://www.mecs-press.org/)
DOI: 10.5815/ijitcs.2014.01.01

Computer Simulation of Theoretical Model of Electromagnetic Transient Processes in Power Transformers

Slobodan Bjelić
Faculty of Technical Sciences, Kosovska Mitrovica, Serbia
E-mail: slobodanbjelić49@yahoo.com

Zorica Bogićević
Advanced Polytechnic School of Professional Sciences, Zvecan, Serbia
E-mail: zocabogi@gmail.com

Abstract — The paper is presenting theoretical analytical model and computer simulation of electromagnetic transient process in a transformer. Transformer parameters in a selected transitional process have been analyzed. Theoretical model refers to an energetic transformer with concentrated parameters with consideration of parameters of mutual inductance M. Simulation was performed on a personal computer using the software program MATLAB SIMULINK. The computer simulation confirmed the possibility of transitional process analysis in transformer's windings with concentrated parameters.

Index Terms— Computer, Simulation, Transient Process, Model, Transformer, Concentrated Parameters

I. Introduction

Transformer's load causes a change in voltage. This change is usually manifested as a decrease (down) of voltage relative to the idle. The transformer should be designed to withstand the possible tension throughout its life. This paper describes changes in the transformer during load changes through computer simulations that indicate a failure or defect.

Computer simulation today presents one of the leading methods for solving, describing, understanding and analysis of complex dynamic systems in the fields of technical sciences.

The MATLAB software is designed for solving various mathematical and engineering problems Which were modeled by the application of linear algebra and a master computer where this simulation has been performed, and that is why the simulation was performed precisely in this software package.

Mathematical model of power transformer is completely derived from the literature [1], and is based on the model derived in the software package MATLAB and Simulink[7]. The simulation's goal of the set theoretical models is to obtain relevant information about the behavior of the power transformer in the transition processes.

In the first example of a transformer of high power and a second example of a transformer of low power it has been shown that such a computer simulation in a qualitative way can describe the transient processes and by doing so a better analysis of their behavior can be performed.

The remainder of this paper is organized as follows: Section 2 gives a theoretical model of the transition process in the ideal and the real transformer with linear magnetic characteristics. Section 3 describes dynamic changes in real power transformer in MATLAB Simulink. It describes two simulations: Simulation *1*- Simulates idling of transformer. Secondary current is equal to zero. Simulation was performed for the real three-phase power transformer SIEMENS - ONAF / ONAN.

Simulation 2- Simulates a short circuit and controls a secondary current set to the rated current for the real three-phase power transformer SIEMENS - ONAF. Graphs fluxes, current magnetization and voltages on the primary and the secondary are given at the end of the simulation process. Conclusion and future work are presented in the final section.

II. Theoretical Model of Transient Process in Transformer

The transformer consists of ferromagnetic circuit and operates as electromagnetic connection of two electric windings: 1. primary is connected to the source of alternating current and 2. the secondary winding to which electric loading is connected. The task of the magnetic circuits is to create spatial distribution of magnetic excitation forces and magnetic fluxes which produce currents in winding at a circuit or strange magnetic fields[6]. The processes in a three-phase transformer are equivalent to processes in single-phase one if the influences of the other two phases are considered according to assumptions as in the model of an ideal and symmetric three-phase transformer, where according to figure 1:

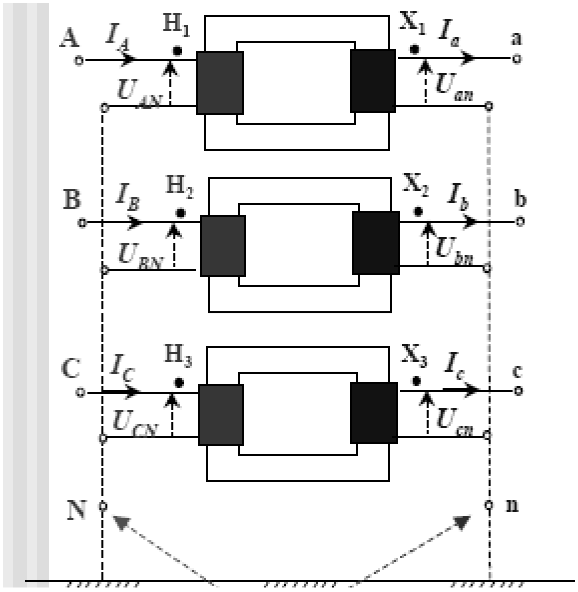

Fig. 1: a)

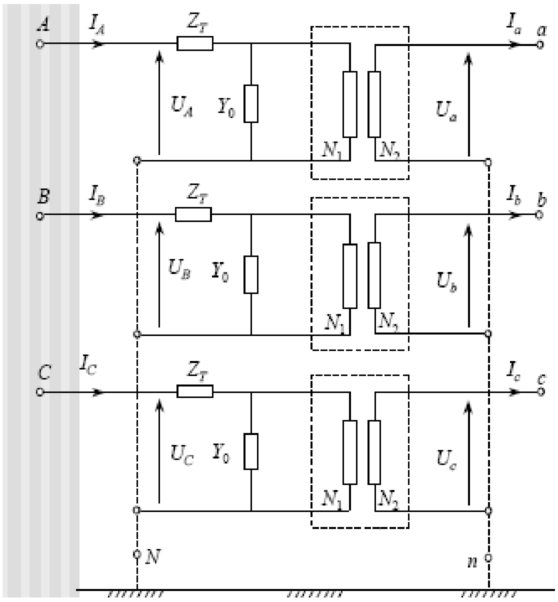

Fig. 1: b)

Fig. 1: a) Three separated magnetic circuits and b) equivalent scheme of transformer with concentrated parameters

1. Conductance of magnetic circuits for all three phases are equal: $\lambda_a \cong \lambda_b \cong \lambda_c \cong \lambda$
2. Sum of magnetic excitation forces of three phases equals to zero $\vec{M}_{\mu a} + \vec{M}_{\mu b} + \vec{M}_{\mu c} = 0$
3. According to IV Maxwell equation $div\vec{B} = 0$, $\phi_{\mu a} + \phi_{\mu b} + \phi_{\mu c} = 0$

Currents and voltages in windings can oscillate propagation through coils if the winding includes; The theory of wave spreading through coil's threads covers parameters of capacitance towards mass, winding inductance, capacitances between adjacent coils, mutual inductivity of one thread to another, reaction of primary coil on secondary and vice versa, turbulent currents in magnetic core , losses caused by hysteresis.

