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A Study on Discrete Model of Three Species Syn-Eco-System with Limited Resources

(One Period Equilibrium States)

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Abstract—In this paper, the system comprises of a commensal (S_1), two hosts S_2 and S_3 ie., S_2 and S_3 both benefit S_1, without getting themselves effected either positively or adversely. Further S_2 is a commensal of S_3, S_3 is a host of both S_1, S_2 and all the three species have limited resources. The basic equations for this model constitute as three first order non-linear ordinary difference equations. All possible equilibrium points are identified based on the model equations and criteria for their stability are discussed. Further the numerical solutions are computed for specific values of the various parameters and the initial conditions.

Index Terms—Commensal, Discrete model, Equilibrium state, Integer, Host, Species, Stable.

AMS Classification: 92D25, 92D40.

I. Introduction

Ecology, a branch evolutionary biology, deals with living species that coexist in a physical environment sustain themselves on common resources. It is a common observations that the species of same nature can not flourish is isolation without any interaction with species of different kinds. Syn-ecology is an ecosystem comprising of two or more distinct species. Species interact with each other in one way or other. The Ecological interactions can be broadly classified as Ammensalism, Competition, Commensalism, Neutralism, Mutualism, Predation Parasitism and so on. Lotka[1] and Volterra [2] pioneered theoretical ecology significantly and opened new eras in the field of life and biological sciences.

Mathematical Modeling is a vital role in providing insight in to the mutual relationships (positive, negative) between the interacting species. The general concepts of modeling have been discussed by several authors, viz., [3-6]. Srinivas[7] studied competitive ecosystem of two species and three species with limited and unlimited resources. Later, Laxminarayan et al [8] studied prey-predator ecological models with partial cover for the prey and alternate food for the predator. Stability analysis of competitive species was carried out by [9-10], while Reddy [11] investigated mutualism between two species. Acharyulu et al [12-13] derived some productive results on various mathematical models of ecological Ammensalism with multifarious resources in the manifold directions. Further Kumar [14] studied some mathematical models of ecological commensalism. The present author Prasad et al [15-22] investigated on the stability of a three and four species syn-ecosystems. Recently, Din et al [23] discussed the basic concepts on stability analysis of discrete ecological models and Kiran et al [24] worked on discrete model of three species eco-system by considering interactions like prey-predator and commensalism.

The present investigation is a discrete model of three species (S_1, S_2, S_3) syn-eco system with limited resources. The system comprises of a commensal (S_1), two hosts S_2 and S_3. Further S_2 is a commensal of S_3, S_3 is a host of both S_1 and S_2. *Commensalism* is a symbiotic interaction between two populations where one population (S_1) gets benefit from (S_2) while the other (S_2) is neither harmed nor benefited due to the interaction with (S_1). The benefited species (S_1) is called the commensal and the other, the helping one (S_2) is called the host species. A common example is an animal using a tree for shelter-tree (host) does not get any benefit from the animal (commensal).

II. Basic Equations Of The Model

The model equations for the three species syn-eco-system is given by the following system of first order non-linear difference equations employing the following notation.

A. Notation Adopted

$N_i(t)$: The population strength of S_i at time t, i = 1, 2, 3
t : Time instant
a_i : Natural growth rate of S_i, i = 1, 2, 3
a_{ii} : Self inhibition coefficient of S_i, i = 1, 2, 3
a_{12}, a_{13} : Interaction coefficients of S_1 due to S_2 and S_1 due to S_3
a_{23} : Interaction coefficient of S_2 due to S_3

Further the variables N_1, N_2, N_3 are non-negative and the model parameters a_1, a_2, a_3, a_{11}, a_{12}, a_{13}, a_{22}, a_{23}, $a_{33}$are assumed to be non-negative constants.

