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A Numerical Approach for Solving High-Order Boundary Value Problems

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Abstract

In this paper, a numerical method which produces an approximate solution is presented for the numerical solutions of sixth,eighth,ninth and twelfth order boundary value problems .With the aid of derivatives of power series which slightly perturbe and collocate, eventually converts boundary value problems into the square matrix equations with the unknown coefficients obtain using MAPLE 18 software. This method gives the approximate solutions and compare with the exact solutions. Finally, some examples and their numerical solutions are given by comparing the numerical results obtained to other methods available in the literature, show a good agreement and efficiency.

Index Terms: Sixth, eighth, ninth and twelfth order boundary value problems, power series, slightly perturbe, collocate, MAPLE 18 software.

1. Introduction

Higher boundary value problems have wide applications in various engineering and applied sciences. In general, modelling of the variation of a physical quantity, such as mechanical vibration or structural dynamics, heat transfer, theory of electric circuits are founded on the theories of boundary value problems. The boundary value problems of higher order have been examined due to their mathematical importance and applications in mathematical physics and engineering. In [1] contains theorems which detail the conditions for existence and uniqueness of solutions of such boundary value problems.

In this paper, we consider the higher-order boundary value problems of the form:

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$$\frac{d^n y}{dt^n} + \ldots\ldots + \beta(t)y(t) = g(t) \quad t_0 \le t \le t_1 \tag{1}$$

Subject to the boundary conditions

$$\begin{cases} y^k(t_0) = A_0, & y^k(t_1) = A_1 & when & k = 0 \\ y^k(t_0) = B_0, & y^k(t_1) = B_1 & when & k = 1 \\ y^k(t_0) = C_0, & y^k(t_1) = C_1 & when & k = 2 \\ \vdots & \vdots & \vdots & \vdots \\ y^k(t_0) = L_0, & y^k(t_1) = L_1 & when & k = n \end{cases} \tag{2}$$

Where $A_0, A_1, B_0, B_1, C_0, C_1 \ldots L_0, L_1$ are finite real constants, $\beta(t)$ is a constant or variable coefficient, $k = 0,1,2,\ldots n$ are order of boundary value problems, $g(t)$ are smooth functions and t_0, t_1 are domain of the boundary value problems.

Several authors have proposed and employed several numerical techniques to obtain numerical solution of higher order boundary value problems. [2] proposed two numerical algorithms spectral Galerkin and spectral collocation methods, were applied independently to address the numerical issues related to this type of this problem, fifth-order boundary value problem was investigated by [3] using finite-difference method, in [4] the Adomain decomposition method and a modified form of the Adomain decomposition method were used to investigate the fifth-order boundary value problem,[5] employed Homotopy perturbation method for solving numerical solution fifth-order boundary value problems, Chebychev Polynomial Solutions of Twelfth-order Boundary-value Problems proposed by [6], Differential Transform Technique for Higher Order Boundary value problems was employed in [7],Numerical Solution of eighth order boundary value problems by Galerkin method with Quintic B-spline proposed by [8] and Legendre wavelet collocation method was employed to solve sixth-order boundary-value problems by [9].

In this study, we extend and employ numerical technique proposed in [10] and compare the results with techniques used in [6, 7, 8, 9].

2. The Description of the Technique

The Exponentially Fitted Collocation Approximation Method (EFCAM) was formulated by [10] and it has been shown that this numerical approach is a powerful tool for solving singular initial value problems [11]. The whole idea of the method is to use power series as a basis function and its derivative substitute into a slightly perturbed equation, with perturbation term added to the right hand side of the equation. The addition of the perturbation term is to minimize the error of the problems in consideration.

To illustrate the basic concept of the EFCAM, we consider boundary value problems (1)-(2) and power series as a basic function of the form:

$$y(t) = \sum_{q=0}^{N} s_q t^q \tag{3}$$

And the approximate solution of the form:

$$y(t) \approx \sum_{q=0}^{N} s_q t^q + \tau_n \ell^t \tag{4}$$

where t represents the dependent variables in the problem, n represents the order of the dependent variable and $y(t)$ $s_q, (q \geq 0)$ are the unknown constants to be determined, N is the length of computation.