In the model, values of primary and secondary winding are $R_p = R_1[\Omega]$, $R_s = R_2[\Omega]$ and values of inductance are $L_p = L_1[H]$, $L_s = L_2[H]$

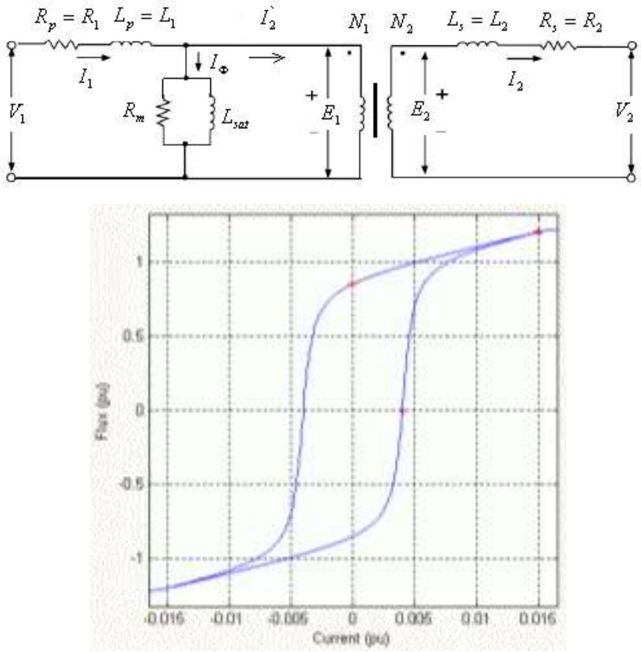

Fig. 2: a) One equivalent scheme of three-phase transformer, with Hysteresis curve

Nonlinear characteristic of magnet biasing, figure 2 is dependence $B = f(H_{Fe})$ which has an hysteresis form of a distorted rectangle and high values of magnetic penetrability $\mu = dB / dH$.

High values of magnetic penetrability $\mu = dB / dH$ help to minimize influences of air gaps and magnetic contours on which magnetic fluxes close.

Magnetic induction and strength of the fields B , H have different values in different points of cross section of the core. Values are variable due to three reasons:

a) due to the value change of cross section

b) due to the heterogeneity of magnetic domains (ferromagnetic, gap filled with air, etc), and

c) due to the dissipation of magnetic fluxes between adjacent coils.

Magnetic characteristics of magnetic circuits of devices such are transformer, dependence $B_{sr} = f(H)$, differs from magnetic characteristics of material $B = f(H)$ due to following reasons [1]:

- Lengths of contours where magnetic fluxes close are not equal, and there are also fluxes of dissipation. Influence of unequal lengths of contours can be lessened only by assumption that a magnetic circuit is composed of elementary parts with the same magnetic characteristics which are packed in order in the direction of a planned contour.
- Inequalities of cross sections along contours of fluxes. In calculations that influence, this can be lessened with the usage of the smallest value of a cross section S_m along contours of the magnetic circuit.
- Influence of connections, cracks and presence of magnetic gaps in the circuit filled with air along contours of flux closures. According to the second of Kirchhoff's laws on magnetic circuits [2]:

$$\sum_j H_j l_j = h_{Fe} \cdot l + \frac{\delta \cdot B_\delta}{\mu_0} , \quad (1)$$
$$= MPS = H(l+\delta)$$

h_{Fe} is the intensity of the field which acts as the magnetic core, $B_\delta = b = B$ - is the inductance value if there is no dissipation in the gap on the cross section, H - mean value or real value of field intensity which acts on the cross section , H_p - value of field intensity which would exist on the cross section if gaps would not exist.

$$H = \frac{MPS}{l+\delta} \approx \frac{F}{l} = h_{Fe} + \frac{\delta}{l}\frac{B}{\mu_0} = h_{Fe} + H_p , \quad (2)$$

where $\mu_e = B/\mu_0 H$ is the relative value of magnetic conductivity of a magnetic circuit, $\mu = B/H$ is the absolute value of magnetic penetrability of material, $\frac{l}{\delta} = m_p$ is the magnetic penetrability in the form of a magnetic $\frac{l}{\delta} = m_p$ circuit.

For magnetic circuits where $\frac{l}{\delta} = m_p \approx 1$, $l \approx \delta$ applies $\frac{1}{\mu_e} = \frac{1}{\mu} + \frac{1}{1} \Rightarrow \mu_e = \frac{\mu}{\mu+1} \approx 1$.

This procedure leads to the expression for differential magnetic penetrability:

$$\frac{1}{\mu_{de}} = \frac{\Delta H}{\Delta B}, \ \frac{1}{\mu_{de}} = \frac{1}{\mu_d} + \frac{1}{m_p} , \quad (3)$$

Differences in the process of magnetic biasing, which occur on hysteresis curve.

Magnetic parameters must be determined on a finished magnetic circuit which has its own geometric shape and winding position.

According to the Faraday law, the electro motor force of primary and secondary winding, u_1 and u_2 are consequences of changes of fluxes:

$$\Phi = -\frac{1}{N_p}\int_0^t u_p dt = -\frac{1}{N_1}\int_0^t u_1 dt , \quad (4)$$

$$\Phi = -\frac{1}{N_s}\int_0^t u_s dt = -\frac{1}{N_2}\int_0^t u_2 dt , \quad (5)$$

2.1 Theoretical Model Of A Transitional Process in an Ideal Transformer with Linear Magnetization Characteristic

$$u_{iz.p} = U_{iz.m}\sin(\omega \cdot t + \theta) \quad (6)$$

Voltage equations are:

$$u_{iz.p} - L_p \frac{di_p}{dt} - M\frac{di_s}{dt} = 0 , \quad (7)$$

$$L_s \frac{di_s}{dt} + M\frac{di_p}{dt} = 0 \Leftrightarrow \frac{di_s}{dt} = -\frac{M}{L_s}\frac{di_p}{dt} \quad (8)$$

Connection coefficient $k_c^2 = \frac{M}{L_p L_s}$ has the value $k_c = \frac{M}{\sqrt{L_p L_s}} \approx 1$ if the fluxes include the winding of the primary and secondary are equal in value. In approximate calculations, the following expressions apply:

$$L_p = \mu_0\mu_r(\frac{N_1}{l})^2 V, L_s = \mu_0\mu_r(\frac{N_2}{l})^2 V$$
$$M = \sqrt{L_p L_s} = \mu_0\mu_r\frac{N_1 N_2}{l^2}V , \quad (9)$$

$$M = \sqrt{L_p L_s} = \mu_0\mu_r\frac{N_1 N_2}{l^2}V , \quad (10)$$

where $\mu_0 = 4\pi \cdot 10^{-7}\frac{H}{m}$, μ_r is the magnetic penetrability of the air and core, N_1 , N_2 are the numbers of the primary and secondary coils and

V volume as the part of the space where the windings are placed, figure 3. In the primary transformer circuit active losses presented with resistance exist R_p and in the secondary circuit with resistance R_s, if the power transformer is directly connected to voltage source:

$$u_{iz.p} - L_p \frac{di_p}{dt} - M \frac{di_s}{dt} = R_p \cdot i_p, \tag{11}$$

$$L_s \frac{di_s}{dt} + M \frac{di_p}{dt} + R_s \cdot i_s = 0 / \frac{\partial}{\partial t}, \tag{12}$$

$$R_s \frac{di_s}{dt} + M \frac{d^2 i_p}{dt^2} + L_s \frac{d^2 i_s}{dt^2} = 0 \tag{13}$$

In the secondary circuit of single-phase transformer, three typical regimes can occur.

a) Idle operation:

$$R_s = \infty \Rightarrow i_s = 0 \Rightarrow u_s = -M \frac{di_p}{dt}$$

b) Short circuit:

$$R_s = 0 \Rightarrow -L_s \frac{di_s}{dt} + M \frac{di_s}{dt} = 0 \Rightarrow \frac{i_s}{i_p} = \frac{L_s}{M} = \frac{N_2}{N_1}$$

c) Loading on secondary is small value but is not equal to zero $u_s = i_s Z_{load}$, $R_s \approx Z_{load} \to 0$:

$$R_s \to 0 \Rightarrow i_s \approx \frac{L_s}{M} i_p \; \frac{di_s}{dt} = -\frac{M}{L_s}\frac{di_p}{dt} - \frac{R_s}{L_s} i_s$$

