

New RLS Wiener Smoother for Colored Observation Noise in Linear Discrete-time Stochastic Systems

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Abstract— In the estimation problems, rather than the white observation noise, there are cases where the observation noise is modeled by the colored noise process. In the observation equation, the observed value $y(k)$ is given as a sum of the signal $z(k) = Hx(k)$ and the colored observation noise $v_c(k)$. In this paper, the observation equation is converted to the new observation equation for the white observation noise. In accordance with the observation equation for the white observation noise, this paper proposes new RLS Wiener estimation algorithms for the fixed-point smoothing and filtering estimates in linear discrete-time wide-sense stationary stochastic systems. The RLS Wiener estimators require the following information: (a) the system matrix for the state vector $x(k)$; (b) the observation matrix H ; (c) the variance of the state vector $x(k)$; (d) the system matrix for the colored observation noise $v_c(k)$; (e) the variance of the colored observation noise.

Index Terms— Discrete-Time Stochastic Systems; RLS Wiener Fixed-Point Smoother; Colored Observation Noise; Covariance Information; Filter

I. Introduction

In comparison with the Kalman estimators, the recursive least-squares (RLS) Wiener estimators have advantages on the point that the RLS Wiener estimators do not require the information of the input noise variance and the input matrix in the state equation. The less information used in the estimators might lead to avoid the degradation of the estimation accuracy instead of using the inaccurate information in the state-space model. In [1], the RLS Wiener filter and fixed-point smoother are developed in linear discrete-time stochastic systems. The estimators require the information of the system matrix, the observation vector, the variance of the state vector in the state equation for the signal and the variance of white observation noise. Also, the Chandrasekhar-type RLS Wiener fixed-point smoother, filter and predictor [2], the square-root RLS

Wiener fixed-point smoother and filter [3] and the RLS Wiener FIR filter [4], etc. are devised in linear discrete-time stochastic systems.

Although almost estimators are designed for the white observation noise, there might be actual cases where the observation noise is colored. The estimation problem for the colored observation noise has been treated in the detection and estimation problems for communication systems [5]-[11]. In [9], an alternative method is proposed with regards to the traditional handling of the autoregressive colored observation noise in Kalman filter based speech enhancement algorithm. In [11], based on the autoregressive moving average (ARMA) innovation model, the reduced-order Wiener state estimators are proposed for descriptor system with MA colored observation noise and multi-observation lags.

In [12], the estimation problem of the signal for the white observation noise is considered in linear continuous-time stochastic systems. The spectral factorization method, for the system matrix, the input matrix and the observation matrix, is discussed. Then the innovations state-space model for the colored observation model is developed.

In [13], based on the improved least-squares (ILS) method, the parameter estimation technique is proposed for the signal observed with additional colored noise. In [14], in order to diagnose faults, the optimal state filter is designed for stochastic systems subject to both colored observation noise and unknown inputs.

In [15], starting with the Wiener-Hopf equation, the RLS Wiener fixed-point smoother and filter are presented for the colored observation noise. In [16], with the relation of estimation technique to the innovation theory [17], [18], assuming that the smoothing estimate is given as a linear transformation of the innovation process, the RLS Wiener fixed-interval smoothing algorithm for the colored observation noise, is proposed.

This paper considers on the RLS Wiener estimation problems for the colored observation noise. In the observation equation, the observed value $y(k)$ is given as a sum of the signal $z(k) = Hx(k)$ and the colored

observation noise $v_c(k)$. In this paper, according to the method in [7], [8], the observation equation is converted to the new observation equation for the white observation noise. In accordance with the observation equation for the white observation noise, this paper proposes new RLS Wiener estimation algorithms for the fixed-point smoothing and filtering estimates in linear discrete-time wide-sense stationary stochastic systems. The RLS Wiener estimators require the following information: (a) the system matrix for the state vector $x(k)$; (b) the observation matrix H ; (c) the variance of the state vector $x(k)$; (d) the system matrix for the colored observation noise $v_c(k)$; (e) the variance of the colored observation noise.

The remainder of this paper is organized as follows. In section 2, the least-squares fixed-point smoothing problem is introduced for the colored observation noise. In section 3, the RLS Wiener fixed-point smoothing and filtering algorithms are presented. Also, the variance of the innovation process is formulated with regard to the current RLS Wiener filter. In a numerical simulation example of section 4, the estimation characteristics of the proposed RLS Wiener fixed-point smoother and filter are compared with those of [15].

II. RLS Wiener Smoothing Problem

Let an m -dimensional observation equation be specified by

$$y(k) = z(k) + v_c(k), \quad z(k) = Hx(k) \quad (1)$$

in linear discrete-time stochastic systems. Here, H is an $m \times n$ observation matrix, $z(k)$ is the zero-mean signal vector. The process $\{v_c(k), k \geq 0\}$ represents the zero-mean colored observation noise sequence. It is assumed that the signal is uncorrelated with the colored observation noise as

$$E[z(k)v_c^T(s)] = 0, \quad 0 \leq k, s < \infty. \quad (2)$$

Let $K_x(k, s) = K_x(k - s)$ represent the auto-covariance function of the state vector $x(k)$ in wide-sense stationary stochastic systems [1], and let $K_x(k, s)$ be expressed in the form of

$$K_x(k, s) = \begin{cases} A(k)B^T(s), & 0 \leq s \leq k, \\ B(s)A^T(k), & 0 \leq k \leq s, \end{cases} \quad (3)$$

$$A(k) = \Phi^k, \quad B^T(s) = \Phi^{-s}K_x(s, s).$$

Here, Φ is the transition matrix of $x(k)$.

Let the state-space model for $x(k)$ be described as

$$x(k+1) = \Phi x(k) + Gw(k),$$

$$E[w(k)w^T(s)] = R_w(k)\delta_K(k-s), \quad (4)$$

where G is an $n \times l$ input matrix and $w(k)$ is the white input noise with the auto-covariance function of (4). It is assumed that the stochastic processes of $w(\cdot)$ and $u(\cdot)$ are mutually uncorrelated.

Let $K_c(\cdot, \cdot)$ denote the auto-covariance function of $v_c(k)$. The auto-covariance function $K_c(k, s)$ is given by

$$K_c(k, s) = \begin{cases} A_c(k)B_c^T(s), & 0 \leq s \leq k, \\ B_c(s)A_c^T(k), & 0 \leq k \leq s, \end{cases} \quad (5)$$

$$A_c(k) = \Phi_c^k, \quad B_c^T(s) = \Phi_c^{-s}K_c(s, s).$$

Let the state equation for $v_c(k)$ be given by

$$v_c(k+1) = \Phi_c v_c(k) + u(k), \quad (6)$$

$$E[u(k)u^T(s)] = R_u(k)\delta_K(k-s),$$

in terms of the white input noise vector $u(k)$ with the variance R_u . It is found that, for the expressions $K_c(k+1, k+1) = E[v_c(k+1)v_c^T(k+1)]$, $K_c(k, k) = E[v_c(k)v_c^T(k)]$, in the wide-sense stationary stochastic systems, the following relationships hold.

$$R_u(k) = K_c(k+1, k+1) - \Phi_c K_c(k, k)\Phi_c^T, \quad (7)$$

$$K_c(k+1, k+1) = K_c(k, k) = K_c(0)$$

In the estimation problem for the colored observation noise, the method of transforming the observation equation into the form of the white observation noise is used [7], [8]. Namely, by introducing

$$\bar{y}(k) = y(k+1) - \Phi_c y(k),$$

from (1), the following relationship is obtained.

$$\begin{aligned} \bar{y}(k) &= Hx(k+1) + v_c(k+1) - \Phi_c(Hx(k) + v_c(k)) \\ &= (H\Phi - \Phi_c H)x(k) + (HGw(k) + u(k)) \\ &= \bar{H}x(k) + \bar{v}(k), \end{aligned} \quad (8)$$

$$\bar{H} = H\Phi - \Phi_c H, \quad \bar{v}(k) = HGw(k) + u(k).$$

Since $w(k)$ and $u(k)$ are the white noise processes, which are mutually independent, $\bar{v}(k)$ is regarded as the

white observation noise in the transformed observation equation (8).