Putting $n = 12$ as order of equation (1) and obtain 12th derivative of (3) and substitute in (1), we have

$$\left\{\begin{array}{l} s_{12} + s_{13}t + s_{14}t^2 + \cdots + \left\{\begin{array}{l} N(N-1)(N-2)(N-3)(N-4) \\ (N-5)(N-6)(N-7)(N-8) \\ (N-9)(N-10)(N-11) \end{array}\right\} s_N t^{N-12} + \\ \beta(t)\left(s_0 + s_1 t + s_2 t^2 + \cdots + s_N t^N\right) = g(t) \end{array}\right\} \tag{5}$$

Collect the likes terms in (5), leads to

$$\left\{ \beta(t)s_0 + \beta(t)s_1 t + \beta(t)s_2 t^2 + \cdots + \left\{\begin{array}{l} N(N-1)(N-2)(N-3)(N-4) \\ (N-5)(N-6)(N-7)(N-8) \\ (N-9)(N-10)(N-11)t^{N-12} + \beta(t)t^N \end{array}\right\} s_N = g(t) \right\} \tag{6}$$

Slightly perturbe and collocate (6), we have,

$$\left\{ \beta(t)s_0 + \beta(t)s_1 t + \beta(t)s_2 t^2 + \cdots + \left\{\begin{array}{l} N(N-1)(N-2)(N-3)(N-4) \\ (N-5)(N-6)(N-7)(N-8) \\ (N-9)(N-10)(N-11)t^{N-12} + \beta(t)t^N \end{array}\right\} s_N = g(t) \right\} \tag{7}$$

Where $H(t) = \tau_{12}(t_r)T_N(t_r) + \tau_{11}(t_r)T_{N-1}(t_r) + \tau_{10}(t_r)T_{N-2}(t_r) + \cdots + \tau_1(t_r)T_1(t_r)$

Here $\tau_1, \tau_2, \tau_3, \cdots\cdots \tau_{12}$ are free tau parameters to be determined and $T_N(t_r), T_{N-1}(t_r), T_{N-2}(t_r), \cdots T_1(t_r)$ are the Chebyshev Polynomials defined [11], N is the computational length and

$$t_r = t_0 + \frac{(t_0 + t_1)_r}{N+2}, \qquad r = 1, 2, 3, \cdots N+1 \tag{8}$$

The above process was repeated for $n = 9, 8$ *and* 6 as order of equation (1) and obtain the corresponding equations as follows:

When $n = 9$

$$\left\{\begin{array}{l}\beta(t_r)s_0+\beta(t_r)s_1t_r+\beta(t_r)s_2{t_r}^2+\cdots+\\ \left\{\begin{array}{l}N(N-1)(N-2)(N-3)(N-4)\\ (N-5)(N-6)(N-7)(N-8){t_r}^{N-9}+\beta(t_r){t_r}^N\end{array}\right\}s_N=g(t)+H(t)\end{array}\right\} \tag{9}$$

Where $H(t)=\tau_9(t_r)T_N(t_r)+\tau_8(t_r)T_{N-1}(t_r)+\tau_7(t_r)T_{N-2}(t_r)+\cdots+\tau_1(t_r)T_1(t_r)$

When $n=8$

$$\left\{\begin{array}{l}\beta(t_r)s_0+\beta(t_r)s_1t_r+\beta(t_r)s_2{t_r}^2+\cdots+\left\{\begin{array}{l}N(N-1)(N-2)(N-3)(N-4)\\ (N-5)(N-6)(N-7){t_r}^{N-8}+\beta(t_r){t_r}^N\end{array}\right\}s_N\\ =g(t)+H(t)\end{array}\right\} \tag{10}$$

Where $H(t)=\tau_8(t_r)T_N(t_r)+\tau_7(t_r)T_{N-1}(t_r)+\tau_6(t_r)T_{N-2}(t_r)+\cdots+\tau_1(t_r)T_1(t_r)$

When $n=6$

$$\left\{\begin{array}{l}\beta(t_r)s_0+\beta(t_r)s_1t_r+\beta(t_r)s_2{t_r}^2+\cdots+\\ \left\{N(N-1)(N-2)(N-3)(N-4)(N-5){t_r}^{N-6}+\beta(t_r){t_r}^N\right\}s_N\\ =g(t)+H(t)\end{array}\right\} \tag{11}$$

Where $H(t)=\tau_6(t_r)T_N(t_r)+\tau_5(t_r)T_{N-1}(t_r)+\tau_4(t_r)T_{N-2}(t_r)+\cdots+\tau_1(t_r)T_1(t_r)$

