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A Robust Autofocusing Approach for Estimating Directions-of-Arrival of Wideband Signals

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Abstract

In this paper, we introduce a robust method for wideband DOA estimation which does not require any preliminary DOA estimation or calibration of the array during the whole estimating process with the performance similar to the autofocusing approach. Our method is based on the autofocusing approach and uses the ESPRIT algorithm during the narrowband processing stage. Meanwhile, we suggest the method of selecting the best focusing frequency which minimizes the mean square error.

Index Terms: direction-of-arrival(DOA); wideband sources; focusing matrix; MUSIC; ESPRIT

1. Introduction

Array processing is a powerful tool for detecting and locating the signals arriving at a set of sensors which are distributed in spaces and receive signals.

This paper addresses the problem of detecting multiple wideband sources and estimating their directions-of-arrival (DOAs), based on the signals received by a sensor array in the presence of a noise field. Wideband processing arises in many applications such as audio conferencing, spread spectrum transmission, and passive sonar. A common approach to wideband array processing is based on sampling the signal spectrum at the output of the sensors. In the so-called incoherent signal subspace method (ISSM) [1], the wideband frequency band is divided into non-overlapping narrow bands, and then narrowband signal-subspace processing is performed on each individual band. Some form of averaging procedure then follows to combine the results from individual narrowband processing. However, perfectly correlated (coherent) sources cannot be handled by this approach. Furthermore, the efficiency of this method deteriorates for closely separated sources and low signal-to-noise ratio (SNR).

To overcome these drawbacks of ISSM, several coherent have been proposed. Of these, the coherent signal-subspace method (CSSM) [2] was the earliest that improves the efficiency of the estimation by condensing the energy of narrowband signal in a predefined subspace. This process is called focusing. Then a high-resolution method such as MUSIC [3] is used to find the DOA's. And the unitary focusing matrices have better

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performances than that are not for they have no focusing loss. However, construction of the focusing matrices mentioned above needs preliminary estimates of DOAs, and the estimation performance is sensitive to the error in the preliminary estimates. In order to overcome this disadvantage, several improved methods have been proposed, such as the weighted average of signal subspaces (WAVES) [4] and test of orthogonality of projected subspaces (TOPS)[5]. WAVES is another coherent method which also requires focusing matrices. Although it can avoid the initial-value requirement, its performance is worse than when it uses focusing matrices with good initial values. However, TOPS shows false peaks which may induce error. Recently, a novel autofocusing approach for wideband DOA estimation has been proposed [6]. Though it constructs the focusing matrices entirely by processing the received signal and does not require any preliminary DOA estimates at the focusing stage, it uses MUSIC algorithm to estimate the final DOAs after getting the universal focused sample correlation matrix at the narrowband processing stage, which needs accurate calibration of the array that may lead errors due to mismatches. In addition, the influence of the selection of different focusing frequency on the performance of estimation is not presented.

In this paper, we introduce a robust method for wideband DOA estimation which does not require any preliminary DOA estimation or calibration of the array during the whole estimating process with the performance similar to the autofocusing approach. Our method is based on the autofocusing approach and uses the ESPRIT [7] algorithm during the narrowband processing stage. Meanwhile, we suggest the method of selecting the best focusing frequency which minimizes the mean square error.

The rest of the paper is organized as follows. In the following section, we formulate the problem and review the coherent methods of wideband DOA estimation. In Section III, we discuss the autofocusing method and introduce the proposed method. Simulation results that illustrate the performance of the proposed method are presented in section IV and finally section V concludes the paper.

2. Preniminaries

2.1. Problem Formulation

Consider an N element linear array with unambiguous. Assume now that D ($D<N$) far-field wideband sources $s_1(t),\ldots,\ s_D(t)$, with identical bandwidth B, impinge on the array from directions $\theta_1,\ldots,\theta_D$ respectively. The signals emitted by the sources are assumed to be ergodic and stationary zero-mean stochastic processes. Then, the signal received at the ith sensor can be expressed as

$$x_n(t) = \sum_{n=1}^{D} a_{nd} s_n(t - \tau_n(\theta_d)) + \eta_n(t) \quad 1 \le n \le N \tag{1}$$

where $s_n(t)$ is the signal radiated by the nth source as observed at an arbitrarily chosen reference point, $\tau_n(\theta_d)$ is the propagation delay between the nth sensor and the reference point for the dth source, a_{nd} is the amplitude response of the nth sensor to the dth source, and $\eta_n(t)$ is the additive noise at the nth sensor with variance σ_n^2 assumed to be uncorrelated with the signal sources and white both temporally and spatially. For a linear array with uniform spacing,

$$\tau_n(\theta_d) = (n-1)\frac{\Delta}{c}\cos\theta_d . \tag{2}$$

where Δ is the spacing between two consecutive sensors, and c is the propagation velocity. The incoming signal at each sensor is first sampled at frequency f_s, and the samples are then partitioned into segments of $K = \Delta T f_s$ samples each. The samples at a segment with a time interval ΔT is called one "snapshot". Then, a K-point DFT is applied to the snapshot to get the DFT coefficients which can be expressed as