Based on the transformed observation equation (8) from the observed values $y(k+1)$ and $y(k)$, let the fixed-point smoothing estimate $\hat{x}(k, L)$ of $x(k)$ be given by

$$\hat{x}(k, L) = \sum_{i=1}^L h(k, i, L) \bar{y}(i), \quad (9)$$

in terms of the linear transformation of the data $\{\bar{y}(i), 1 \leq i \leq L\}$. Here, $h(k, i, L)$ is called the impulse response function.

Let us consider the least-squares estimation problem, which minimizes the mean-square value (MSV)

$$J = E[\|x(k) - \hat{x}(k, L)\|^2] \quad (10)$$

of the fixed-point smoothing error. From an orthogonal projection lemma [19],

$$x(k) - \sum_{i=1}^L h(k, i, L) \bar{y}(i) \perp \bar{y}(s), \quad 1 \leq s \leq L, \quad (11)$$

the impulse response function satisfies the Wiener-Hopf equation

$$E[x(k) \bar{y}^T(s)] = \sum_{i=1}^L h(k, i, L) E[\bar{y}(i) \bar{y}^T(s)]. \quad (12)$$

Here ' \perp ' denotes the notation of the orthogonality. The left hand side of (12) is developed as

$$\begin{aligned} K_{x\bar{y}}(k, s) &= E[x(k) \bar{y}^T(s)] \\ &= E[x(k)(\bar{H}x(s) + \bar{v}(s))^T] \\ &= K_x(k, s) \bar{H}^T + E[x(k)(HGw(s) + u(s))^T] \quad (13) \\ &= K_x(k, s) \bar{H}^T + E[x(k)w^T(s)]G^T H^T \\ &= K_x(k, s) \bar{H}^T + K_{xw}(k, s)G^T H^T. \end{aligned}$$

Here, $K_{xw}(k, s) = E[x(k)w^T(s)]$ represents the cross-covariance function of $x(k)$ with $w(s)$. Also, the auto-covariance function $K_y(i, s) = E[\bar{y}(i) \bar{y}^T(s)]$ is written as

$$\begin{aligned} K_y(i, s) &= E[(\bar{H}x(i) + \bar{v}(i))(\bar{H}x(s) + \bar{v}(s))^T] \\ &= E[(\bar{H}x(i) + HGw(i) \\ &\quad + u(i))(\bar{H}x(s) + HGw(s) + u(s))^T] \quad (14) \\ &= \bar{H}K_x(i, s) \bar{H}^T + \bar{H}K_{xw}(i, s)G^T H^T \\ &\quad + HGK_{wx}(i, s) \bar{H}^T \\ &\quad + (R_u + HGR_w(i)G^T H^T) \delta_K(i - s), \end{aligned}$$

where the cross-covariance function $K_{wx}(i, s) = E[w(i)x^T(s)]$ is introduced.

Substitution of (13) and (14) into (12) yields the equation

$$\begin{aligned} h(k, s, L) \bar{R}(s) &= K_x(k, s) \bar{H}^T \\ &\quad + K_{xw}(k, s)G^T H^T \\ &\quad - \sum_{i=1}^L h(k, i, L) [\bar{H}K_x(i, s) \bar{H}^T \\ &\quad + \bar{H}K_{xw}(i, s)G^T H^T + HGK_{wx}(i, s) \bar{H}^T], \quad (15) \end{aligned}$$

$$\bar{R}(s) = R_u(s) + HGR_w(s)G^T H^T,$$

which the optimal impulse response function satisfies.

In section 3, by starting with (15), the RLS Wiener estimation equations for the fixed-point smoothing and filtering estimates are derived via the invariant imbedding method.

III. RLS Wiener Estimation Algorithms

Under the preliminary assumptions on the linear least-squares estimation problem of the signal $z(k)$ in section 2, Theorem 1 presents the RLS Wiener fixed-point smoothing and filtering equations, which use the covariance information of the signal and the colored observation noise.

Theorem 1

Let the auto-covariance function $K_x(k, s)$ of the state vector $x(k)$ be given by (3), let the variance of the colored observation noise $v_c(k)$ be $K_c(k, k)$, let the variance of the white input noise $w(k)$ be $R_w(k)$ and let the variance of $u(k)$ in the state equation (6) for the colored observation noise $v_c(k+1)$ be $R_u(k)$. Then, the RLS Wiener algorithms for the fixed-point smoothing and filtering estimates of the signal $z(k)$ consist of (16)-(30) in linear discrete-time stochastic systems with the wide-sense stationarities.

Fixed-point smoothing estimate of the signal $z(k)$: $\hat{z}(k, L)$

$$\hat{z}(k, L) = H\hat{x}(k, L) \quad (16)$$

Fixed-point smoothing estimate of $x(k)$: $\hat{x}(k, L)$

$$\begin{aligned} \hat{x}(k, L) &= \hat{x}(k, L-1) \\ &\quad + h(k, L, L)(\bar{y}(L) - \bar{H}\Phi\hat{\alpha}_1(L-1, L-1) \\ &\quad - \bar{H}\Phi\hat{\alpha}_2(L-1, L-1)) \quad (17) \end{aligned}$$

Smoother gain: $h(k, L, L)$

$$\begin{aligned}
 h(k, L, L) &= (K_x(k, k)(\Phi^T)^{L-k} \bar{H}^T \\
 &- q_1(k, L-1)\Phi^T \bar{H}^T - q_2(k, L-1)\bar{H}^T) \\
 &\times [\bar{R}(L) + \bar{H}(K_x(L, L) - \Phi S_{11}(L-1)\Phi^T \\
 &- \Phi S_{21}(L-1)\Phi^T) \bar{H}^T \\
 &+ HGR_w^T(L)G^T(\Phi^T)^{-1} \bar{H}^T \\
 &- \bar{H}\Phi S_{12}(L-1)\bar{H}^T \\
 &- \bar{H}\Phi S_{22}(L-1)\bar{H}^T]^{-1}
 \end{aligned} \tag{18}$$

Fixed-point smoothing estimate of $x(k)$: $\hat{x}(k, L)$

$$\begin{aligned}
 \hat{x}(k, L) &= \hat{x}(k, L-1) \\
 &+ h(k, L, L)(\bar{y}(L) \\
 &- \bar{H}\Phi \hat{\alpha}_1(L-1, L-1) \\
 &- \bar{H}\Phi \hat{\alpha}_2(L-1, L-1))
 \end{aligned} \tag{19}$$

$$\begin{aligned}
 q_1(k, L) &= q_1(k, L-1)\Phi^T \\
 &+ h(k, L, L)(\bar{H}K_x(L, L) \\
 &- \bar{H}\Phi S_{11}(L-1)\Phi^T \\
 &- \bar{H}\Phi S_{21}(L-1)\Phi^T),
 \end{aligned} \tag{20}$$

$$q_1(k, k) = S_{11}(k) + S_{21}(k)$$

$$\begin{aligned}
 q_2(k, L) &= q_2(k, L-1)\Phi^T \\
 &+ h(k, L, L)(HGR_w^T(L)G^T \\
 &- \bar{H}\Phi S_{12}(L-1)\Phi^T \\
 &- \bar{H}\Phi S_{22}(L-1)\Phi^T),
 \end{aligned} \tag{21}$$

$$q_2(k, k) = S_{12}(k) + S_{22}(k)$$

Filtering estimate of the signal $z(L)$: $\hat{z}(L, L)$

$$\hat{z}(L, L) = H\hat{x}(L, L) \tag{22}$$

Filtering estimate of $x(L)$: $\hat{x}(L, L)$

$$\begin{aligned}
 \hat{x}(L, L) &= \Phi \hat{x}(L-1, L-1) \\
 &+ (G_1(L, L) + G_2(L, L))(\bar{y}(L) \\
 &- \bar{H}\Phi \hat{x}(L-1, L-1)),
 \end{aligned} \tag{23}$$