Hence, equations (8),(9),(10),(11) gives rise to algebraic linear system of equations in (N+1) unknown constants. Twenlve,nine,eight and five extra equations are obtain by fitting one tau-parameter to the boundary condition as suggested in [12] to the boundary conditions given in equation (2):

$$\left\{\begin{array}{llll}y^k(t_0)=A_0+\tau_{12}\ell^{t_0}, & y^k(t_1)=A_1+\tau_{12}\ell^{t_1} & when & k=0\\ y^k(t_0)=B_0+\tau_{12}\ell^{t_0}, & y^k(t_1)=B_1+\tau_{12}\ell^{t_1} & when & k=1\\ y^k(t_0)=C_0+\tau_{12}\ell^{t_0}, & y^k(t_1)=C_1+\tau_{12}\ell^{t_1} & when & k=2\\ \vdots \quad\quad \vdots & \vdots \quad\quad \vdots & & \\ y^k(t_0)=L_0+\tau_{12}\ell^{t_0}, & y^k(t_1)=L_1+\tau_{12}\ell^{t_1} & when & k=12\end{array}\right. . \tag{12}$$

$$\begin{cases} y^k(t_0) = A_0 + \tau_9 \ell^{t_0}, & y^k(t_1) = A_1 + \tau_9 \ell^{t_1} \quad when \quad k=0 \\ y^k(t_0) = B_0 + \tau_9 \ell^{t_0}, & y^k(t_1) = B_1 + \tau_9 \ell^{t_1} \quad when \quad k=1 \\ y^k(t_0) = C_0 + \tau_9 \ell^{t_0}, & y^k(t_1) = C_1 + \tau_9 \ell^{t_1} \quad when \quad k=2 \\ \vdots \quad \vdots & \vdots \quad \vdots \\ y^k(t_0) = L_0 + \tau_9 \ell^{t_0}, & y^k(t_1) = L_1 + \tau_9 \ell^{t_1} \quad when \quad k=9 \end{cases} \tag{13}$$

$$\begin{cases} y^k(t_0) = A_0 + \tau_8 \ell^{t_0}, & y^k(t_1) = A_1 + \tau_8 \ell^{t_1} \quad when \quad k=0 \\ y^k(t_0) = B_0 + \tau_8 \ell^{t_0}, & y^k(t_1) = B_1 + \tau_8 \ell^{t_1} \quad when \quad k=1 \\ y^k(t_0) = C_0 + \tau_8 \ell^{t_0}, & y^k(t_1) = C_1 + \tau_8 \ell^{t_1} \quad when \quad k=2 \\ \vdots \quad \vdots & \vdots \quad \vdots \\ y^k(t_0) = L_0 + \tau_8 \ell^{t_0}, & y^k(t_1) = L_1 + \tau_8 \ell^{t_1} \quad when \quad k=8 \end{cases} \tag{14}$$

$$\begin{cases} y^k(t_0) = A_0 + \tau_5 \ell^{t_0}, & y^k(t_1) = A_1 + \tau_5 \ell^{t_1} \quad when \quad k=0 \\ y^k(t_0) = B_0 + \tau_5 \ell^{t_0}, & y^k(t_1) = B_1 + \tau_5 \ell^{t_1} \quad when \quad k=1 \\ y^k(t_0) = C_0 + \tau_5 \ell^{t_0}, & y^k(t_1) = C_1 + \tau_5 \ell^{t_1} \quad when \quad k=2 \\ \vdots \quad \vdots & \vdots \quad \vdots \\ y^k(t_0) = L_0 + \tau_6 \ell^{t_0}, & y^k(t_1) = L_1 + \tau_5 \ell^{t_1} \quad when \quad k=6 \end{cases} \tag{15}$$

Altogether, we obtain $(N+13),(N+10),(N+9)$ and $(N+7)$ algebraic linear equations in $(N+13),(N+10),(N+9)$ and $(N+7)$ unknown constants.

Thus, we put the equations in Matrix form as follows:

$$M_{order12} S_{order12} = G_{order12} \tag{16}$$

Where

$M_{order12}$ = System of equations of $N+13$

$$S_{order12} = \left(s_0, s_1, s_2, s_3, s_4, \cdots, s_{12}, \tau_{12}, \tau_{11}, \tau_{10}, \cdots, \tau_1\right)$$

$$G_{order12} = \left(g(t_1), g(t_2), g(t_3), \cdots g(t_N), A_0, A_1, B_0, B_1, C_0, C_1, \cdots L_0, L_1,\right)$$

$$M_{order9} S_{order9} = G_{order9} \tag{17}$$

Where

M_{order9} = System of equations of $N+10$

$$S_{order9}=\left(s_0,s_1,s_2,s_3,s_4,\cdots,s_9,\tau_9,\tau_8,\tau_7,\cdots,\tau_1\right)$$

$$G_{order9}=\left(g(t_1),g(t_2),g(t_3),\cdots g(t_N),A_0,A_1,B_0,B_1,C_0,C_1,\cdots L_0,L_1,\right)$$

$$M_{order8}S_{order8}=G_{order8} \tag{18}$$

Where

M_{order8} = System of equations of $N+9$

$$S_{order8}=\left(s_0,s_1,s_2,s_3,s_4,\cdots,s_8,\tau_8,\tau_7,\tau_6,\cdots,\tau_1\right)$$