Substituting the values for derivative (di_s / dt) in (6) of the system of (9), the following is obtained:

$$u_p - L_p (1 - \frac{M^2}{L_p \cdot L_s}) \frac{di_p}{dt} = [R_p + (\frac{N_1}{N_2})^2 R_s] \cdot i_p \tag{14}$$

From the first equation of the system of (6) is also:

$$\frac{di_s}{dt} = \frac{1}{M}(u_{iz.p} - R_p i_p - L_p \frac{di_p}{dt}) \tag{15}$$

When the primary transformer is connected to external electric circuit with a voltage source $u_{iz.p} = U_{iz.m} \, sin(\omega \cdot t + \theta)$ and the parameters of the circuit are R_{iz}, L_{iz} and the secondary transformer winding is connected, loading with the parameters R_{load}, L_{load} in the primary transformer are: total active resistance $R_{p\Sigma} = R_p + R_{iz}$, total inductance $L_{p\Sigma} = L_p + L_{iz}$, and in the secondary transformer the active resistance is $R_{s\Sigma} = R_s + R_{load}$, and the total inductance $L_{s\Sigma} = L_s + L_{load}$ while time constants gain other values:

$$\tau_{p\Sigma} = \frac{R_{p\Sigma}}{L_{p\Sigma}} = \frac{R_p + R_{iz}}{L_p + L_{iz}}; \quad \tau_{s\Sigma} = \frac{R_{s\Sigma}}{L_{s\Sigma}} = \frac{R_s + R_{load}}{L_p + L_{load}}, \tag{16}$$

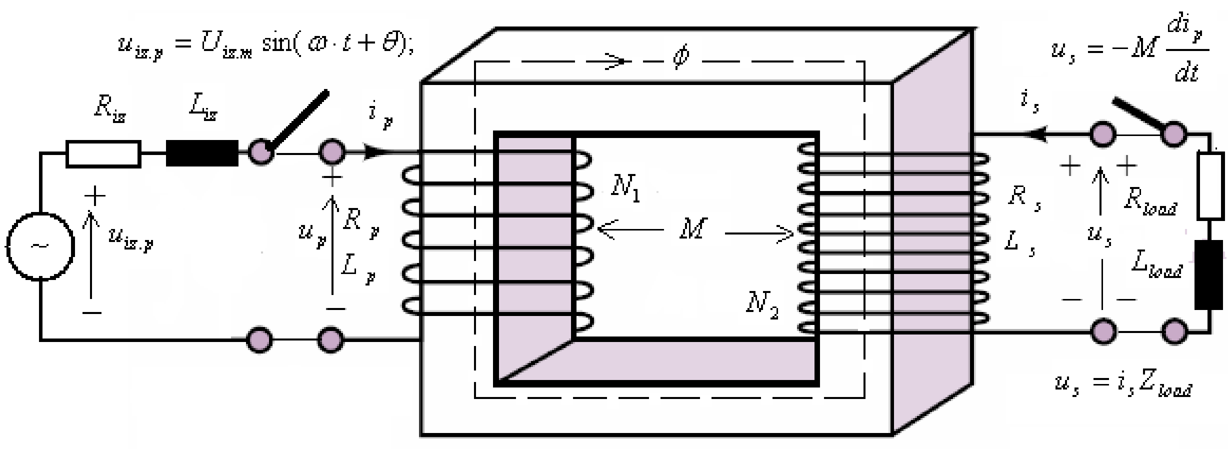

Fig. 3: Electromagnetic connection of transformer winding with concentrated parameters

With strongly paired magnetic circuits, as in the transformer here, the following applies: $M^2 = k_c^2 L_p L_s$. With these substitutions which refer to a new connection coefficient $k_{c\Sigma}$, effective inductance of the primary $L_{ef.p}$ and the mutual inductance M and the introduction of time constants to the primary circuit $\tau_{p\Sigma}$ and the secondary circuit $\tau_{s\Sigma}$ is:

$$k_{c\Sigma}^2 = \frac{M^2}{L_{p\Sigma} L_{s\Sigma}}, \; L_{ef.p} = (1-k_c^2) L_{p\Sigma}, \quad (17)$$

For which an homogenous part according to convolution theorem $(\partial/\partial t \Leftrightarrow j\omega = p)$ applies to the equation:

$$\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}{}^2)p^2 + (\tau_{p\Sigma}+\tau_{s\Sigma})p + 1 = 0, \quad (18)$$

Roots of characteristic equation, when decrement $D=0$, are:

$$p_{1,2} = \frac{-(\tau_{p\Sigma}+\tau_{s\Sigma}) \pm \sqrt{(\tau_{p\Sigma}+\tau_{s\Sigma})^2 - 4\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}^2)}}{2\tau_{p\Sigma}\tau_{s\Sigma}\left(1-k_c^2\right)}, \quad (19)$$

and the decrement of the equation is greater than zero, the roots are multiple and different:

$$\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}{}^2)p^2 + (\tau_{p\Sigma}+\tau_{s\Sigma})p + 1 > 0, \quad (20)$$

The current of the primary is the sum of a free component i_{psl} and a forced component i_{ppr} of the primary current:

$$i_p = i_{psl} + i_{ppr} = A_1 e^{p_1 t} + A_2 e^{p_2 t} + i_{ppr}, \quad (21)$$

The forced component is created by the voltage of the resource $u_{iz.p} = U_m \sin(\omega \cdot t + \theta)$ as well as the harmonic function:

$$i_{ppr} = I_{pm} \sin\left(\omega \cdot t + \theta - \varphi_p\right), \quad (22)$$

If the expression for the forced component i_{ppr} substitutes in (19) values for I_{pm}, then φ_p shall be determined:

Rising to the second power and then extracting of the root of the left and right side of the last equation is obtained:

$$I_{pm}\sqrt{[1-\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}^2)\omega^2]^2 + (\tau_{p\Sigma}+\tau_{s\Sigma})^2\omega^2} \cdot \cdot \sin(\omega \cdot t + \theta - \varphi_p - \alpha) = \frac{U_{iz.m}}{R_{p\Sigma}}\sqrt{1+\tau_{s\Sigma}^2\omega^2}\,\sin(\omega \cdot t + \theta + \beta), \quad (23)$$

$$\alpha = arctg\frac{(\tau_{p\Sigma}+\tau_{s\Sigma})\omega}{1-\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}^2)\omega^2}, \beta = arctg(\tau_{s\Sigma}\omega), \quad (24)$$

$$\varphi_p = arctg\frac{(\tau_{p\Sigma}+\tau_{s\Sigma})\omega}{1-\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}^2)\omega^2} - arctg(\tau_{s\Sigma}\omega), \quad (25)$$

$$I_{pm} = \frac{U_{iz.m}}{R_{p\Sigma}} \frac{\sqrt{1+\tau_{s\Sigma}^2\omega^2}}{\sqrt{[1-\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}^2)\omega^2]^2 + (\tau_{p\Sigma}+\tau_{s\Sigma})^2\omega^2}}, \quad (26)$$

In the moment of closing of both switches, figure 3 (in the circuit of primary and secondary) values are: $i_p(0) = 0$ for $i_s(0) = 0$ and $u_p(0) = U_{iz.m}\sin(\omega t)$.