$$\hat{x}(0, 0) = 0$$

$$\begin{aligned}
 G_1(L, L) &= (K_x(L, L)\bar{H}^T \\
 &- \Phi S_{11}(L-1)\Phi^T \bar{H}^T \\
 &- \Phi S_{12}(L-1)\bar{H}^T) \\
 &\times [\bar{R}(L) + \bar{H}(K_x(L, L) \\
 &- \Phi S_{11}(L-1)\Phi^T \\
 &- \Phi S_{21}(L-1)\Phi^T) \bar{H}^T \\
 &+ HGR_w^T(L)G^T(\Phi^T)^{-1} \bar{H}^T \\
 &- \bar{H}\Phi S_{12}(L-1)\bar{H}^T \\
 &- \bar{H}\Phi S_{22}(L-1)\bar{H}^T]^{-1}
 \end{aligned} \tag{24}$$

$$\begin{aligned}
 G_2(L, L) &= (\Phi^{-1}GR_w^T G^T H^T \\
 &- \Phi S_{21}(L-1)\Phi^T \bar{H}^T \\
 &- \Phi S_{22}(L-1)\bar{H}^T) \\
 &\times [\bar{R}(L) + \bar{H}(K_x(L, L) \\
 &- \Phi S_{11}(L-1)\Phi^T \\
 &- \Phi S_{21}(L-1)\Phi^T) \bar{H}^T \\
 &+ HGR_w^T(L)G^T(\Phi^T)^{-1} \bar{H}^T \\
 &- \bar{H}\Phi S_{12}(L-1)\bar{H}^T \\
 &- \bar{H}\Phi S_{22}(L-1)\bar{H}^T]^{-1}
 \end{aligned} \tag{25}$$

$$\begin{aligned}
 S_{11}(L) &= \Phi S_{11}(L-1)\Phi^T \\
 &+ G_1(L, L)(\bar{H}K_x(L, L) \\
 &- \bar{H}\Phi S_{11}(L-1)\Phi^T \\
 &- \bar{H}\Phi S_{21}(L-1)\Phi^T),
 \end{aligned} \tag{26}$$

$$S_{11}(0) = 0$$

$$\begin{aligned}
 S_{12}(L) &= \Phi S_{12}(L-1)\Phi^T \\
 &+ G_1(L, L)(HGR_w^T(L)G^T \\
 &- \bar{H}\Phi S_{12}(L-1)\Phi^T \\
 &- \bar{H}\Phi S_{22}(L-1)\Phi^T),
 \end{aligned} \tag{27}$$

$$S_{12}(0) = 0$$

$$\begin{aligned}
 S_{21}(L) &= \Phi S_{21}(L-1)\Phi^T \\
 &+ G_2(L, L)(\bar{H}K_x(L, L) \\
 &- \bar{H}\Phi S_{11}(L-1)\Phi^T \\
 &- \bar{H}\Phi S_{21}(L-1)\Phi^T),
 \end{aligned} \tag{28}$$

$$S_{21}(0) = 0$$

$$\begin{aligned}
 S_{22}(L) &= \Phi S_{22}(L-1)\Phi^T \\
 &+ G_2(L, L)(HGR_w^T(L)G^T \\
 &\quad - \bar{H}\Phi S_{12}(L-1)\Phi^T \\
 &\quad - \bar{H}\Phi S_{22}(L-1)\Phi^T), \\
 S_{22}(0) &= 0
 \end{aligned} \tag{29}$$

$$\begin{aligned}
 \bar{R}(L) &= R_u(L) + HGR_w(L)G^T H^T, \\
 R_u(L) &= K_c(L+1, L+1) - \Phi_c K_c(L, L)\Phi_c^T \\
 &= K_c(0) - \Phi_c K_c(0)\Phi_c^T, \\
 GR_w(L)G^T &= K_x(L+1, L+1) - \Phi K_x(L, L)\Phi^T \\
 &= K_x(0) - \Phi K_x(0)\Phi^T, \\
 \bar{H} &= H\Phi - \Phi_c H
 \end{aligned} \tag{30}$$

Proof of Theorem 1 is deferred to the appendix.

From (22) and (23), the variance of the innovation process $\bar{v}(L) = \bar{y}(L) - \bar{H}\Phi\hat{x}(L-1, L-1)$ is given by

$$\begin{aligned}
 \Pi(L) &= \bar{R}(L) + \bar{H}(K_x(L, L) \\
 &\quad - \Phi S_{11}(L-1)\Phi^T \\
 &\quad - \Phi S_{21}(L-1)\Phi^T)\bar{H}^T \\
 &+ HGR_w^T(L)G^T(\Phi^T)^{-1}\bar{H}^T \\
 &\quad - \bar{H}\Phi S_{12}(L-1)\bar{H}^T \\
 &\quad - \bar{H}\Phi S_{22}(L-1)\bar{H}^T.
 \end{aligned} \tag{31}$$

IV. A Numerical Simulation Example

In this section, to show the efficiency of the estimation characteristics of the proposed RLS Wiener fixed-interval smoother and filter, a numerical example is demonstrated.

Let a scalar observation equation be specified by

$$y(k) = z(k) + v_c(k), \quad z(k) = Hx(k). \tag{32}$$

Here, $v_c(k)$ is the zero-mean colored observation noise. Let the signal $z(k)$ be generated by the second-order AR model.

$$\begin{aligned}
 z(k+1) &= -a_1 z(k) - a_2 z(k-1) + w(k), \\
 E[w(k)w(s)] &= \sigma^2 \delta_K(k-s), \\
 a_1 &= -0.1, \quad a_2 = -0.8, \quad \sigma = 0.5.
 \end{aligned} \tag{33}$$

The corresponding state-space model for $z(k)$ can be written as

$$\begin{aligned}
 z(k) &= Hx(k) = x_1(k), \quad H = [1 \ 0], \\
 x(k) &= \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \\
 \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \\
 &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(k), \\
 E[w(k)w(s)] &= R_u \delta_K(k-s).
 \end{aligned} \tag{34}$$

The auto-covariance function of the signal $z(k)$ is given by [15]

$$\begin{aligned}
 K(0) &= \sigma^2 \\
 K(m) &= \sigma^2 \left\{ \frac{\alpha_1(\alpha_2^2 - 1)\alpha_1^m}{[(\alpha_2 - \alpha_1)(\alpha_1\alpha_2 + 1)]} \right. \\
 &\quad \left. - \frac{\alpha_2(\alpha_1^2 - 1)\alpha_2^m}{[(\alpha_2 - \alpha_1)(\alpha_1\alpha_2 + 1)]} \right\},
 \end{aligned} \tag{35}$$

$m > 0,$

$$\alpha_1, \alpha_2 = (-a_1 \pm \sqrt{a_1^2 - 4a_2}) / 2.$$

From (34) and (35), it is found that

$$\begin{aligned}
 K_x(k, k) &= \begin{bmatrix} K(0) & K(1) \\ K(1) & K(0) \end{bmatrix}, \\
 \Phi &= \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix},
 \end{aligned} \tag{36}$$

$$K(0) = 0.25, \quad K(1) = 0.125.$$

Let the state equation for $v_c(k)$ be given by

$$\begin{aligned}
 v_c(k+1) &= \Phi_c v_c(k) + u(k), \\
 E[u(k)u^T(s)] &= R_u \delta_K(k-s), \\
 \Phi_c &= 0.91, \quad R_u = 0.01,
 \end{aligned} \tag{37}$$

where $u(k)$ is the white input noise in the state equation (37) for the colored observation noise process. The auto-variance function of the colored observation noise $v_c(\cdot)$ satisfies the relationships $K_c(k+1, k+1) = K_c(k, k) = K_c(0)$ and hence $K_c(0) = \frac{R_u}{1-a^2}$ in wide-sense stationary stochastic systems.

Substituting $H, \Phi, K_x(L, L) = K_x(0), \Phi_c,$

$K_c(L, L) = K_c(0)$ and R_u into the RLS Wiener estimation algorithms of Theorem 1, the fixed-point smoothing and filtering estimates are calculated recursively.