$$G_{order9}=\left(g(t_1),g(t_2),g(t_3),\cdots g(t_N),A_0,A_1,B_0,B_1,C_0,C_1,\cdots L_0,L_1,\right)$$

$$M_{order6}S_{order5}=G_{order6} \tag{19}$$

Where

M_{order6} = System of equations of $N+7$

$$S_{order6}=\left(s_0,s_1,s_2,s_3,s_4,\cdots,s_6,\tau_6,\tau_5,\tau_4,\cdots,\tau_1\right)$$

$$G_{order5}=\left(g(t_1),g(t_2),g(t_3),\cdots g(t_N),A_0,A_1,B_0,B_1,C_0,C_1,\cdots L_0,L_1,\right)$$

MAPLE 18 software is use to obtain the unknown constants, which substitute into the exponentially fitted approximate solution (4).

3. Numerical Application

In this section, we implement the proposed method to verify the accuracy of the method. Four examples are investigated and the computation results are compare to the works in [6, 7, 8, 9].

Example 1

Consider Twelfth-order boundary value problem given in [6]

$$\frac{d^{12}y}{dt^{12}}+y(t)=-(120+23t+t^3)\ell^t \qquad 0\le t\le 1 \tag{20}$$

Subject to boundary conditions:

$$\begin{cases} y(0)=0, & y(1)=0 \\ y^{/}(0)=1, & y^{/}(1)=-\ell \\ y^{//}(0)=0, & y^{//}(1)=-4\ell \\ y^{///}(0)=-3, & y^{///}(1)=-9\ell \\ y^{/v}(0)=-8, & y^{/v}(1)=-16\ell \\ y^{v}(0)=-15, & y^{v}(1)=-25\ell \end{cases} \tag{21}$$

Exact solution is

$$y(t)=t(1-t)\ell^t \tag{22}$$

Comparing equations (20) and (21) with equations (7) and (12) respectively. Taking computational length $N=18$, $\beta(t)=1$ and collocate the resulting equations as follows:

$$\left\{ t_1=\frac{1}{20}, t_2=\frac{2}{20}, t_3=\frac{3}{20}, t_4=\frac{4}{20}, t_5=\frac{5}{20}, \cdots\cdots, t_{19}=\frac{19}{20} \right.$$

Consider Matrix equation (16). Thus using MAPLE 18 software to obtain thirty one (31) unkown constants, we have the following constants:

$$\begin{cases} s_0=\text{-}0.003279086633, s_1=0.9996720913, s_2=-0.001639543316, s_3=-0.5000546514 \\ s_4=-0.3333469962,\ s_5=-0.1250027326,\ s_6=-0.033333789,\ s_7=-0.006944507111 \\ s_8=-0.001190490291, s_9=-0.0001736041663, s_{10}=-0.00002205156570 \\ s_{11}=-0.000002478290090, s_{12}=-0.0000002505155836, s_{13}=0.00000002313343416 \\ s_{14}=-0.0000000189586198, s_{15}=-0.0000000000151624004, s_{16}=-0.00000000009316488741 \\ s_{17}=-0.000000000001017724638, s_{18}=-0.000000000006288458066 \\ \tau_1=\text{-}0.00001095375166, \tau_2=0.00000198369330, \tau_3=-0.000003912546619 \\ \tau_4=0.00001166070304, \tau_5=-0.00001694418078, \tau_6=0.00004662514308 \\ \tau_7=-0.00004450541740, \tau_8=0.0001081373581, \tau_9=-0.00008424614546 \\ \tau_{10}=0.0001979827102, \tau_{11}=0.0001347960774,\ \tau_{12}=0.0003279086633 \end{cases}$$

Substitute the above values into approximation solution (4), thus, the approximate solution of 12[th] boundary value problem (20) is given in a close form as:

$$y_{18}(t)\approx\begin{cases}-0.003279086633+0.9996720913t-0.001639543316t^2-0.5000546514t^3\\-0.3333469962t^4-0.1250027326t^5-0.033333789t^6-0.006944507111t^7\\-0.001190490291t^8-0.0001736041663t^9-0.00002205156570t^{10}\\-0.000002478290090t^{11}-0.0000002505155836t^{12}+0.00000002313343416t^{13}\\-0.0000000189586198t^{14}-0.0000000000151624004t^{15}-0.00000000009316488741t^{16}\\-0.00000000000101772 4638t^{17}-0.00000000006288458066t^{18}+0.0003279086633\ell^{t}\end{cases}\tag{23}$$