From the (7) of the system we obtain the first derivative:

$$\frac{di_p}{dt}\Big|_{t=0} = \frac{L_{s\Sigma}}{L_{p\Sigma}L_{s\Sigma} - M^2}u_p(0) = \frac{u_{iz.p}(0)}{L_{p\Sigma}(1-k_{c\Sigma}^2)}, \quad (27)$$

Integrating constants are determined from the boundary conditions:

$$A_1 p + A_2 p + \omega \cdot I_{pm}\cos(\theta - \phi_p) = \frac{u_{iz.p}(0)}{L_{p\Sigma}(1-k_{c\Sigma}^2)}, \quad (28)$$

The value of the current i_s is obtained from the differential equation:

$$\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}^2)\frac{d^2 i_s}{dt^2} + (\tau_{p\Sigma}+\tau_{s\Sigma})\frac{di_s}{dt} + i_s = -\frac{M}{R_{p\Sigma}R_{s\Sigma}}\left(\frac{du_{iz.p}}{dt}\right), \quad (29)$$

For the homogenous part of differential equation the same decrement applies:

$$\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}{}^2)p^2 + (\tau_{p\Sigma}+\tau_{s\Sigma})p + 1 = 0, \quad (30)$$

The solution for the secondary current is the sum of free and the forced component:

$$i_s = i_{ssl} + i_{spr} = B_1 e^{p_1 t} + B_2 e^{p_2 t} + i_{spr}, \quad (31)$$

The forced component is defined by a time form:

$$i_{spr} = I_{sm}\sin(\omega \cdot t + \theta - \varphi_s), \quad (32)$$

If the expression for the forced component i_{spr} is substituted in for (34) the values for I_{sm} i φ_s will be determined:

$$I_{sm} = \frac{\omega M U_{iz.m}}{R_{p\Sigma}R_{s\Sigma}} \frac{1}{\sqrt{[-\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}^2)\omega^2]^2 + (\tau_{p\Sigma}+\tau_{s\Sigma})^2\omega^2}}, \quad (33)$$

$$\varphi_s = arctg\frac{(\tau_{p\Sigma}+\tau_{s\Sigma})\omega}{1-\tau_{p\Sigma}\tau_{s\Sigma}(1-k_{c\Sigma}^2)\omega^2}+\frac{\pi}{2}, \qquad (34)$$

Integrated constants are obtained from two conditions: 1. At the moment of closing of both switches (on the primary and the secondary) the value is $i_s(0)=0$, and 2. from (7) the first derivative is:

$$\frac{di_s}{dt}\Big|_{t=0} = -\frac{M}{L_{s\Sigma}}\frac{di_p}{dt}\Big|_{t=0} = \frac{k_{c\Sigma}^2 u_{iz.p}(0)}{M(1-k_{c\Sigma}^2)}, \qquad (35)$$

If there are no magnetic dissipations the values are:

$$k_c = 1,\ p_1 = -\frac{1}{\tau_{p\Sigma}+\tau_{s\Sigma}},\ p_2 = -\infty, \qquad (36)$$

$$i_p = i_{psl} + i_{ppr} = A_1 e^{p_1 t} + i_{ppr}\ , \qquad (37)$$

$$i_s = i_{ssl} + i_{spr} = B_1 e^{p_1 t} + i_{spr}, \qquad (38)$$

For primary the values are:

$$I_{pm} = \frac{U_m}{L_{p\Sigma}}\frac{\tau_{p\Sigma}\sqrt{1+\omega^2\tau_{s\Sigma}^2}}{\sqrt{1+(\tau_{p\Sigma}+\tau_{s\Sigma})^2}}, \qquad (39)$$

$$\frac{U_m}{L_{p\Sigma}}\frac{\tau_{p\Sigma}}{\sqrt{1+(\tau_{p\Sigma}+\tau_{s\Sigma})^2}} = \frac{I_{pm}}{\sqrt{1+\omega^2\tau_{s\Sigma}^2}}, \qquad (40)$$

$$A_1 = \frac{\tau_{p\Sigma}\tau_{s\Sigma}}{\tau_{p\Sigma}+\tau_{s\Sigma}}\frac{u_{iz.p.}(0)}{L_{p\Sigma}} - I_{pm}\sin(\theta-\varphi_p)$$
$$\varphi_p = arctg[(\tau_{p\Sigma}+\tau_{s\Sigma})\omega], \qquad (41)$$

and for secondary the are values:

$$I_{sm} = \frac{\omega M}{L_{s\Sigma}}\frac{\tau_{s\Sigma}}{\sqrt{1+\omega^2\tau_{s\Sigma}^2}} I_{pm}, \qquad (42)$$

$$B_1 = -\frac{\tau_{p\Sigma}\tau_{s\Sigma}}{\tau_{p\Sigma}+\tau_{s\Sigma}}\frac{u_{iz.p}(0)}{M} - I_{sm}\cos(\theta-\varphi_p), \qquad (43)$$

$$\varphi_s = arctg[(\tau_{p\Sigma}+\tau_{\Sigma s})\omega]+\frac{\pi}{2}, \qquad (44)$$

$$\frac{1}{I_{pm}^2}\left|A_1 - \frac{u_{iz.p}(0)}{L_{p\Sigma}}\frac{\tau_{p\Sigma}\tau_{s\Sigma}}{\tau_{p\Sigma}+\tau_{s\Sigma}}\right|^2 = \sin^2(\theta-\varphi_p), \qquad (45)$$

$$\frac{1}{I_{sm}^2}\left|B_1 - \frac{u_{iz.p}(0)}{M}\frac{\tau_{p\Sigma}\tau_{s\Sigma}}{\tau_{p\Sigma}+\tau_{s\Sigma}}\right|^2 = \cos^2(\theta-\varphi_p), \qquad (46)$$

The values of transformer's voltage are calculated of the system of (9):

$$u_p = u_1 = -M\frac{di_s}{dt}, \qquad (47)$$

$$u_s = u_2 = -M\frac{di_p}{dt}, \qquad (48)$$

In transformers of $10 < V_n < 110kV$ the voltages for basic harmonic $f = 50[Hz]$ values of relation of winding parameters are such an error of 0,1% occurs:

$$|X_{X=\omega L}| > 4|R| \Leftrightarrow$$
$$\tau\omega = (\frac{L\omega}{R})\omega \approx 4\times 314 = 1256 >> 1, \qquad (49)$$

$$\sqrt{\frac{1+\tau_{s\Sigma}^2\omega^2}{1+(\tau_{p\Sigma}+\tau_{s\Sigma})\omega^2}} \approx \frac{\tau_{s\Sigma}}{\tau_{p\Sigma}+\tau_{s\Sigma}}, \qquad (50)$$

$$I_{pm} = \frac{U_{iz.m}}{L_{p\Sigma}}\frac{\tau_{p\Sigma}\tau_{s\Sigma}}{\tau_{p\Sigma}+\tau_{s\Sigma}}$$
$$\Leftrightarrow \frac{\tau_{p\Sigma}\tau_{s\Sigma}}{\tau_{p\Sigma}+\tau_{s\Sigma}} = \frac{I_{pm}}{U_{iz.m}}L_{p\Sigma}, \qquad (51)$$