Fig. 1 illustrates the signal $z(k)$ and the fixed-point smoothing estimate $\hat{z}(k, k+5)$ vs. $k, 1 \leq k \leq 250$, for the input noise variance $R_u = 0.01$ in (37). Fig. 2 illustrates the MSVs of the filtering errors $z(k) - \hat{z}(k, k)$ and the fixed-point smoothing errors $z(k) - \hat{z}(k, k + Lag), 1 \leq Lag \leq 10$, for $R_u = 0.0225, 0.01, 0.0225$. The MSVs of the estimation errors by the filtering and fixed-point smoothing algorithms in Theorem 1 are compared with those in the previous estimators [15]. For $Lag = 0$, the MSV of the filtering errors $z(k) - \hat{z}(k, k), 1 \leq k \leq 2000$, is plotted. From Fig. 2, it

is indicated that the estimation accuracy of the proposed filter and the fixed-point smoother is superior to that of the estimators in [15]. For $R_u = 0.01, 0.0225$, in comparison with the MSVs of the filtering errors, the MSVs of the fixed-point smoothing errors are improved slightly. As the variances R_u of the input noise, in the state equation for the colored observation noise, becomes small, the MSVs of the filtering errors and the fixed-point smoothing errors decrease and the estimation accuracy is improved. Here, the MSVs of the fixed-point smoothing and filtering errors are evaluated by $\sum_{k=1}^{2000} (z(k) - \hat{z}(k, k + Lag))^2 / 2000, 1 \leq Lag \leq 10$, and $\sum_{k=1}^{2000} (z(k) - \hat{z}(k, k))^2 / 2000.$

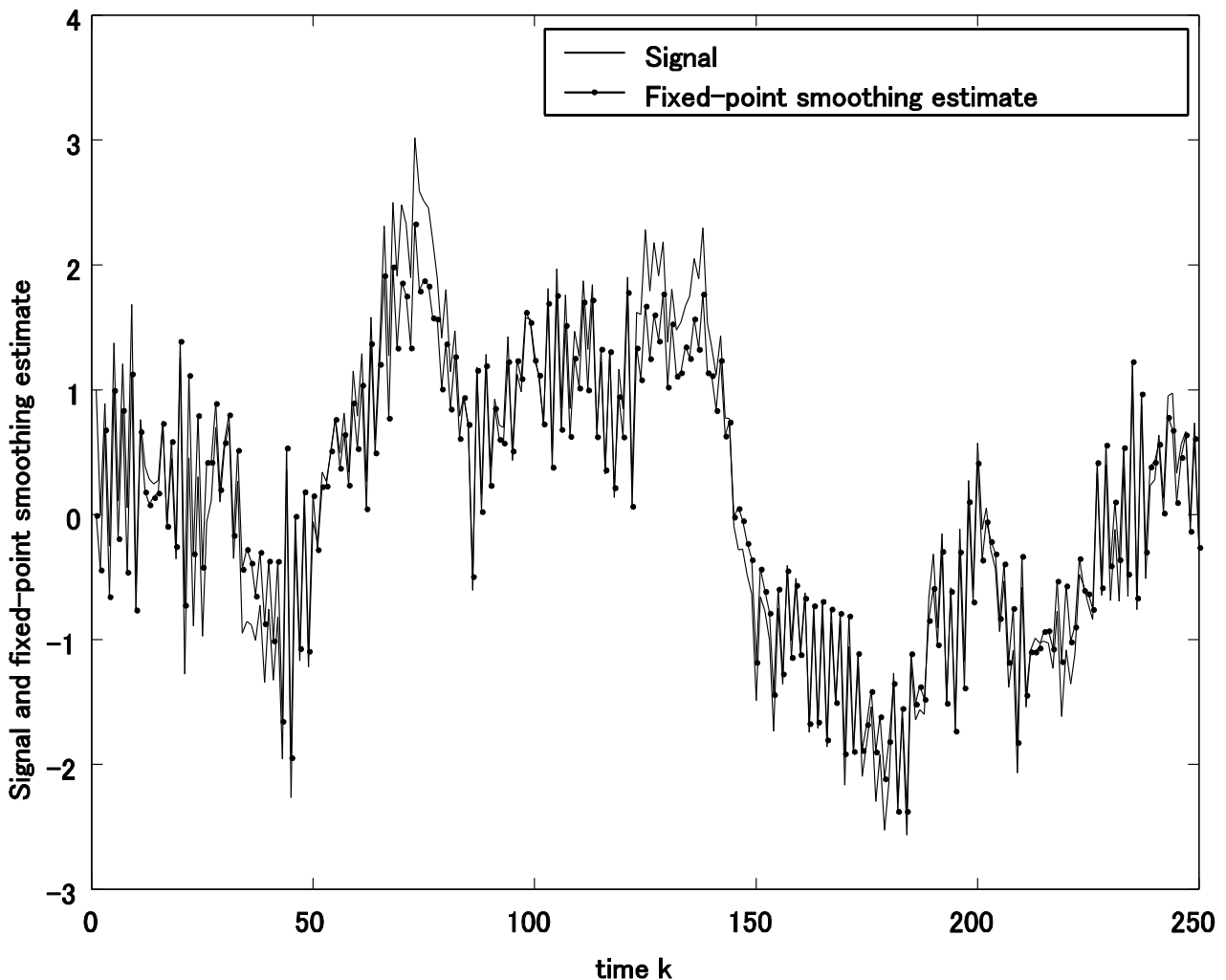


Fig. 1: Signal $z(k)$ and fixed-point smoothing estimate $\hat{z}(k, k+5)$ vs. $k, 1 \leq k \leq 250$, for the input noise variance $R_u = 0.01$ in (37).