B. Basic equations

Consider the growth of the species during the time interval (t, t + 1)

● *Equation for the first species (N_1):*

$$N_1(t+1) = N_1(t) + a_1 N_1(t) - a_{11} N_1^2(t) + a_{12} N_1(t)\, N_2(t) + a_{13} N_1(t)\, N_3(t) \quad (1)$$

● *Equation for the second species (N_2):*

$$N_2(t+1) = N_2(t) + a_2 N_2(t) - a_{22} N_2^2(t) + a_{23} N_2(t) N_3(t) \quad (2)$$

● *Equation for the third species (N_3):*

$$N_3(t+1) = N_3(t) + a_3 N_3(t) - a_{33} N_3^2(t) \quad (3)$$

● *Species-growth equations in the discrete form:*

Consider the nonlinear autonomous system of discrete equations

$$N_1(t+1) = \alpha_1 N_1(t) - a_{11} N_1^2(t) + a_{12} N_1(t)\, N_2(t) + a_{13} N_1(t)\, N_3(t) \quad (4)$$

$$N_2(t+1) = \alpha_2 N_2(t) - a_{22} N_2^2(t) + a_{23} N_2(t) N_3(t) \quad (5)$$

$$N_3(t+1) = \alpha_3 N_3(t) - a_{33} N_3^2(t) \quad (6)$$

where $\alpha_i = a_i + 1$, i = 1, 2, 3

III. Equilibrium States

For a continuous model the equilibrium states are defined by $\frac{dN_i}{dt} = 0,\ i = 1, 2, 3$, the equilibrium states for a discrete model are defined in terms of the period of no growth. i.e, $N_i(t+r) = N_i(t),\ r = 1, 2, 3, ...$, where r is the period of the equilibrium state.

Kapur J.N, Mathematical Modeling through Discrete Mathematics-Fascinating World of Mathematical Sciences, 1989, Volume-II, pp.71-80, Mathematical Science Trust Society, New Delhi.

● *One period equilibrium states (Stage-I)*

$$N_i(t+1) = N_i(t),\ i = 1, 2, 3$$

The system under investigation has fourteen equilibrium states given by

(i) Fully washed out state

$$E_1 : \bar{N}_1 = 0,\ \bar{N}_2 = 0,\ \bar{N}_3 = 0$$

(ii) States in which only one of the tree species is survives while the other two are not

$$E_2 : \bar{N}_1 = 0,\ \bar{N}_2 = 0,\ \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}},\ \text{when } \alpha_3 > 1$$

$$E_3 : \bar{N}_1 = 0,\ \bar{N}_2 = \frac{\alpha_2 - 1}{a_{22}},\ \bar{N}_3 = 0,\ \text{when } \alpha_2 > 1$$

$$E_4 : \bar{N}_1 = \frac{\alpha_1 - 1}{a_{11}},\ \bar{N}_2 = 0,\ \bar{N}_3 = 0,\ \text{when } \alpha_1 > 1$$

(iii) States in which only two of the tree species are survives while the other one is not

$$E_5 : \bar{N}_1 = 0, \bar{N}_2 = \frac{1}{a_{22}}\left[(\alpha_2 - 1) + a_{23}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right], \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}$$

when $\alpha_2 > 1$ and $\alpha_3 > 1$

$$E_6 : \bar{N}_1 = 0, \bar{N}_2 = a_{23}\left(\frac{\alpha_3 - 1}{a_{22} a_{33}}\right), \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}$$

when $\alpha_2 = 1$ and $\alpha_3 > 1$

$$E_7 : \bar{N}_1 = \frac{1}{a_{11}}\left[(\alpha_1 - 1) + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right], \bar{N}_2 = 0, \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}$$

when $\alpha_1 > 1$ and $\alpha_3 > 1$

$$E_8 : \bar{N}_1 = a_{13}\left(\frac{\alpha_3 - 1}{a_{11} a_{33}}\right), \bar{N}_2 = 0, \bar{N}_3 = \frac{\alpha_3 - 1}{a_{33}}$$

when $\alpha_1 = 1$ and $\alpha_3 > 1$

$$E_9 : \bar{N}_1 = \frac{1}{a_{11}}\left[(\alpha_1 - 1) + a_{12}\left(\frac{\alpha_2 - 1}{a_{22}}\right)\right], \bar{N}_2 = \frac{\alpha_2 - 1}{a_{22}}, \bar{N}_3 = 0$$