$$A_1 = I_{pm}[\frac{u_{iz.p}(0)}{U_{iz.m}} - \sin(\theta-\varphi_p)], \qquad (52)$$

$$i_p = I_{pm}\{e^{-\frac{t}{\tau_{p\Sigma}+\tau_s\Sigma}}[\frac{u_{iz.p}(0)}{U_{iz.m}} - \sin(\theta-\phi_p)]$$
$$+\sin(\omega t+\theta-\phi_p)\}, \qquad (53)$$

$$I_{sm} = \frac{U_{iz.m}}{L_{p\Sigma}}\frac{\tau_{p\Sigma}\tau_{s\Sigma}}{(\tau_{p\Sigma}+\tau_{s\Sigma})\omega}\frac{M\omega}{L_{s\Sigma}} = \frac{M}{L_{s\Sigma}}I_{pm}, \qquad (54)$$

$$\varphi_s = \varphi_p + \frac{\pi}{2} \Rightarrow \sin(\theta-\varphi_s) = \cos(\theta-\varphi_p), \qquad (55)$$

$$B_1 = I_{pm}[-\frac{u_{iz.p}(0)}{U_{iz.m}}\frac{L_{p\Sigma}}{M} - \frac{M}{L_{s\Sigma}}\cos(\theta-\varphi_p)], \qquad (56)$$

In the idle mode:

$$R_{s\Sigma} = \infty / L_{s\Sigma} = \infty \Rightarrow i_s = 0 \Rightarrow u_s = -M\frac{di_p}{dt}, \quad (57)$$

$$I_{pm} = \frac{U_{iz.m}}{L_{p\Sigma}} \frac{\tau_{p\Sigma}}{\sqrt{1+(\tau_{p\Sigma}\omega)^2}}, \quad (58)$$

This value also presents the current of magnetization:

$$i_m = i_{10} = I_{pm}\{e^{-\frac{t}{\tau_{p\Sigma}}}[\frac{u_{iz.p}(0)}{U_{iz.m}} - \sin(\theta-\phi_p)] + \sin(\omega t+\theta-\phi_p)\}, \quad (59)$$

$$u_s = MI_{pm}\{\frac{1}{\tau_{p\Sigma}} e^{-\frac{t}{\tau_{p\Sigma}}}[\frac{u_{iz.p}(0)}{U_{iz.m}} - \sin\left(\theta-\phi_p\right)] - \omega\cos(\omega t+\theta-\phi_p)\}, \quad (60)$$

Value of flux from (5) is:

$$\Phi = -\frac{1}{N_2}\int_0^t u_s dt \cdot, \quad (61)$$

In the short-circuit mode conditions are:

$$R_{s\Sigma} \to 0 \Rightarrow -L_s\frac{di_s}{dt} + M\frac{di_s}{dt} = 0 \\ \Rightarrow \frac{i_s}{i_p} = \frac{L_s}{M}, \quad (62)$$

$$i_p = I_{pm}\{e^{-\frac{t}{\tau_{s\Sigma}}}[\frac{u_{iz.p}(0)}{U_{iz.m}} - \sin(\theta-\phi_p)] + \sin(\omega t+\theta-\phi_p)\}, \quad (63)$$

$$i_s = I_{pm}\frac{L_{s\Sigma}}{M}\{e^{-\frac{t}{\tau_{s\Sigma}}}[\frac{u_{iz.p}(0)}{U_{iz.m}} - \sin(\theta-\phi_p)], + \sin(\omega t+\theta-\phi_p)\} \quad (64)$$

2.2 Calculation of The Transformer's Para- Meters

The model of the transformer is accomplished, using the software program MATLAB-SIMULINK in such a way that its inputs are vectors of source voltage and source parameters and windings of the transformer, and outputs are vectors of flux, currents of magnetic biasing and variable equations of voltage value of both primary and secondary.

Simulation and analysis of the transformer's behavior in transient processes was preformed in idle operation and is consisted of the described simulation models of voltages and source parameters and transformers along with blocks which simulate voltages of the transformer.

In accordance with technical practice it is necessary to determine resistance and inductance of windings in the system per unit.

Base values are nominal output $S_n[VA]$, nominal frequency $f_n[Hz]$, and nominal voltages $V_n[V]$ and effective voltage value V_{rms} which corresponds to winding.

The base unit resistance, and base and unit inductance which are used for each winding are:

$$R_{base} = \frac{[V_n]^2}{S_n} \qquad L_{base} = \frac{R_{base}}{2\pi f_n} \\ R(p.u) = \frac{R[\Omega]}{R_{base}[\Omega]} \quad L(p.u) = \frac{L[H]}{L_{base}[H]},$$

The unit value of active resistance of the magnetic circuit R_m is based on indicated power $S_n[VA]$ and on the nominal voltage of the winding 1.

III. Transients in Transformers in MATLAB SIMULINK

If the process is simulated with a residual flux phi0, the second point of a saturation characteristic is on the coordinate and corresponds to zero value of the current, as shown on figure 4.b. The characteristic of saturation is input with (i , phi) values per unit in the system, and with start from couple (0,0). The Power Block (PSB) converts vectors of flux p.u. and vectors of the current I_{pu} into standard units which shall be used in the model of saturation:

$$\Phi = \Phi_{pu}\Phi_{base}, \Phi_{pu} = \Phi / \Phi_{base} \\ I = I_{pu}I_{base}, I_{pu} = I / I_{base},$$

where the base values of flux and current have values which correspond to nominal voltage, power and frequency.

Nominal power and frequency: Nominal apparent power $S_n[VA]$, and frequency $f_n[Hz]$, of transformer.

Winding parameters 1: Nominal effective value of voltage in $[V]$, resistance and drive inductance of winding 1 in p.u.. Winding parameters 2: Nominal effective value of voltage in $[V]$, resistance and drive inductance of winding 2 in p.u.. The characteristic of saturation-denoted as a flow/diagram of current pair (p.u.)-flux (p.u.) which starts from the point (0,0). Active losses in the core of magnetic circuit and initial flux: Active losses of the power in the core which are included into the resistance parameter R_m in p.u.. For example: Denoting loss of active power as 0,2% at nominal voltage nominal value for $R_m = 500$ p.u. also can be denoted as the initial flux phi0 (p.u). The initial flux has special importance in transformer feeding. If phi0 is not denoted, the initial flux is automatically set for simulation of a steady-state start.

The following has been measured: Voltages of windin $U_1 = U_p = U_{N1}(p.u.)$, $U_2 = U_s = U_{N2}(p.u.)$, Currents of winding: I_{N1} , I_{N2} . Magnetizing current: I_{mag} . Flux: Φ .

Outputs and inputs: One input, one output, or three outputs (if they exist) instantly have the same polarity. If on the input, with the third winding equaling 0 is found, then in the block-set it is implemented in the transformer with two windings and magnetic circuit and the display shows an icon which symbolizes a transformer.

Limitations: A winding on the icon can vary. A variable winding is internally and directly connected to the resistance in the circuit, and an invisible connection has no influence on voltage and current measurement. The flux saturation model does not include hysteresis, figure 4.b. The circuit is available in the file psbxfosaturable.mdl file.