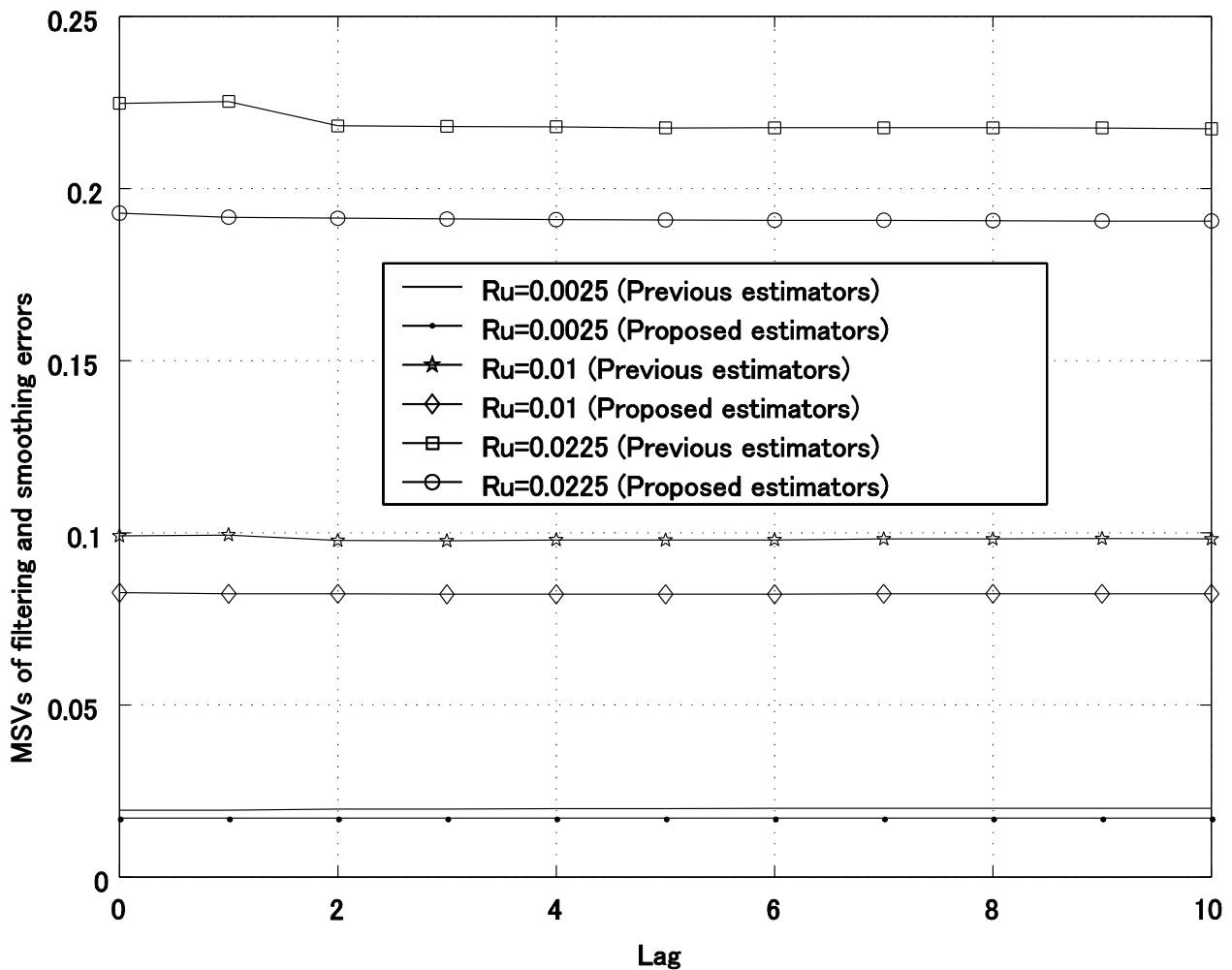


Fig. 2: MSVs of the filtering errors $z(k) - \hat{z}(k, k)$ and the fixed-point smoothing errors $z(k) - \hat{z}(k, k + Lag)$, $1 \leq Lag \leq 10$, for $R_u = 0.0225, 0.01, 0.0225$.

V. Conclusions

In this paper, the RLS Wiener filter and the fixed-point smoother have been designed for the colored observation noise. In particular, the observation equation (1) is converted to the new observation equation (8) by the transformation $\bar{y}(k) = y(k+1) - \Phi_c y(k)$. By this transformation, the observation equation (1) for $y(k)$ with the additive colored observation noise $v_c(k)$ is converted to (8) for $\bar{y}(k)$ with the additive white observation noise $\bar{v}(k)$.

A numerical simulation example has shown that the proposed RLS Wiener algorithms for the fixed-point smoothing and filtering estimates are feasible. From Fig. 2, it is indicated that the estimation accuracy of the proposed filter and the fixed-point smoother is superior to that of the estimators in [15]. For $R_u = 0.01, 0.0225$, in comparison with the MSVs of the filtering errors, the MSVs of the fixed-point smoothing errors are improved slightly.

As in [15], [16], the RLS Wiener estimators do not require the information of the input noise variance $Q(k)$ and the input matrix G in the state equation (4), in comparison with the Kalman estimation technique [19]. In the RLS Wiener estimators, we are not anxious about the degradation in the estimation accuracy caused by the modeling errors for $Q(k)$ and G .

References

- [1] Nakamori S. Recursive estimation technique of signal from output measurement data in linear discrete-time systems [J]. IEICE Trans. Fundamentals, 1995, E-78-A: 600-607.
- [2] Nakamori S. Chandrasekhar-type recursive Wiener estimation technique in linear discrete-time systems [J]. Applied Mathematics and Computation, 2007, 188: 1656-1665.
- [3] Nakamori S. Square-root algorithms of RLS Wiener filter and fixed-point smoother in linear

discrete stochastic systems [J]. Applied Mathematics and Computation, 2008, 203(1): 186-193.

[4] Nakamori S. Design of RLS Wiener FIR filter using covariance information in linear discrete-time stochastic systems [J]. Digital Signal Processing, 2010, 20(5): 1310-1329.

[5] Boll S. Suppression of acoustic noise in speech using spectral subtraction [J]. IEEE Trans. Acoustics, Speech and Signal Processing, 1979, ASSP-27(2): 113-120.

[6] Xiong S. S., Zhou Z. Y. Neural filtering of colored noise based on Kalman filter structure [J]. IEEE Transactions on Instrumentation and Measurement, 2003, 52(3): 742-747.

[7] Bryson A., Henrikson L. Estimation using sampled data containing sequentially correlated noise [J]. J. of Spacecraft and Rockets, 1968, 5(6): 662-665.

[8] Simon D. Optimal state estimation: Kalman, H infinity, and nonlinear approaches [M]. John Wiley & Sons, New Jersey, NJ, 2006.

[9] Must'iere F., Boli'c M., Bouchad M. Improved colored noise handling in Kalman filter-based speech enhancement algorithms [C]. In: Canadian Conference on Electrical Computer Engineering, 2008, CCECE 2008, 497-500.

[10] Park S., Choi S. A constrained sequential EM algorithm for speech enhancement [J]. Neural Networks [J]. 2008, 21: 1401-1409.

[11] Shuli Sun Reduced-order Wiener state estimators for descriptor system with multi-observation lags and MA colored observation noise [C]. In: Control Conference 2008, CCC 2008, 27th Chinese, 2008, 417-420.

[12] Nakamori S. Estimation of signal and parameters using covariance information in linear continuous systems [J]. Mathematical and Computer modeling, 1992, 16(10): 3-15.

[13] Mahmoudi A., Karimi M. Parameter estimation of autoregressive signals from observations corrupted with colored noise [J]. Signal Processing, 2010, 90(1): 157-164.

[14] Sawada Y., Tanikawa A. An improved recursive algorithm of optimal filter for discrete-time linear systems subject to colored observation noise [J]. International J. of Innovative Computing, Information and Control, 2012, 8(3B): 2389-2397.

[15] Nakamori S. Design of RLS Wiener smoother and filter for colored observation noise in linear discrete-time stochastic systems [J]. J. of Signal and Information Processing, 2012, 3(3): 316-329.

[16] Nakamori S. RLS Wiener smoother for colored observation noise with relation to innovation theory in linear discrete-time stochastic systems [J].

I. J. Information Technique and Computer Science, 2013, 5(3): 1-12.

[17] Kailath T. Lectures on Wiener and Kalman filtering [M]. CISM Monographs, No. 140, Springer-Verlag, New York, N Y, 1981.

[18] Kailath T., Frost P. An innovations approach to least-squares estimation Part II: Linear smoothing in additive white noise [J]. IEEE Trans. Automatic Control, 1968, AC-13(6): 655-660.

[19] Sage A. P., Melsa J. L. Estimation theory with applications to communications and control [M]. McGraw-Hill, New York, N Y, 1971.

Appendix Proof of Theorem 1

Let us subtract the equation obtained by putting $L \rightarrow L-1$ in (15) from (15) as follows.

$$\begin{aligned} & (h(k, s, L) - h(k, s, L-1))\bar{R}(s) = -h(k, L, L) \\ & \times (K_x(L, s)\bar{H}^T + K_{xw}(L, s)G^T H^T) \\ & - \sum_{i=1}^L (h(k, i, L) - h(k, i, L-1))[\bar{H}K_x(i, s)\bar{H}^T \\ & + \bar{H}K_{xw}(i, s)G^T H^T + HGK_{wx}(i, s)\bar{H}^T]. \end{aligned} \quad (A-1)$$

By introducing the following equations,

$$\begin{aligned} J_1(s, L-1)\bar{R}(s) &= \Phi^{-s} K_x(s, s)\bar{H}^T \\ & - \sum_{i=1}^{L-1} J_1(i, L-1)[\bar{H}K_x(i, s)\bar{H}^T + \bar{H}K_{xw}(i, s)G^T H^T \\ & + HGK_{wx}(i, s)\bar{H}^T], \end{aligned} \quad (A-2)$$

$$\begin{aligned} J_2(s, L-1)\bar{R}(s) &= \Phi^{-s-1} GR_w(s)G^T H^T \\ & - \sum_{i=1}^{L-1} J_2(i, L-1)[\bar{H}K_x(i, s)\bar{H}^T \\ & + \bar{H}K_{xw}(i, s)G^T H^T + HGK_{wx}(i, s)\bar{H}^T], \end{aligned} \quad (A-3)$$

we notice that the impulse response function satisfies

$$\begin{aligned} & h(k, s, L) - h(k, s, L-1) \\ & = -h(k, L, L)(\bar{H}\Phi^L J_1(s, L-1) \\ & + \bar{H}\Phi^L J_2(s, L-1)). \end{aligned} \quad (A-4)$$