Example 2

Consider Ninth-order boundary value problem given in [7]

$$\frac{d^9y}{dt^9}-y(t)=-9\ell^t \qquad 0\le t\le 1\tag{24}$$

Subject to boundary conditions:

$$\begin{cases}y(0)=1, & y(1)=0\\ y^{/}(0)=0, & y^{/}(1)=-\ell\\ y^{//}(0)=-1, & y^{//}(1)=-2\ell\\ y^{///}(0)=-2, & y^{///}(1)=-3\ell\\ y^{/v}(0)=-3, & \end{cases}\tag{25}$$

Exact solution is

$$y(t)=(1-t)\ell^t\tag{26}$$

Comparing equations (24) and (25) with equations (9) and (13) respectively. Taking computational length $N=18$, $\beta(t)=-1$ and collocate the resulting equation as follows:

$$\left\{t_1=\frac{1}{20},t_2=\frac{2}{20},t_3=\frac{3}{20},t_4=\frac{4}{20},t_5=\frac{5}{20},\cdots\cdots,t_{19}=\frac{19}{20}\right.$$

Consider Matrix equation (17). Thus using MAPLE 18 software to obtain twenty eight (28) unkown constants, we have the following constants:

$$\begin{cases} s_0 = 1.000063190, s_1 = 0.00006319017850, s_2 = -0.4999684049, s_3 = -0.3333228016 \\ s_4 = -0.1249973671, s_5 = -0.03333280675, s_6 = -0.006944356681, s_7 = -0.001190463629 \\ s_8 = -0.0001736098806, s_9 = -0.00002204311524, s_{10} = -0.000002490972670 \\ s_{11} = -0.000002224676687, s_{12} = -0.0000007059534367, s_{13} = 0.00000005287747827 \\ s_{14} = -0.00000004319807404, s_{15} = 0.0000002268106370, \cdots s_{16} = -0.0000000763691797 \\ s_{17} = 0.000000001467428163, s_{18} = -0.0000000001202474314, \tau_1 = -0.0000005763155686 \\ \tau_2 = -0.00000008432547828, \tau_3 = -0.000003199289682, \tau_4 = 0.0000007055954231 \\ \tau_5 = -0.00001251461328, \tau_6 = 0.000003560561622 \quad \tau_7 == 0.00003229483688 \\ \tau_8 = 0.000008833210026, \quad \tau_9 = -0.000006319017850 \end{cases}$$

Substitute the above values into approximation solution (4), hence, the approximate solution of 9th boundary value problem (24) is given in a close form

$$y_{18}(t) \approx \begin{cases} 1.000063190 + 0.00006319017850t - 0.4999684049t^2 - 0.3333228016\text{t}^3 \\ -0.1249973671t^4 - 0.03333280675t^5 - 0.006944356681t^6 - 0.001190463629\text{t}^7 \\ -0.0001736098806t^8 - 0.00002204311524t^9 - 0.000002490972670\text{t}^{10} \\ -0.000002224676687t^{11} - 0.0000007059534367\text{t}^{12} + 0.00000005287747827\text{t}^{13} \\ -0.00000004319807404\text{t}^{14} + 0.0000002268106370t^{15} - 0.0000000763691797\text{t}^{16} \\ +0.000000001467428163\text{t}^{17} - 0.0000000001202474314\text{t}^{18} - 0.000006319017850\ell^{\text{t}} \end{cases} \tag{27}$$

Example 3

Consider Eighth-order boundary value problem given in [8]

$$\frac{d^8 y}{dt^8} - ty(t) = -(48 + 15t + t^3)\ell^t \qquad 0 \le t \le 1 \tag{28}$$

Subject to boundary conditions:

$$\begin{cases} y(0) = 0, & y(1) = 0 \\ y^{/}(0) = 1, & y^{/}(1) = -\ell \\ y^{//}(0) = 0, & y^{//}(1) = -4\ell \\ y^{///}(0) = -3, & y^{///}(1) = -9\ell \end{cases} \tag{29}$$

Exact solution is

$$y(t) = t(1-t)\ell^t \tag{30}$$

Comparing equations (28) and (29) with equations (10) and (14) respectively. Taking computational length $N = 18$, $\beta(t) = -t$ and collocate the resulting equation as follows:

$$\left\{ t_1 = \frac{1}{20}, t_2 = \frac{2}{20}, t_3 = \frac{3}{20}, t_4 = \frac{4}{20}, t_5 = \frac{5}{20}, \cdots\cdots, t_{19} = \frac{19}{20} \right.$$