3.1 Simulation No. 1

Simulation was performed on the real three-phase power transformer SIMENS-ONAF/ONAN (dimensions: length $7800\,mm$, width $3250\,mm$, height $6100\,mm$, transformer has fans which start to work at higher loadings; oil flow depends only on siphon effect) with the following information: connection Yy, current of idle operation $i_0\% \cong 0{,}5\%$, voltage of short circuit $u_k\% \cong 10{,}5\%$, voltages of primary and secondary $U_{np} = U_{n1} = 71.09[kV]$, $U_{ns} = U_{n2} = 21{,}24[kV]$, $S_n = 100[MVA]$, $P_{Fe} = 49[kW]$, $P_{cun} = 285[kW]$, $V_{np} = V_{n1} = 123 = (110 \pm 8\%)[kV]$, $V_{ns} = V_{n2} = 36{,}75[kV]$.

From the primary side, the reactance of the short circuit is:

$$X_k = \frac{u_k\%}{100}\frac{V_{1n}^2}{S_n} = \frac{10}{100}\frac{123^2 \cdot 10^6}{100 \cdot 10^6} \cong 15.13[\Omega]$$

The ratio of the number of coils of the primary and secondary windings is:

$$k_t = \frac{V_{2n}}{V_{1n}} = \frac{36{,}75}{123} = 0{,}2988 \cong 0{,}3$$

Reactance of the primary and secondary windings are:

$$X_{k1} = \frac{X_k}{2} = \frac{15.13}{2} \cong 7.56[\Omega]$$

$$X_{k2} = \frac{X_k}{2}k_t^2 = \frac{15{,}13}{2}(0{,}3)^2 \cong 0{,}68[\Omega]$$

Dissipative inductances of primary and secondary are:

$$L_1 = L_{\gamma 1} = \frac{X_{k1}}{\omega} = \frac{7.56}{314} \cong 0{,}024[H]$$

$$L_2 = L_{\gamma 2} = \frac{X_{k2}}{\omega} = \frac{0.68}{314} \cong 0.0022[H]$$

Impedance of magnetic biasing is:

$$Z_m \cong X_m = \frac{V_{1n}V_{1n}}{\sqrt{3}\cdot i_0\% I_n V_{1n}} = \frac{V^2{}_{1n}}{i_0 S_n} =$$

$$= \frac{123^2 10^6}{0{,}005 \cdot 100 \cdot 10^6} = 30{,}26[k\Omega]$$

Inductance on the primary side is:

$$L_m = \frac{X_m}{\omega} = \frac{30.26 \cdot 10^3}{314} = 96{,}36[H]$$

Resistance of transformer winding (primary and secondary) is:

$$R_1 = R_p = R_{1k} = \frac{1}{2}\frac{P_{cun}}{3I_{1n}^2} = \frac{1}{2}\frac{P_{cun}}{3 \cdot S_n^2}(\sqrt{3}V_{1n})^2 =$$

$$= \frac{1}{2}\frac{P_{cun}}{S_n^2}V_{1n}^2 = \frac{1}{2}\frac{285 \cdot 10^3}{100^2 10^{12}}123^2 10^6 \cong 0{,}2156[\Omega]$$

$$R_2 = R_s = R_{2k} = \frac{1}{2}\frac{P_{cun}}{3I_{2n}^2} = \frac{1}{2}\frac{P_{cun}}{3 \cdot S_n^2}(\sqrt{3}V_{2n})^2 =$$

$$= \frac{1}{2}\frac{P_{cun}}{S_n^2}V_{2n}^2$$

$$= \frac{1}{2}\frac{285 \cdot 10^3}{100^2 10^{12}}36{,}75^2 \cdot 10^6 \cong 0{,}0194[\Omega]$$

$$R_{base} = \frac{[V_n]^2}{S_n} = 50.43\Omega$$

$$L_{base} = \frac{R_{base}}{2\pi f_n} = \frac{50.43}{314} = 0{,}1606$$

$$\Phi_{base} = \frac{V_1}{2\pi f_n}\sqrt{2} = \frac{123e3/sqrt(3)}{2\pi \cdot 50} sqrt(2) = 3{,}19[Wb]$$

$$I_{base} = \frac{S_n}{V_1}\sqrt{2} = \frac{100e6}{123e3/sqrt(3)} sqrt(2) = 1983[A]$$

The three-phase transformer 100 $[MVA]$, 123/36.75 $[kV]$ is single-phase fed from the resource 50 $[Hz]$. Transformer: Nominal power 100e6,50 $[Hz]$, parameters of winding 1 (primary): 123e3 Vrms/sqrt(3), $R = 0{,}00427$ p.u., $L = 0{,}145$ p.u., parameters of winding 2 (secondary): 36.7e3. Active losses in the magnetic circuit: 1000p.u. Characteristic of saturation: [0 0; 0 1.0; 1.0 1.22], residual flux = 0.6 p.u. is presented as a partially linear curve of flux dependence from the magnetization current

Input values which correspond to dialog box are:

$$R_1 = \frac{0.2156[\Omega]}{50.43.[\Omega]} = 0.004275\, p.u.$$

$$L_1 = \frac{0.024[H]}{0.1606[H]} = 0{,}1494\, p.u.$$

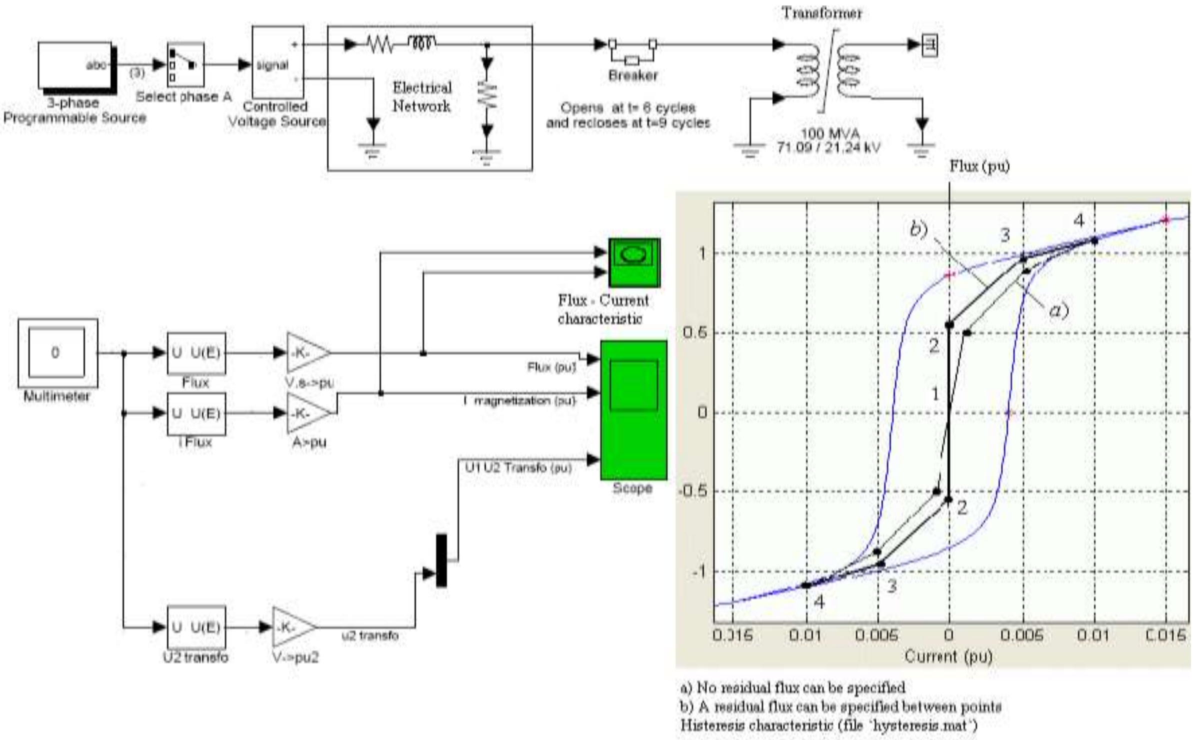

Fig. 4: a) Simulation of idle operation of transformer and b) characteristics of block saturation of saturated transformer