when $\alpha_1 > 1$ and $\alpha_2 > 1$

$$E_{10} : \bar{N}_1 = a_{12}\left(\frac{\alpha_2 - 1}{a_{11} a_{22}}\right), \bar{N}_2 = \frac{\alpha_2 - 1}{a_{22}}, \bar{N}_3 = 0$$

when $\alpha_1 = 1$ and $\alpha_2 > 1$

(iv) The co-existent states (or) normal steady states

$$E_{11}:\bar{N}_1=\frac{1}{a_{11}}\left\{(\alpha_1-1)+\frac{a_{12}}{a_{22}}\left[(\alpha_2-1)+a_{23}\left(\frac{\alpha_3-1}{a_{33}}\right)\right]+a_{13}\left(\frac{\alpha_3-1}{a_{33}}\right)\right\},$$

$$\bar{N}_2=\frac{1}{a_{22}}\left[(\alpha_2-1)+a_{23}\left(\frac{\alpha_3-1}{a_{33}}\right)\right],\bar{N}_3=\frac{\alpha_3-1}{a_{33}},$$

when α_1,α_2 and $\alpha_3>1$

$$E_{12}:\bar{N}_1=\frac{1}{a_{11}}\left\{\frac{a_{12}}{a_{22}}\left[(\alpha_2-1)+a_{23}\left(\frac{\alpha_3-1}{a_{33}}\right)\right]+a_{13}\left(\frac{\alpha_3-1}{a_{33}}\right)\right\},$$

$$\bar{N}_2=\frac{1}{a_{22}}\left[(\alpha_2-1)+a_{23}\left(\frac{\alpha_3-1}{a_{33}}\right)\right],\bar{N}_3=\frac{\alpha_3-1}{a_{33}}$$

when $\alpha_1=1$ and $\alpha_2,\alpha_3>1$

$$E_{13}:\bar{N}_1=\frac{1}{a_{11}}\left[(\alpha_1-1)+a_{12}a_{23}\left(\frac{\alpha_3-1}{a_{22}a_{33}}\right)+a_{13}\left(\frac{\alpha_3-1}{a_{33}}\right)\right],$$

$$\bar{N}_2=a_{23}\left(\frac{\alpha_3-1}{a_{22}a_{33}}\right),\bar{N}_3=\frac{\alpha_3-1}{a_{33}}$$

when $\alpha_2=1$ and $\alpha_1,\alpha_3>1$

$$E_{14}:\bar{N}_1=\frac{1}{a_{11}}\left[a_{12}a_{23}\left(\frac{\alpha_3-1}{a_{22}a_{33}}\right)+a_{13}\left(\frac{\alpha_3-1}{a_{33}}\right)\right],$$

$$\bar{N}_2=a_{23}\left(\frac{\alpha_3-1}{a_{22}a_{33}}\right),\bar{N}_3=\frac{\alpha_3-1}{a_{33}}$$

when $\alpha_1,\alpha_2=1$ and $\alpha_3>1$

IV. Stability Analysis

A. Stability of $E_1(0,0,0)$

$$N_1(t)=N_1(t+1)=N_1(t+2)=.................=0$$
$$N_2(t)=N_2(t+1)=N_2(t+2)=.................=0$$
$$N_3(t)=N_3(t+1)=N_3(t+2)=.................=0$$

i.e, $N_i(t+r)=0$, where r is an integer and i = 1, 2, 3
Hence, $E_1(0,0,0)$ is stable.

B. Stability of E_2

$$N_1(t)=N_1(t+1)=N_1(t+2)=.................=0$$
$$N_2(t)=N_2(t+1)=N_2(t+2)=.................=0$$
$$N_3(t)=N_3(t+1)=N_3(t+2)=.................=\frac{\alpha_3-1}{a_{33}}$$

i.e, $N_i(t+r)=0,\ N_3(t+r)=\frac{\alpha_3-1}{a_{33}}$ where r is an integer and i = 1, 2
Hence, E_2 is stable.