Consider Matrix equation (18). Thus using MAPLE 18 software to obtain twenty seven (27) unkown constants, we have the following constants

$$\begin{cases} s_0 = -0.0004633615280, s_1 = 0.9999536638, s_2 = -0.0000231680764, s_3 = -0.5000077227 \\ s_4 = -0.3333352613, s_5 = -0.1250003964, s_6 = -0.03333338274, s_7 = -0.00694447288 \\ s_8 = -0.001190371912, s_9 = -0.0001742139247, s_{10} = -0.0000197890363 \\ s_{11} = -0.00000820890103, s_{12} = 0.000009915454113, s_{13} = -0.0000128119032 \\ s_{14} = 0.00001138177296, s_{15} = -0.00000702442084, s_{16} = 0.000002862560157 \\ s_{17} = -0.0000006938261405, s_{18} = 0.00000007582206908, \tau_1 = -0.00000246910167 \\ \tau_2 = -0.000000282505572, \tau_3 = -0.00001324886894, \tau_4 = 0.000003821952984 \\ \tau_5 = -0.00005114975339, \tau_6 = 0.00001867559447, \tau_7 = -0.0001310777287 \\ \tau_8 = 0.00004633615280 \end{cases}$$

Substitute the above values into approximation solution (4), thus, the approximate solution of 8th boundary value problem (28) is given in a close form as,

$$y_{18}(t) \approx \begin{cases} -0.0004633615280 + 0.9999536638t - 0.0000231680764t^2 - 0.5000077227\mathrm{t}^3 \\ -0.3333352613t^4 - 0.1250003964t^5 - 0.03333338274t^6 - 0.00694447288\mathrm{t}^7 \\ -0.001190371912t^8 - 0.0001742139247t^9 - 0.0000197890363\mathrm{t}^{10} \\ -0.00000820890103\mathrm{t}^{11} + 0.000009915454113t^{12} - 0.0000128119032\mathrm{t}^{13} \\ +0.00001138177296t^{14} - 0.00000702442084\mathrm{t}^{15} + 0.000002862560157\mathrm{t}^{16} \\ -0.0000006938261405\mathrm{t}^{17} + 0.00000007582206908t^{18} + 0.00004633615280\ell^{\mathrm{t}} \end{cases} \tag{31}$$

Example 4

Consider Sixth-order boundary value problem given in [9]

$$\frac{d^6 y}{dt^6} - y(t) = -6\ell^t \qquad 0 \le t \le 1 \tag{32}$$

Subject to boundary conditions:

$$\begin{cases} y(0)=1, & y(1)=0 \\ y^{//}(0)=-1, & y^{//}(1)=-2\ell \\ y^{/v}(0)=-3, & y^{/v}(1)=-4\ell \end{cases} \tag{33}$$

Exact solution is

$$y(t)=(1-t)\ell^{t} \tag{34}$$

Comparing equations (32) and (33) with equations (11) and (15) respectively. Taking computational length $N = 13$, $\beta(t) = -1$ and collocate the resulting equation as follows:

$$\left\{ t_1=\frac{1}{15}, t_2=\frac{2}{15}, t_3=\frac{3}{15}, t_4=\frac{4}{15}, t_5=\frac{5}{15}, \cdots\cdots, t_{14}=\frac{14}{15} \right.$$

Consider Matrix equation (19), Thus using MAPLE 18 software to obtain twenty (20) unkown constants, we have the following constants

$$\begin{cases} s_0=0.9999998251, s_1=-0.0000001748965074, s_2=-0.5000000875 \\ s_3=-0.3333333624, s_4=-0.1250000073, s_5=-0.03333333684 \\ s_6=-0.006944426916, s_7=-0.001190557616, s_8=-0.0001733917044 \\ s_9=-0.00002241249726, s_{10}=-0.000002094177955, s_{11}=-0.0000005007641358 \quad - \\ s_{12}=-0.00000006893451398, s_{13}=-0.000000001689300519 \\ \tau_1=0.00000006994350377, \tau_2=0.00000001389193444, \tau_3=0.0000003732339330 \\ \tau_4=0.00000006544212469, \tau_5=0.000001037681477, \tau_6=0.000000174901092 \end{cases}$$

Substitute the above values into approximation solution (4), thus, the approximate solution of 6^{th} boundary value problem (32) is given in a close form

$$y_{13}(t)\approx\begin{cases} 0.9999998251-0.0000001748965074t-0.5000000875t^2 \\ -0.3333333624t^3-0.1250000073t^4-0.03333333684t^5 \\ -0.006944426916t^6-0.001190557616t^7-0.0001733917044t^8 \\ -0.00002241249726t^9-0.000002094177955t^{10}-0.0000005007641358t^{11} \\ -0.00000006893451398t^{12}-0.000000001689300519t^{13}+0.000000174901092\ell^{t} \end{cases} \tag{35}$$