In this simulation process, figure 4 which presents a regime of idle operation of a transformer shows the influence of the characteristic of saturation and time constants of a transformer. In that moment, the values of the flux and magnetization current start to increase which cause the increase of the voltage in the primary and secondary, where the voltage on the secondary transformer can have a higher unit value from the voltage of primary transformer. During the process the time constants of the primary and secondary as well as the nonlinear characteristic of magnetic saturation have the influence. The voltage of the secondary, due to this influence, has a slight increase after a time period of 0,06 *s* and is permanently stable until the end of simulation, figure 5.

3.2 Simulation No. 2

Figure 6: It simulates the short circuit and only controls the current of the secondary set to the nominal current $I_n'' = 232[A]$, for a real three-phase power transformer SIMENS-ONAF (dimensions: length, width, height 1140x800x1700 mm, transformer has fans which start to work at higher loadings; oil flow depends only on siphon-effect) with the following information: connection $Dy5$, current of idle operation $i_0\% \cong 1\%$, voltage of short circuit $u_k\% \cong 3{,}75\%$, voltages of primary and secondary $U_{np} = U_{n1} = 10.5[kV]$, $U_{ns} = U_{n2} = 230[V]$, $S_n = 160[kVA]$, $P_{Fe} = 500[W]$, $P_{cun} = 3200[W]$, $V_{np} = V_{n1} = 10.5[kV]$, $V_{ns} = V_{n2} = 0.4[kV]$.

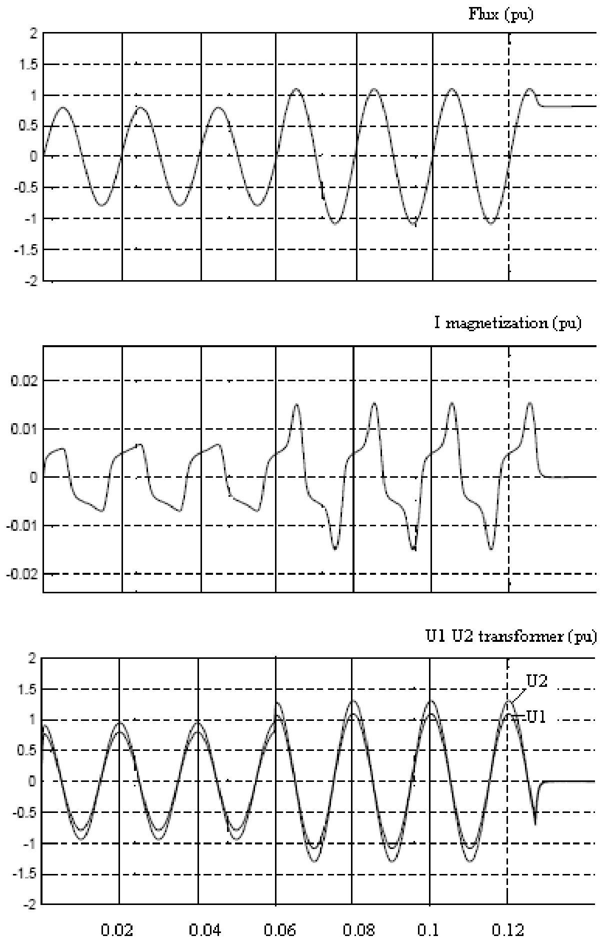

Fig. 5: Results of simulation no. 1: Time flow of flux, magnetization current and voltages of primary and secondary

From the primary side the reactance of short circuit is:

$$X_k = \frac{u_k\%}{100}\frac{V_{1n}^2}{S_n} = \frac{3.75}{100}\frac{10.5^2 \cdot 10^6}{160 \cdot 10^3} \cong 25.8[\Omega]$$

The ratio of number of the coils of the primary and secondary winding is:

$$k_t = \frac{U_{1n}}{U_{2n}} = \frac{10{,}5}{0.230} = 45.652 \cong 46$$

The reactance and dissipative inductance of primary winding is:

$$X_{k1} = \frac{X_k}{2} = \frac{25.8}{2} \cong 12.9[\Omega]$$

$$L_1 = L_{\gamma 1} = \frac{X_{k1}}{\omega} = \frac{12.9}{314} \cong 0{,}043[H]$$

Magnetization impedance is:

$$Z_m \cong X_m = \frac{V_{1n}V_{1n}}{\sqrt{3} \cdot i_0\% I_n V_{1n}} = \frac{V^2{}_{1n}}{i_0 S_n} =$$
$$= \frac{10.5^2 10^6}{0{,}01 \cdot 160 \cdot 10^3} = 68{,}9[k\Omega]$$

Inductance of magnetization from the primary side is:

$$L_m = \frac{X_m}{\omega} = \frac{68.9 \cdot 10^3}{314} = 219{,}36[H]$$

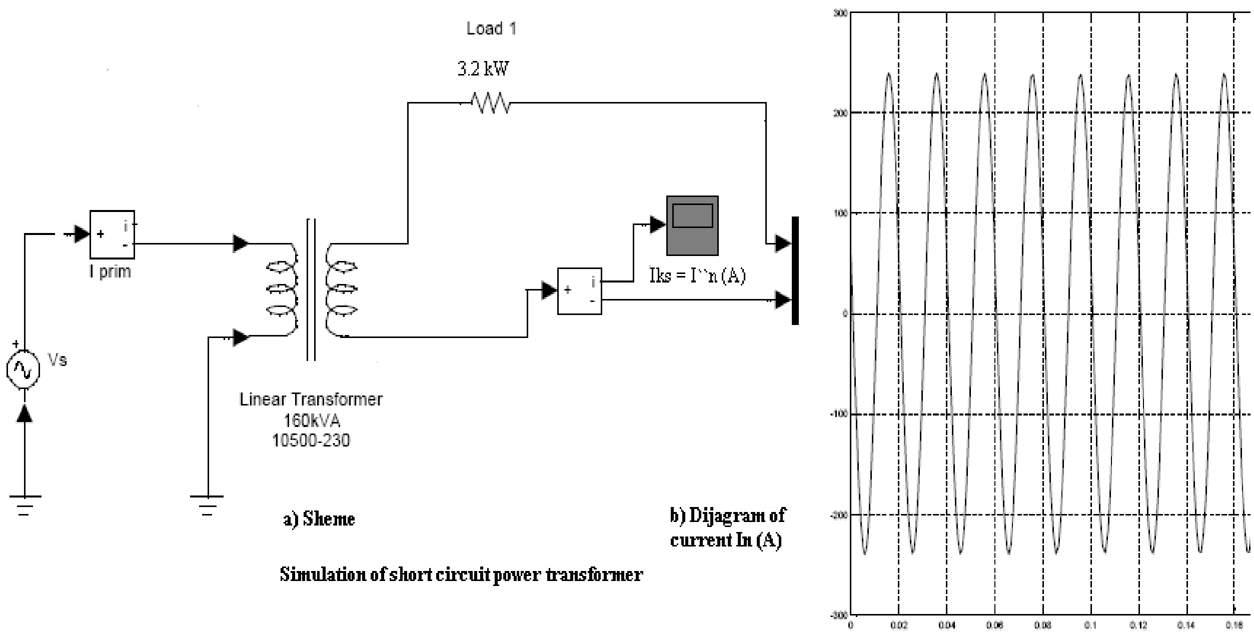

Fig 6: Results of simulation no. 2: Short circuit is made by adjustment of loading in secondary $P_{cun} = 3200[W]$ which was the goal of this simulation