C. Stability of E_3

$$N_1(t)=N_1(t+1)=N_1(t+2)=.................=0$$
$$N_2(t)=N_2(t+1)=N_2(t+2)=.................=\frac{\alpha_2-1}{a_{22}}$$
$$N_3(t)=N_3(t+1)=N_3(t+2)=.................=0$$

i.e, $N_i(t+r)=0,\ N_2(t+r)=\frac{\alpha_2-1}{a_{22}}$ where r is an integer and i = 1, 3
Hence, E_3 is stable.

D. Stability of E_4

$$N_1(t)=N_1(t+1)=N_1(t+2)=.................=\frac{\alpha_1-1}{a_{11}}$$
$$N_2(t)=N_2(t+1)=N_2(t+2)=.................=0$$
$$N_3(t)=N_3(t+1)=N_3(t+2)=.................=0$$

i.e, $N_1(t+r)=\frac{\alpha_1-1}{a_{11}},N_i(t+r)=0$ where r is an integer and i = 2, 3
Hence, E_4 is stable.

E. Stability of E_5

$$N_1(t)=N_1(t+1)=N_1(t+2)=.................=0$$
$$N_2(t)=N_2(t+1)=N_2(t+2)=.................=\frac{1}{a_{22}}\left[(\alpha_2-1)+a_{23}\left(\frac{\alpha_3-1}{a_{33}}\right)\right]$$
$$N_3(t)=N_3(t+1)=N_3(t+2)=.................=\frac{\alpha_3-1}{a_{33}}$$

i.e, $N_1(t+r)=0,N_2(t+r)=\frac{1}{a_{22}}\left[(\alpha_2-1)+a_{23}\left(\frac{\alpha_3-1}{a_{33}}\right)\right],$

$N_3(t+r)=\frac{\alpha_3-1}{a_{33}},$ where r is an integer
Hence, E_5 is stable.

F. Stability of E_6

$$N_1(t)=N_1(t+1)=N_1(t+2)=.................=0$$

$$N_2(t) = N_2(t+1) = N_2(t+2) = ... = a_{23}\left(\frac{\alpha_3 - 1}{a_{22}a_{33}}\right)$$

$$N_3(t) = N_3(t+1) = N_3(t+2) = ... = \frac{\alpha_3 - 1}{a_{33}}$$

i.e, $N_1(t+r) = 0, N_2(t+r) = a_{23}\left(\frac{\alpha_3 - 1}{a_{22}a_{33}}\right),$

$N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}$, where r is an integer

Hence, E_6 is stable.

G. Stability of E_7

$$N_1(t) = N_1(t+1) = N_1(t+2) = ... = \frac{1}{a_{11}}\left[(\alpha_1 - 1) + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right]$$

$$N_2(t) = N_2(t+1) = N_2(t+2) = ... = 0$$

$$N_3(t) = N_3(t+1) = N_3(t+2) = ... = \frac{\alpha_3 - 1}{a_{33}}$$

i.e, $N_1(t+r) = \frac{1}{a_{11}}\left[(\alpha_1 - 1) + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right],$

$N_2(t+r) = 0, N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}$, where r is an integer

Hence, E_7 is stable.

H. Stability of E_8

$$N_1(t) = N_1(t+1) = N_1(t+2) = ... = a_{13}\left(\frac{\alpha_3 - 1}{a_{11}a_{33}}\right)$$

$$N_2(t) = N_2(t+1) = N_2(t+2) = ... = 0$$

$$N_3(t) = N_3(t+1) = N_3(t+2) = ... = \frac{\alpha_3 - 1}{a_{33}}$$

i.e, $N_1(t+r) = a_{13}\left(\frac{\alpha_3 - 1}{a_{11}a_{33}}\right), N_2(t+r) = 0,$

$N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}$, where r is an integer

Hence, E_8 is stable.