Table 1: Numerical Results

	12th Order BVP Example 1		**9th Order BVP Example 2**	
t	*Exact Solution*	*EFCAM*	*Exact Solution*	*EFCAM*
0.0	0.0000000000	0.0000000000	1.0000000000	0.9999999998
0.1	0.0994653826	0.0994653826	0.9946538262	0.9946538263
0.2	0.1954244413	0.1954244412	0.9771222064	0.9771222064
0.3	0.2834703497	0.2834703496	0.9449011656	0.9449011652
0.4	0.3580379275	0.3580379273	0.8950948188	0.8950948182
0.5	0.4121803178	0.4121803174	0.8243606355	0.8243606351
0.6	0.4373085120	0.4373085120	0.7288475200	0.7288475197
0.7	0.4228880685	0.4228880685	0.6041258121	0.6041258120
0.8	0.3560865485	0.3560865485	0.4451081856	0.4451081854
0.9	0.2213642800	0.2213642801	0.2459603111	0.2459603105
1.0	0.0000000000	0.0000000003	0.0000000000	0.0000000003

Table 2: Numerical Results

	8th Order BVP Example 3		**6th Order BVP Example 4**	
t	*Exact Solution*	*EFCAM*	*Exact Solution*	*EFCAM*
0.0	0.0000000000	0.0000000000	1.0000000000	1.0000000000
0.1	0.0994653826	0.0994653826	0.9946538262	0.9946538263
0.2	0.1954244413	0.1954244414	0.9771222064	0.9771222066
0.3	0.2834703497	0.2834703496	0.9449011656	0.9449011652
0.4	0.3580379275	0.3580379274	0.8950948188	0.8950948185
0.5	0.4121803178	0.4121803176	0.8243606355	0.8243606354
0.6	0.4373085120	0.4373085120	0.7288475200	0.7288475201
0.7	0.4228880685	0.4228880686	0.6041258121	0.6041258123
0.8	0.3560865485	0.3560865487	0.4451081856	0.4451081857
0.9	0.2213642800	0.2213642799	0.2459603111	0.2459603111
1.0	0.0000000000	0.0000000001	0.0000000000	0.0000000004

Table 3: Absolute Error

	12th Order BVP Example 1		**9th Order BVP Example 2**	
t	$\lvert \text{Error}_{\text{EFCAM}} \rvert$	$\lvert \text{Error}_{[6]} \rvert$	$\lvert \text{Error}_{\text{EFCAM}} \rvert$	$\lvert \text{Error}_{[7]} \rvert$
0.0	0.0E-10	0.0E-10	2.0E-10	0.0E-10
0.1	0.0E-10	0.0E-10	1.0E-10	6.4E-13
0.2	0.0E-10	0.0E-10	0.0E-10	1.9E-11
0.3	1.0E-10	1.0E-10	4.0E-10	6.8E-11
0.4	2.0E-10	1.0E-10	6.0E-10	1.7E-10
0.5	4.0E-10	1.0E-10	4.0E-10	2.9E-10
0.6	0.0E-10	0.0E-10	3.0E-10	3.5E-10
0.7	0.0E-10	0.0E-10	1.0E-10	3.0E-10
0.8	0.0E-10	0.0E-10	2.0E-10	1.0E-10
0.9	1.0E-10	0.0E-10	6.0E-10	4.2E-10
1.0	3.0E-10	0.0E-10	3.0E-10	2.1E-09

Table 4: Absolute Error

8th Order BVP Example 3			6th Order BVP Example 4	
t	$\lvert Error_{EFCAM} \rvert$	$\lvert Error_{[8]} \rvert$	$\lvert Error_{EFCAM} \rvert$	$\lvert Error_{[9]} \rvert$
0.0	0.0E-10	------	0.0E-10	0.0E-10
0.1	0.0E-10	5.2E-08	1.0E-10	1.3E-10
0.2	1.0E-10	2.2E-06	2.0E-10	2.4E-10
0.3	1.0E-10	7.0E-06	4.0E-10	3.3E-10
0.4	1.0E-10	1.1E-05	3.0E-10	3.7E-10
0.5	2.0E-10	1.2E-05	1.0E-10	3.9E-10
0.6	0.0E-10	8.8E-06	1.0E-10	3.6E-10
0.7	1.0E-10	2.5E-06	2.0E-10	3.0E-10
0.8	2.0E-10	1.8E-06	1.0E-10	2.8E-10
0.9	1.0E-10	2.0E-06	0.0E-10	1.1E-10
1.0	1.0E-10	------	4.0E-11	3.2E-16