Resistance of transformer's primary winding:

$$R_1 = R_p = R_{1k} = \frac{1}{2}\frac{P_{cun}}{3I_{1n}^2} = \frac{1}{2}\frac{P_{cun}}{S_n^2}V_{1n}^2 =$$

$$= \frac{1}{2}\frac{3200}{160^2 10^6}10.5^2 10^6 \cong 6{,}89[\Omega]$$

$$R_{base} = \frac{[V_n]^2}{S_n} = \frac{10.5^2 \cdot 10^6}{160 \cdot 10^3} = 689[\Omega]$$

$$L_{base} = \frac{R_{base}}{2\pi f_n} = \frac{689}{314} = 2.194[H]$$

For this simulation a three-phase transformer ONAN was used, $Dy5$ 160 $[kVA]$, 12(10.5)/0.4 $[kV]$ is single-phase fed from the resource 50 $[Hz]$. Transformer: Nominal power 250e6,50 $[Hz]$. Parameters of winding 1 (primary): 10.5e3 Vrms, $R = 0{,}01$ p.u., $L = 0{,}02$ p.u., parameters of winding 2 (secondary): 0.4e3 Vrms/sqrt(3), active losses in the magnetic circuit: 50p.u. Characteristic of saturation: [0 0;0 1.0;1.0 1.22], residual flux = 0.6 p.u.

Input values which correspond to dialog box are:

$$R_{1pu} = \frac{6.89[\Omega]}{689[\Omega]} = 0.01 p.u.$$

$$L_{1pu} = \frac{0.043[H]}{2.194[H]} \cong 0{,}02 p.u.$$

The short circuit was simulated with adjusted current of secondary:

$$I_n^{''} = \frac{S_n}{\sqrt{3}V_n} = \frac{160 \cdot 10^3}{\sqrt{3} \cdot 0.4 \cdot 10^3} = 232[A]$$

IV. Conclusion

The application of a computer simulation of dynamic behavior of a power transformer in the software program MATLAB-SIMULINK at transient processes, the validity of a given theoretical model in extreme regimes as idle operation has been confirmed.

Measurement results a) in idle operation- time change of the flux verifies the value of expression (61) time flow of a current of magnetization verifies the value of expression (59), and time flows of voltages of primary and secondary relations (47) and (48) at b) short circuit diagram verifies the value, figure 5.

In the first example of a transformer of high power and a second example of a transformer of low power it has been shown that such a computer simulation in a qualitative way can describe the transient processes and by doing so a better analysis of their behavior can be performed. This paper described dynamic changes in real and ideal power transformer through MATLAB Simulink, which indicates possible malfunction or defect.

References

[1] B. Mitraković: Transformatori, Beograd, 1985.

[2] Datasheet, online: March 10th 2011, http://www.eps.et.tudelft.nl

[3] Van der Sluis, L., Rutgers, W.R., Koreman, C.G.A.: A physical arc model for the simulation of current zero behaviour of high-voltage circuit breakers, *IEEE T-PWRD* 7(2), 1992, p. 1016–1022.

[4] Akademik Antonin Veverka: TECHNIKA VYSOKÝCH NAPĚTÍ SNTL/ALFA, DT621.3027 3:621.315.31:II, *Elektrickě razově jevy ve vinutih transforma torů a točivýh strojů*, Praha, 1982.

[5] Mayer, D.: ŮVOD DO TEORIE ELEKTRICKÝH OBVODŮ, SNTL/ALFA, *16. Nelinearni obvody sa soustrednimi parametry*, Praha, 1978.

[6] Kvasnica, J.: Teorie elektromagnetickeho pole, Kapitola, V.: Kvayistacionárni elektromagneticke polé, *ACADEMIA PRAHA*, 1985.

[7] MATLAB SIMULINK Sim Power System, Copyright 1984-2002 The Math Works, Version 6.5.0,180913a, June 2, 2000.

[8] Bjelić, S., Marković, N. Jakšić, U.: The simplified procedure for calculation of influence of thermal losses on decrease of technical endurance of electric equipment, *III Energy efficiency in application of electricity www.society-of thermal engineering of Serbia*, IEEP `2011, ISBN 978-86-7877-022-7, COBISS.SR-ID 184481804, 2011, Abs. p. 28.

[9] Розенблат М.А.: Магнитные элементы автоматики й вычислительной техники, УДК 538, *Наука*, Москва, 1974, p. 85–101.

[10] Bjelic, S., Markovic N.: Multisliced-level type phase-comparison carrier relaying system for multi-terminal lines, *16th International Telekommunication forum*, TELFOR 2008, Belgrade, Session 7: AEE, work 29, Proceedings, 2008, p. 585.

[11] Bjelic, S., Jaksic, U., Markovic, N.: Informatical Model of the System of Automatic Breakdown Control in Energetic Systems, *ELEKTRONIKA IR ELEKTROTECHNIKA*, Vol. 1, 2011, p. 87–92.

[12] Markovic, N., Bjelic, S., Jaksic, U.: Development of new measuring systems based on symmetric components in electric networks, Submission

ID:37, Journal Arhive Kaunas University Lithuania, ISSN 1392-1215, *ELECTRONICS&ELECTRICAL ENGINEERING*, T 120, No 8(104), 2010, p. 57–62.

[13] Marković, N., Bjelić, S., Jakšić, U.: Calculation of influence of additional losses on decrease of time period of exploitation of power transformer, *Inovacije i razvoj*, Bor, br. 1, 2011, p. 75–86.

[14] Bjelić, S. et al: Estimation of currents flow, loss of power and voltage fall down in distribution network by using PDM KMp.q, *19 International Conference of Electrical Distribution*, Paper 0907, Block 4.2, Method and tools, Cired, Wien, 2007.

[15] Bjelić, S.: Introduction in middle voltage networks and low voltage installations, BOOK, ISBN 978-86-83561-15-5, Cobiss SR-ID 14100 0204 *SVEN*, Nis, 2007.

Authors' Profiles

Slobodan Bjelić: Professor of electrical engineering at Faculty of Technical Sciences in Kosovska Mitrovica, Serbia

Zorica Bogićević: Post-graduate student for doctor degree in electrical engineering at Faculty of Technical Sciences in Kosovska Mitrovica, Serbia

How to cite this paper: Slobodan Bjelić, Zorica Bogićević,"Computer Simulation of Theoretical Model of Electromagnetic Transient Processes in Power Transformers", International Journal of Information Technology and Computer Science(IJITCS), vol.6, no.1, pp.1-12, 2014. DOI: 10.5815/ijitcs.2014.01.01