Subtraction of $J_1(s, L-1)$ from $J_1(s, L)$ yields

$$\begin{aligned} & (J_1(s, L) - J_1(s, L-1))\bar{R}(s) = \\ & -J_1(L, L)(\bar{H}K_x(L, s)\bar{H}^T \\ & + \bar{H}K_{xw}(L, s)G^T H^T + HGK_{wx}(L, s)\bar{H}^T) \\ & - \sum_{i=1}^L (J_1(i, L) - J_1(i, L-1))[\bar{H}K_x(i, s)\bar{H}^T \\ & + \bar{H}K_{xw}(i, s)G^T H^T + HGK_{wx}(i, s)\bar{H}^T]. \end{aligned} \quad (A-5)$$

Here, the cross-covariance function of $w(L)$ with $x(s)$ is given by $K_{wx}(L, s) = E[w(L)x^T(s)] = 0$. From (A-2), (A-3) and (A-5), the equation for $J_1(s, L)$ is reduced to

$$\begin{aligned} J_1(s, L) - J_1(s, L-1) = \\ -J_1(L, L)(\bar{H}\Phi^L J_1(s, L-1) \\ + \bar{H}\Phi^L J_2(s, L-1)). \end{aligned} \quad (\text{A-6})$$

Subtraction of $J_2(s, L-1)$ from $J_2(s, L)$ yields

$$\begin{aligned} (J_2(s, L) - J_2(s, L-1))\bar{R}(s) = -J_2(L, L) \\ \times (\bar{H}K_x(L, s)\bar{H}^T + \bar{H}K_{xw}(L, s)G^T H^T) \\ - \sum_{i=1}^L (J_2(i, L) - J_2(i, L-1))[\bar{H}K_x(i, s)\bar{H}^T \\ + \bar{H}K_{xw}(i, s)G^T H^T + HGK_{wx}(i, s)\bar{H}^T]. \end{aligned} \quad (\text{A-7})$$

From (A-2), (A-3) and (A-7), the equation for $J_2(s, L)$ holds as follows.

$$\begin{aligned} J_2(s, L) - J_2(s, L-1) = \\ -J_2(L, L)(\bar{H}\Phi^L J_1(s, L-1) \\ + \bar{H}\Phi^L J_2(s, L-1)) \end{aligned} \quad (\text{A-8})$$

From (A-2), $J_1(L, L)$ satisfies

$$\begin{aligned} J_1(L, L)\bar{R}(L) = \Phi^{-L}K_x(L, L)\bar{H}^T \\ - \sum_{i=1}^L J_1(i, L)[\bar{H}K_x(i, L)\bar{H}^T + \bar{H}K_{xw}(i, L)G^T H^T \\ + HGK_{wx}(i, L)\bar{H}^T]. \end{aligned} \quad (\text{A-9})$$

In terms of the expression of $K_x(i, L)$ in (3), $K_{xw}(i, L) = 0$ and $K_{wx}(i, L) = R_w^T(i)G^T(\Phi^T)^{-i}(\Phi^T)^{L-1}$, (A-9) is written as

$$\begin{aligned} J_1(L, L)\bar{R}(L) = \Phi^{-L}K_x(L, L)\bar{H}^T \\ - \sum_{i=1}^L J_1(i, L)[\bar{H}B(i)A^T(L)\bar{H}^T \\ + HGR_w^T(i)G^T(\Phi^T)^{-i}(\Phi^T)^{L-1}\bar{H}^T]. \end{aligned}$$

Introducing the functions

$$r_{11}(L) = \sum_{i=1}^L J_1(i, L)\bar{H}B(i), \quad (\text{A-10})$$

$$r_{12}(L) = \sum_{i=1}^L J_1(i, L)HGR_w^T(i)G^T(\Phi^T)^{-i}, \quad (\text{A-11})$$

we obtain

$$\begin{aligned} J_1(L, L)\bar{R}(L) = \Phi^{-L}K_x(L, L)\bar{H}^T \\ - r_{11}(L)A^T(L)\bar{H}^T - r_{12}(L)(\Phi^T)^{L-1}\bar{H}^T. \end{aligned} \quad (\text{A-12})$$

From (A-3), $J_2(L, L)$ satisfies

$$\begin{aligned} J_2(L, L)\bar{R}(s) = \Phi^{-L-1}GR_w(L)G^T H^T \\ - \sum_{i=1}^L J_2(i, L)[\bar{H}K_x(i, L)\bar{H}^T \\ + \bar{H}K_{xw}(i, L)G^T H^T \\ + HGK_{wx}(i, L)\bar{H}^T]. \end{aligned} \quad (\text{A-13})$$

In terms of the expression of $K_x(i, L)$ in (3), $K_{xw}(i, L) = 0$ and $K_{wx}(i, L) = R_w^T(i)G^T(\Phi^T)^{-i}(\Phi^T)^{L-1}$, (A-13) is written as

$$\begin{aligned} J_2(L, L)\bar{R}(L) = \Phi^{-L-1}GR_w(L)G^T H^T \\ - \sum_{i=1}^L J_2(i, L)[\bar{H}B(i)A^T(L)\bar{H}^T \\ + HGR_w^T(i)G^T(\Phi^T)^{-i}(\Phi^T)^{L-1}\bar{H}^T]. \end{aligned}$$

Introducing the functions

$$r_{21}(L) = \sum_{i=1}^L J_2(i, L)\bar{H}B(i), \quad (\text{A-14})$$

$$r_{22}(L) = \sum_{i=1}^L J_2(i, L)HGR_w^T(i)G^T(\Phi^T)^{-i}, \quad (\text{A-15})$$

we obtain

$$\begin{aligned} J_2(L, L)\bar{R}(L) = \Phi^{-L-1}GR_w(L)G^T H^T \\ - r_{21}(L)A^T(L)\bar{H}^T - r_{22}(L)(\Phi^T)^{L-1}\bar{H}^T. \end{aligned} \quad (\text{A-16})$$

Subtraction of $r_{11}(L-1)$ from $r_{11}(L)$ yields

$$\begin{aligned} r_{11}(L) - r_{11}(L-1) = J_1(L, L)\bar{H}B(L) \\ + \sum_{i=1}^{L-1} (J_1(i, L) - J_1(i, L-1))\bar{H}B(i). \end{aligned} \quad (\text{A-17})$$

Substituting (A-6) into (A-17), from (A-10) and (A-14), we obtain

$$\begin{aligned} r_{11}(L) - r_{11}(L-1) = \\ J_1(L, L)(\bar{H}B(L) - \bar{H}\Phi^L r_{11}(L-1) \\ - \bar{H}\Phi^L r_{21}(L-1)). \end{aligned} \quad (\text{A-18})$$

Subtraction of $r_{12}(L-1)$ from $r_{12}(L)$ yields

$$\begin{aligned} r_{12}(L) - r_{12}(L-1) = J_1(L, L)HGR_w^T(L)G^T(\Phi^T)^{-L} \\ + \sum_{i=1}^{L-1} (J_1(i, L) - J_1(i, L-1))HGR_w^T(i)G^T(\Phi^T)^{-i}. \end{aligned} \quad (\text{A-19})$$

Substituting (A-6) into (A-19), from (A-11) and (A-15), we obtain

$$r_{12}(L) - r_{12}(L-1) = J_1(L, L)(HGR_w^T(L)G^T(\Phi^T))^{-L} - \bar{H}\Phi^L r_{12}(L-1) - \bar{H}\Phi^L r_{22}(L-1). \quad (\text{A-20})$$

Subtraction of $r_{21}(L-1)$ from $r_{21}(L)$ yields

$$r_{21}(L) - r_{21}(L-1) = J_2(L, L)\bar{H}B(L) + \sum_{i=1}^{L-1} (J_2(i, L) - J_2(i, L-1))\bar{H}B(i). \quad (\text{A-21})$$

Substituting (A-8) into (A-21), from (A-10) and (A-14), we obtain

$$r_{21}(L) - r_{21}(L-1) = J_2(L, L)(\bar{H}B(L) - \bar{H}\Phi^L r_{11}(L-1) - \bar{H}\Phi^L r_{21}(L-1)). \quad (\text{A-22})$$