I. Stability of E_9

$$N_1(t) = N_1(t+1) = N_1(t+2) = ... = \frac{1}{a_{11}}\left[(\alpha_1 - 1) + a_{12}\left(\frac{\alpha_2 - 1}{a_{22}}\right)\right]$$

$$N_2(t) = N_2(t+1) = N_2(t+2) = ... = \frac{\alpha_2 - 1}{a_{22}}$$

$$N_3(t) = N_3(t+1) = N_3(t+2) = ... = 0$$

i.e, $N_1(t+r) = \frac{1}{a_{11}}\left[(\alpha_1 - 1) + a_{12}\left(\frac{\alpha_2 - 1}{a_{22}}\right)\right],$

$N_2(t+r) = \frac{\alpha_2 - 1}{a_{22}}, N_3(t+r) = 0$, where r is an integer

Hence, E_9 is stable.

J. Stability of E_{10}

$$N_1(t) = N_1(t+1) = N_1(t+2) = ... = a_{12}\left(\frac{\alpha_2 - 1}{a_{11}a_{22}}\right)$$

$$N_2(t) = N_2(t+1) = N_2(t+2) = ... = \frac{\alpha_2 - 1}{a_{22}}$$

$$N_3(t) = N_3(t+1) = N_3(t+2) = ... = 0$$

i.e, $N_1(t+r) = a_{12}\left(\frac{\alpha_2 - 1}{a_{11}a_{22}}\right), N_2(t+r) = \frac{\alpha_2 - 1}{a_{22}},$

$N_3(t+r) = 0$, where r is an integer

Hence, E_{10} is stable.

K. Stability of E_{11}

$$N_1(t) = N_1(t+1) = N_1(t+2) = ... = \frac{1}{a_{11}}\left\{(\alpha_1 - 1) + \frac{a_{12}}{a_{22}}\left[(\alpha_2 - 1) + a_{23}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right] + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right\}$$

$$N_2(t) = N_2(t+1) = N_2(t+2) = ... = \frac{1}{a_{22}}\left[(\alpha_2 - 1) + a_{23}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right]$$

$$N_3(t) = N_3(t+1) = N_3(t+2) = ... = \frac{\alpha_3 - 1}{a_{33}}$$

i.e, $N_1(t+r) = \frac{1}{a_{11}}\left\{\begin{array}{l}(\alpha_1 - 1) + \frac{a_{12}}{a_{22}}\left[(\alpha_2 - 1) + a_{23}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right] \\ + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\end{array}\right\},$

$N_2(t+r) = \frac{1}{a_{22}}\left[(\alpha_2 - 1) + a_{23}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right], N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}$

where r is an integer

Hence, E_{11} is stable.

L. Stability of E_{12}

$$N_1(t) = N_1(t+1) = N_1(t+2) = ... = \frac{1}{a_{11}}\left\{\frac{a_{12}}{a_{22}}\left[(\alpha_2 - 1) + a_{23}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right] + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right\}$$

$$N_2(t) = N_2(t+1) = N_2(t+2) = ... = \frac{1}{a_{22}}\left[(\alpha_2 - 1) + a_{23}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right]$$

$$N_3(t) = N_3(t+1) = N_3(t+2) = \dots\dots\dots\dots\dots = \frac{\alpha_3 - 1}{a_{33}}$$

i.e, $N_1(t+r) = \frac{1}{a_{11}}\left\{\frac{a_{12}}{a_{22}}\left[(\alpha_2 - 1) + a_{23}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right] + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right\}$,

$$N_2(t+r) = \frac{1}{a_{22}}\left[(\alpha_2 - 1) + a_{23}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right],\ N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}$$

where r is an integer
Hence, E_{12} is stable.

M. Stability of E_{13}

$$N_1(t) = N_1(t+1) = N_1(t+2) = \dots\dots\dots$$

$$= \frac{1}{a_{11}}\left[(\alpha_1 - 1) + a_{12}a_{23}\left(\frac{\alpha_3 - 1}{a_{22}a_{33}}\right) + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right]$$

$$N_2(t) = N_2(t+1) = N_2(t+2) = \dots\dots\dots = a_{23}\left(\frac{\alpha_3 - 1}{a_{22}a_{33}}\right)$$

$$N_3(t) = N_3(t+1) = N_3(t+2) = \dots\dots\dots\dots\dots = \frac{\alpha_3 - 1}{a_{33}}$$

i.e, $N_1(t+r) = \frac{1}{a_{11}}\left[(\alpha_1 - 1) + a_{12}a_{23}\left(\frac{\alpha_3 - 1}{a_{22}a_{33}}\right) + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right]$,

$$N_2(t+r) = a_{23}\left(\frac{\alpha_3 - 1}{a_{22}a_{33}}\right),\ N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}$$

where r is an integer
Hence, E_{13} is stable.