------ Not Available

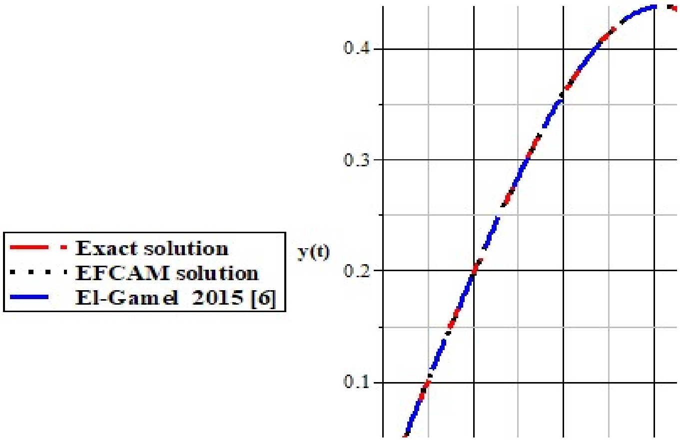

Fig.1. 12th Order Boundary Value Problem Example 1

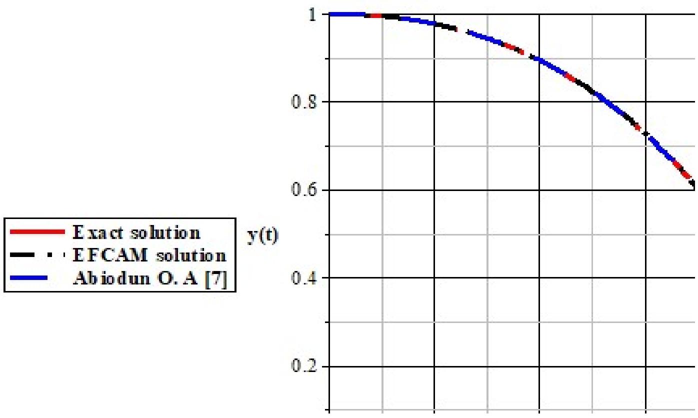

Fig.2. 9th Order Boundary Value Problem Example 2

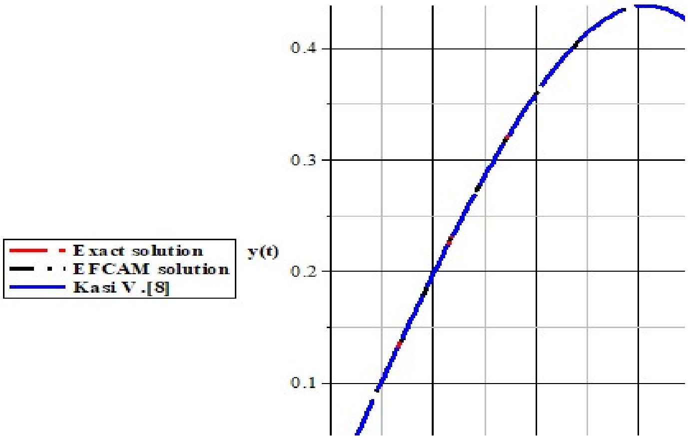

Fig.3. 8th Order Boundary Value Problem Example

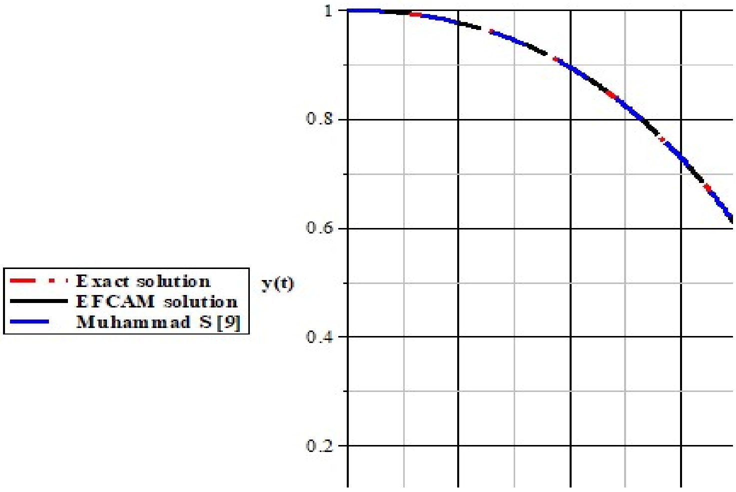

Fig.4. 6th Order Boundary Value Problem Example 4

4. Conlusion

In this paper, we aim to extend and employ numerical technique proposed in [10] to solve higher order boundary value problems.The results obtained are compared with exact solution and existing techniques available in the literature which show good agreement (see table 3 and table 4). Fig1,Fig2,Fig3 and Fig4 show good agreement with exact solutions and existing methods in terms of efficiency and accurancy.It may be concluded that this technique is realiable and efficient in finding numerical solutions for boundary value problems.Consequently,the flexibilty and adaptation provided by the method have made the method a good numerical technique for approximate analytical solution for other higher order differential equations occur in various applied mathematical physics.

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