Subtraction of $r_{22}(L-1)$ from $r_{22}(L)$ yields

$$r_{22}(L) - r_{22}(L-1) = J_2(L, L)HGR_w^T(L)G^T(\Phi^T)^{-L} + \sum_{i=1}^{L-1} (J_2(i, L) - J_2(i, L-1))HGR_w^T(i)G^T(\Phi^T)^{-i}. \quad (\text{A-23})$$

Substituting (A-8) into (A-23), from (A-11) and (A-15), we obtain

$$r_{22}(L) - r_{22}(L-1) = J_2(L, L)(HGR_w^T(L)G^T(\Phi^T))^{-L} - \bar{H}\Phi^L r_{12}(L-1) - \bar{H}\Phi^L r_{22}(L-1). \quad (\text{A-24})$$

Let us introducing the functions $S_{11}(L) = \Phi^L r_{11}(L)(\Phi^T)^L$, $S_{12}(L) = \Phi^L r_{12}(L)(\Phi^T)^L$,

$$S_{21}(L) = \Phi^L r_{21}(L)(\Phi^T)^L, \quad S_{22}(L) = \Phi^L r_{22}(L)(\Phi^T)^L.$$

Substituting (A-18) into $S_{11}(L) = \Phi^L r_{11}(L)(\Phi^T)^L$, we have

$$S_{11}(L) = \Phi^L r_{11}(L-1)(\Phi^T)^L + \Phi^L J_1(L, L)(\bar{H}B(L) - \bar{H}\Phi^L r_{11}(L-1) - \bar{H}\Phi^L r_{21}(L-1))(\Phi^T)^L.$$

Introducing the function $G_1(L, L) = \Phi^L J_1(L, L)$, we obtain (26).

Substituting (A-20) into $S_{12}(L) = \Phi^L r_{12}(L)(\Phi^T)^L$, we have

$$S_{12}(L) = \Phi^L r_{12}(L-1)(\Phi^T)^L + \Phi^L J_1(L, L)(HGR_w^T(L)G^T(\Phi^T))^{-L} - \bar{H}\Phi^L r_{12}(L-1) - \bar{H}\Phi^L r_{22}(L-1)(\Phi^T)^L.$$

Hence, (27) is obtained.

Substituting (A-22) into $S_{21}(L) = \Phi^L r_{21}(L)(\Phi^T)^L$, we have

$$S_{21}(L) = \Phi^L r_{21}(L-1)(\Phi^T)^L + \Phi^L J_2(L, L)(\bar{H}B(L) - \bar{H}\Phi^L r_{11}(L-1) - \bar{H}\Phi^L r_{21}(L-1))(\Phi^T)^L.$$

Introducing the function $G_2(L, L) = \Phi^L J_2(L, L)$, we obtain (28).

Substituting (A-24) into $S_{22}(L) = \Phi^L r_{22}(L)(\Phi^T)^L$, we have

$$S_{22}(L) = \Phi^L r_{22}(L-1)(\Phi^T)^L + \Phi^L J_2(L, L)(HGR_w^T(L)G^T(\Phi^T))^{-L} - \bar{H}\Phi^L r_{12}(L-1) - \bar{H}\Phi^L r_{22}(L-1)(\Phi^T)^L.$$

Hence, (29) is obtained.

Substituting (A-12) into $G_1(L, L) = \Phi^L J_1(L, L)$, we have

$$G_1(L, L) = (K_x(L, L)\bar{H}^T - \Phi^L r_{11}(L)(\Phi^T)^L \bar{H}^T - \Phi^L r_{12}(L)(\Phi^T)^{L-1} \bar{H}^T) \bar{R}^{-1}(L) = (K_x(L, L)\bar{H}^T - S_{11}(L)\bar{H}^T - S_{12}(L)(\Phi^T)^{-1} \bar{H}^T) \bar{R}^{-1}(L). \quad (\text{A-25})$$

Substituting (26) and (27) into (A-25), we obtain (24).

Substituting (A-16) into $G_2(L, L) = \Phi^L J_2(L, L)$, we have

$$G_2(L, L) = (\Phi^{-1}GR_w(L)G^T H^T - \Phi^L r_{21}(L)A^T(L)\bar{H}^T - \Phi^L r_{22}(L)(\Phi^T)^{L-1} \bar{H}^T) \bar{R}^{-1}(L) = (\Phi^{-1}GR_w(L)G^T H^T - S_{21}(L)\bar{H}^T - S_{22}(L)(\Phi^T)^{-1} \bar{H}^T) \bar{R}^{-1}(L). \quad (\text{A-26})$$

Substituting (28) and (29) into (A-26), we obtain (25).

From (15), $h(k, L, L)$ satisfies

$$h(k, L, L)\bar{R}(L) = K_x(k, L)\bar{H}^T + K_{xw}(k, L)G^T H^T - \sum_{i=1}^L h(k, i, L)[\bar{H}K_x(i, L)\bar{H}^T + \bar{H}K_{xw}(i, L)G^T H^T + HGK_{wx}(i, L)\bar{H}^T]. \quad (\text{A-27})$$

Noting that $K_x(k, L) = B(k)A^T(L)$, $K_{xw}(k, L) = 0$, $0 \leq k \leq L$,

$K_{wx}(i, L) = R_w^T(i)G^T(\Phi^T)^{-i}(\Phi^T)^{L-1}$, (A-27) is transformed into

$$\begin{aligned} h(k, L, L)\bar{R}(L) &= B(k)A^T(L)\bar{H}^T \\ &- \sum_{i=1}^L h(k, i, L)[\bar{H}B(i)A^T(L)\bar{H}^T \\ &+ HGR_w^T(i)G^T(\Phi^T)^{-i}(\Phi^T)^{L-1}\bar{H}^T] \\ &= K_x(k, k)(\Phi^T)^{L-k}\bar{H}^T \\ &- P_1(k, L)(\Phi^T)^L\bar{H}^T - P_2(k, L)(\Phi^T)^{L-1}\bar{H}^T. \end{aligned} \quad (A-28)$$

Here, the functions $P_1(k, L)$ and $P_2(k, L)$ are given by

$$P_1(k, L) = \sum_{i=1}^L h(k, i, L)\bar{H}B(i), \quad (A-29)$$

$$P_2(k, L) = \sum_{i=1}^L h(k, i, L)HGR_w^T(i)G^T(\Phi^T)^{-i}. \quad (A-30)$$

Subtraction of $P_1(k, L-1)$ from $P_1(k, L)$ yields, and using (A-4), (A-10) and (A-14), we obtain

$$\begin{aligned} P_1(k, L) - P_1(k, L-1) &= h(k, L, L)\bar{H}B(L) \\ &+ \sum_{i=1}^L (h(k, i, L) - h(k, i, L-1))\bar{H}B(i) \\ &= h(k, L, L)\bar{H}B(L) \\ &- h(k, L, L)\sum_{i=1}^L (\bar{H}\Phi^L J_1(i, L-1) \\ &+ \bar{H}\Phi^L J_2(i, L-1))\bar{H}B(i) \\ &= h(k, L, L)(\bar{H}B(L) \\ &- \bar{H}\Phi^L r_{11}(L-1) - \bar{H}\Phi^L r_{21}(L-1)). \end{aligned} \quad (A-31)$$

Subtraction of $P_2(k, L-1)$ from $P_2(k, L)$ yields, and using (A-4), (A-11) and (A-15), we obtain

$$\begin{aligned} P_2(k, L) - P_2(k, L-1) &= \\ h(k, L, L)HGR_w^T(L)G^T(\Phi^T)^{-L} \\ &+ \sum_{i=1}^L (h(k, i, L) \\ &- h(k, i, L-1))HGR_w^T(i)G^T(\Phi^T)^{-i} \\ &= h(k, L, L)HGR_w^T(L)G^T(\Phi^T)^{-L} \\ &- h(k, L, L)\sum_{i=1}^L (\bar{H}\Phi^L J_1(i, L-1) \\ &+ \bar{H}\Phi^L J_2(i, L-1))HGR_w^T(i)G^T(\Phi^T)^{-i} \\ &= h(k, L, L)(HGR_w^T(L)G^T(\Phi^T)^{-L} \\ &- \bar{H}\Phi^L r_{12}(L-1) \\ &- \bar{H}\Phi^L r_{22}(L-1)). \end{aligned} \quad (A-32)$$

By introducing the functions $q_1(k, L)$ and $q_2(k, L)$ as

$$q_1(k, L) = P_1(k, L)(\Phi^T)^L, \quad (A-33)$$

$$q_2(k, L) = P_2(k, L)(\Phi^T)^L. \quad (A-34)$$

From (A-31), (A-33), together with the expressions of $S_{11}(L) = \Phi^L r_{11}(L)(\Phi^T)^L$ and $S_{21}(L) = \Phi^L r_{21}(L)(\Phi^T)^L$,

(20) is obtained. Similarly, from (A-32), (A-34), together with the expressions of $S_{12}(L) = \Phi^L r_{12}(L)(\Phi^T)^L$ and $S_{22}(L) = \Phi^L r_{22}(L)(\Phi^T)^L$, (21) is obtained.