N. Stability of E_{14}

$$N_1(t) = N_1(t+1) = N_1(t+2) = \dots\dots\dots$$

$$= \frac{1}{a_{11}}\left[a_{12}a_{23}\left(\frac{\alpha_3 - 1}{a_{22}a_{33}}\right) + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right]$$

$$N_2(t) = N_2(t+1) = N_2(t+2) = \dots\dots\dots = a_{23}\left(\frac{\alpha_3 - 1}{a_{22}a_{33}}\right)$$

$$N_3(t) = N_3(t+1) = N_3(t+2) = \dots\dots\dots\dots\dots = \frac{\alpha_3 - 1}{a_{33}}$$

$$N_1(t+r) = \frac{1}{a_{11}}\left[a_{12}a_{23}\left(\frac{\alpha_3 - 1}{a_{22}a_{33}}\right) + a_{13}\left(\frac{\alpha_3 - 1}{a_{33}}\right)\right],$$

$$N_2(t+r) = a_{23}\left(\frac{\alpha_3 - 1}{a_{22}a_{33}}\right),\ N_3(t+r) = \frac{\alpha_3 - 1}{a_{33}}$$

where r is an integer
Hence, E_{14} is stable.

At this stage all the fourteen equilibrium states are stable.

● *Two period equilibrium states (Stage-II)*

$$N_i(t+2) = N_i(t),\ i = 1, 2, 3$$

At this stage the system has 125 equilibrium states. The present paper deals only the stability of one period equilibrium states and the stability of the two period equilibrium states will be presented in the forth coming communications.

V. Numerical Examples

The numerical solutions of the discrete model equations computed for specific values of the various parameters and the initial conditions. The results are illustrated in Figures 1 to 6.

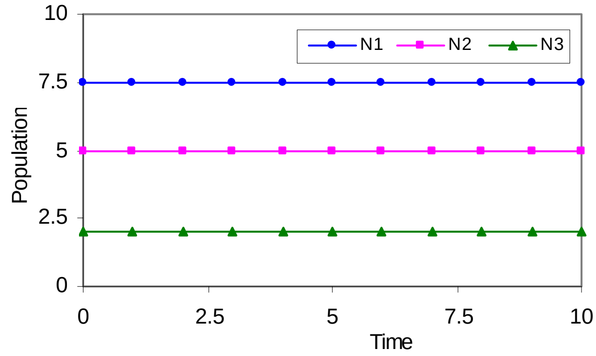

Fig.1. Variation of population against time for α_1=3.9, α_2=2.6, α_3=4.4, a_{11}=1.18, a_{12}=0.6, a_{13}=1.5, a_{22}=0.8, a_{23}=1.2, a_{33}=1.7, N_{10}=7.5, N_{20}=5, N_{30}=2

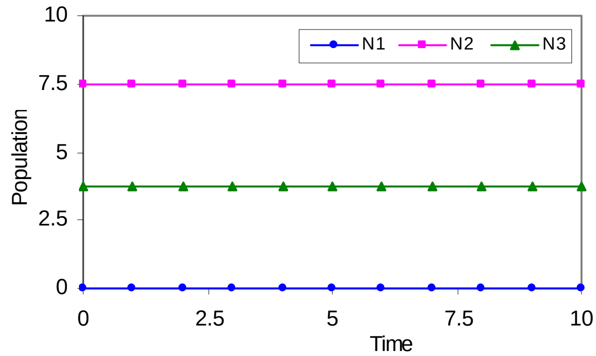

Fig.2. Variation of population against time for α_2=1, α_3=2.5, a_{22}=2.3, a_{23}=4.4, a_{33}=0.4, N_{10}=0, N_{20}=7.5, N_{30}=3.75