It is also noticed that $h(k, L, L)$ in (A-28) satisfies

$$\begin{aligned} h(k, L, L)\bar{R}(L) &= K_x(k, k)(\Phi^T)^{L-k}\bar{H}^T \\ &- q_1(k, L)\bar{H}^T \\ &- q_2(k, L)(\Phi^T)^{-1}\bar{H}^T. \end{aligned} \quad (A-35)$$

Substitution of (20) and (21) into (A-35), we obtain (18) for the smoother gain $h(k, L, L)$.

From (A-29), the initial value of the difference equation (20) for $q_1(k, L)$ at $L = k$ is written as follows.

$$\begin{aligned} q_1(k, k) &= P_1(k, k)(\Phi^T)^k \\ &= \sum_{i=1}^k h(k, i, k)\bar{H}B(i)(\Phi^T)^k. \end{aligned} \quad (A-36)$$

From (15), $h(k, s, k)$ satisfies

$$\begin{aligned} h(k, s, k)\bar{R}(k) &= K_x(k, s)\bar{H}^T + K_{xw}(k, s)G^T H^T \\ &- \sum_{i=1}^k h(k, i, k)[\bar{H}K_x(i, s)\bar{H}^T \\ &+ \bar{H}K_{xw}(i, s)G^T H^T + HGK_{wx}(i, s)\bar{H}^T]. \end{aligned}$$

From (A-2) and (A-3), $h(k, s, k)$ is given by

$$h(k, s, k) = \Phi^k J_1(s, k) + \Phi^k J_2(s, k). \quad (A-37)$$

Substituting (A-37) into (A-36) and using (A-10) and (A-14), we obtain

$$q_1(k, k) = S_{11}(k) + S_{21}(k). \quad (A-38)$$

Also, substituting (A-37) into

$$\begin{aligned} q_2(k, k) &= P_2(k, k)(\Phi^T)^k \\ &= \sum_{i=1}^k h(k, i, k)HGR_w^T(i)G^T(\Phi^T)^{-i}(\Phi^T)^k, \end{aligned}$$

from (A-11) and (A-15), we obtain

$$q_2(k, k) = S_{12}(k) + S_{22}(k). \quad (A-39)$$

The filtering estimate $\hat{x}(L, L)$ of $x(L)$ is expressed by

$$\hat{x}(L, L) = \sum_{i=1}^L h(L, i, L) \bar{y}(i). \quad (\text{A-40})$$

Substituting (A-37) into (A-40) and introducing the functions

$$O_1(L) = \sum_{i=1}^L J_1(i, L) \bar{y}(i), \quad (\text{A-41})$$

$$O_2(L) = \sum_{i=1}^L J_2(i, L) \bar{y}(i), \quad (\text{A-42})$$

we have

$$\hat{x}(L, L) = \Phi^L O_1(L) + \Phi^L O_2(L). \quad (\text{A-43})$$

Subtracting $O_1(L-1)$ from $O_1(L)$ and using (A-6), we have

$$\begin{aligned} O_1(L) - O_1(L-1) &= J_1(L, L) \bar{y}(L) \\ &+ \sum_{i=1}^{L-1} (J_1(i, L) - J_1(i, L-1)) \bar{y}(i) \\ &= J_1(L, L) \bar{y}(L) - J_1(L, L) (\bar{H} \Phi^L O_1(L-1) \\ &+ \bar{H} \Phi^L O_2(L-1)). \end{aligned} \quad (\text{A-44})$$

From (A-43), (A-44) is written as

$$\begin{aligned} O_1(L) - O_1(L-1) &= \\ J_1(L, L) (\bar{y}(L) - \bar{H} \Phi \hat{x}(L-1, L-1)). \end{aligned} \quad (\text{A-45})$$

Subtracting $O_2(L-1)$ from $O_2(L)$ and using (A-8) and (A-43), we have

$$\begin{aligned} O_2(L) - O_2(L-1) &= J_2(L, L) \bar{y}(L) \\ &+ \sum_{i=1}^{L-1} (J_2(i, L) - J_2(i, L-1)) \bar{y}(i) \\ &= J_2(L, L) \bar{y}(L) - J_2(L, L) (\bar{H} \Phi^L O_1(L-1) \\ &+ \bar{H} \Phi^L O_2(L-1)) \\ &= J_2(L, L) (\bar{y}(L) - \bar{H} \Phi \hat{x}(L-1, L-1)). \end{aligned} \quad (\text{A-46})$$

Substitution of (A-45) and (A-46) into (A-43) yields

$$\begin{aligned} \hat{x}(L, L) &= \Phi^L (O_1(L-1) + J_1(L, L) (\bar{y}(L) \\ &- \bar{H} \Phi \hat{x}(L-1, L-1)) \\ &+ \Phi^L (O_2(L-1) + J_2(L, L) (\bar{y}(L) \\ &- \bar{H} \Phi \hat{x}(L-1, L-1))) \\ &= \Phi \hat{x}(L-1, L-1) + (G_1(L, L) + G_2(L, L)) (\bar{y}(L) \\ &- \bar{H} \Phi \hat{x}(L-1, L-1)). \end{aligned} \quad (\text{A-47})$$

Subtracting $\hat{x}(k, L-1)$ from the fixed-point smoothing estimate $\hat{x}(k, L)$ in (9) and using (A-4), (A-41)-(A-43), we obtain

$$\begin{aligned} \hat{x}(k, L) - \hat{x}(k, L-1) &= h(k, L, L) \bar{y}(L) \\ &+ \sum_{i=1}^{L-1} (h(k, i, L) - h(k, i, L-1)) \bar{y}(i) \\ &= h(k, L, L) \bar{y}(L) \\ &- h(k, L, L) \sum_{i=1}^{L-1} (\bar{H} \Phi^L J_1(s, L-1) \\ &+ \bar{H} \Phi^L J_2(s, L-1)) \bar{y}(i) \\ &= h(k, L, L) (\bar{y}(L) \\ &- \bar{H} \Phi^L O_1(L-1) - \bar{H} \Phi^L O_2(L-1)) \\ &= h(k, L, L) (\bar{y}(L) - \bar{H} \Phi \hat{x}(L, L-1)). \end{aligned} \quad (\text{A-48})$$

The initial value of (23) for the filtering estimate $\hat{x}(L, L)$ at $L=0$ is $\hat{x}(L, L)=0$ from (A-41)-(A-43). Also, the initial values of (26)-(29) for $S_{11}(L)$, $S_{12}(L)$, $S_{21}(L)$ and $S_{22}(L)$, at $L=0$ are $S_{11}(L)=0$, $S_{12}(L)=0$, $S_{21}(L)=0$ and $S_{22}(L)=0$ from (A-10), (A-11), (A-14) and (A-15).

(Q.E.D.)

Author's Profile



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How to cite this paper: Seichi Nakamori, "New RLS Wiener Smoother for Colored Observation Noise in Linear Discrete-time Stochastic Systems", International Journal of Information Technology and Computer Science(IJITCS), vol.6, no.1, pp.13-24, 2014. DOI: 10.5815/ijitcs.2014.01.02