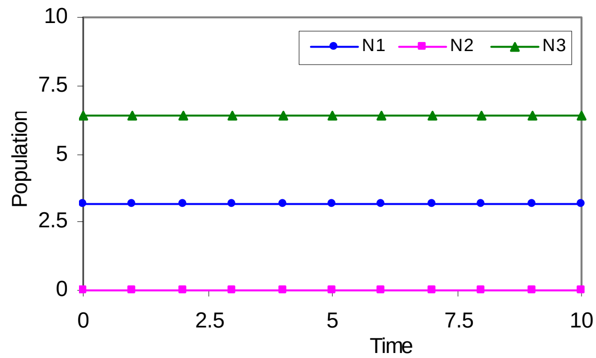

Fig.3. Variation of population against time for α2 =1, α3 =4.2, a11 =3.2, a13 =1.6, a33 =0.5, N10 =3.2, N20 =0, N30 =6.4

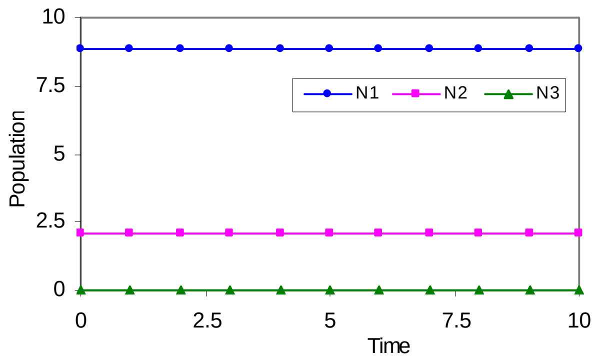

Fig.4 Variation of population against time for α2 =1, α2 =2.7, a11 =1.4, a12 =5.85, a22 =0.8, N10 =8.9, N20 =2.1, N30 =0

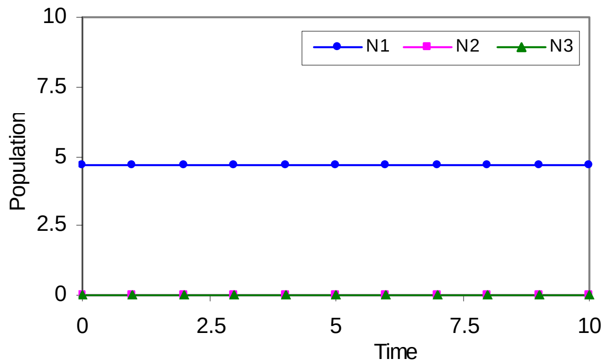

Fig.5. Variation of population against time for α1=3.8, a11 =0.6, N10 =4.7, N20 =0, N30 =0

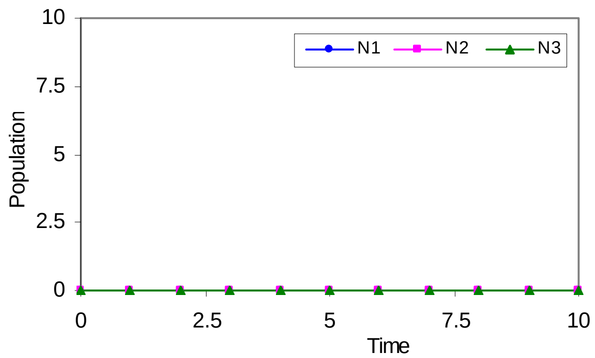

Fig.6. Variation of population against time for N10 =0, N20 =0, N30 =0

VI. Conclusion

The present paper deals with an investigation on a discrete model of three species syn eco-system with limited resources. The basic equations for this model constitute as three first order non-linear coupled ordinary difference equations. All possible equilibrium points of the model are identified at $N_i\left(t+1\right)=N_i\left(t\right);\ i=1,2,3$. It is observe that, all the fourteen equilibrium points are found to be stable. Further the numerical solutions for the discrete model equations are computed.

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