$$\mathbf{X}(i) = \mathbf{A}(f_i)\mathbf{S}(i) + \mathbf{N}(i), i = 0, \cdots, K-1 \tag{3}$$

where

$$f_i = \frac{i}{K} f_s \tag{4}$$

and according to that the signals are limited in the bandwidth $B = f_{\max} - f_{\min}$, we can get the range of i related to B is

$$i = [\lfloor f_{\min} / f_s \rfloor, \lfloor f_{\max} / f_s \rfloor] = [i_{\min}, i_{\max}] \tag{5}$$

where $f_{\max}$ and $f_{\min}$ denote the maximum and minimum value, respectively. Symbol $\lfloor \cdot \rfloor$ denotes the maximum integer that less than re. In (3), $\mathbf{X}(i) = [\mathbf{X}_1(i)\mathbf{X}_2(i)\cdots\mathbf{X}_N(i)]^T$ with $\mathbf{X}_n(i)$ denoting the ith DFT coefficient of samples of $s_n(t)$, where subscript T denotes transpose. And,

$$\mathbf{A}(f_i) = [\mathbf{a}_{\theta_1}(f_i)\mathbf{a}_{\theta_2}(f_i)\cdots\mathbf{a}_{\theta_D}(f_i)], \tag{6}$$

with

$$\mathbf{a}_{\theta_d}(f_i) = [e^{-j2\pi f_i \tau_1(\theta_d)}, \cdots, e^{-j2\pi f_i \tau_N(\theta_d)}]^T \tag{7}$$

In addition,

$$\mathbf{N}(i) = [N_1(i)N_2(i)\cdots N_N(i)]^T \tag{8}$$

$$\mathbf{S}(i) = [S_1(i)S_2(i)\cdots S_D(i)]^T \tag{9}$$

where $N_n(i)$ and $S_d(i)$ are the ith DFT coefficients of samples of $\eta_n(t)$ and $s_d(t)$ respectively. Under the assumption that ΔT is long enough compared to the correlation time of the signals and the noise so that the DFT coefficients are uncorrelated, we can get the covariance matrix of the received array data

$$\mathbf{R}_{xx}(i) = E(\mathbf{X}(i)\mathbf{X}^H(i)) = \mathbf{A}(f_i)\mathbf{R}_{ss}(i)\mathbf{A}^H(f_i) + \sigma_n^2\mathbf{I},$$

where $\mathbf{R}_{ss}(i) = E(\mathbf{S}(i)\mathbf{S}^H(i))$ and subscript H denotes conjugate transpose. Assume that the D sources are uncorrelated, then $\mathbf{R}_{ss}(i)$ has full rank. Then, if the eigenvectors of $\mathbf{R}_{ss}(i)$ are ordered in descending order with respect to their eigenvalues, the first D eigenvectors span the same subspace of dimension D (known as signal subspace at frequency $f(i)$)as $\Re(\mathbf{A}(f_i))$ (range space of $\mathbf{A}(f_i)$)and are know as the signal subspace eigenvectors. The last *N-D* eigenvectors are called the noise subspace eigenvectors and they span a subspace of dimension *N-D* orthogonal to signal subspace.

2.2. Review of Coherent Methods of Wideband DOA Estimation

For the advantages of coherent methods compared to ISSM, coherent methods have been paid more attention to and lots of effective methods have been proposed. The key problem is the construction of the focusing matrix $\mathbf{T}(f_i)$ which transforms the array manifold at frequency f_i to that at a common frequency f_0 as follows [2]

$$\mathbf{T}(f_i)\mathbf{A}(f_i) = \mathbf{A}(f_0).$$

Then the coherent autocorrelation matrix is expressed as

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$$R_{coh} = \sum_{i=i_{\min}}^{i_{\max}} \mathbf{T}(f_i)\mathbf{R}_{ss}(i)\mathbf{T}^H(f_i)$$
$$= \mathbf{A}(f_0)\mathbf{R}_s(i)\mathbf{A}^H(f_0) + \sigma_n^2 \sum_{i=i_{\min}}^{i_{\max}} \mathbf{T}(f_i)\mathbf{T}^H(f_i)$$

where $\mathbf{R}_s = \sum_{i=i_{\min}}^{i_{\max}} \mathbf{R}_{ss}(i)$ .Now any standard narrowband DOA estimation method can be applied by computing the eigenvectors of the matrix pencil $(\mathbf{R}_{coh}, \sum_{i=i_{\min}}^{i_{\max}} T(f_i)T^H(f_i))$. The most popular way to generate focusing matrices is to construct a unitary matrix by minimizing the Frobenius norm of the manifold mismatches, viz

$$\min_{\mathbf{T}(f_i)} = \| \mathbf{A}(f_0) - \mathbf{T}(f_i)\mathbf{A}(f_i) \|$$

subject to $\mathbf{T}(f_i)\mathbf{T}^H(f_i) = \mathbf{I}$.

This class of focusing matrices is called RSS (rotational signal subspace) matrices [8] and the matrix is required to be unitary in order to preserve the SNR before and after focusing.

3. Autofocusing approach and the proposed method

In this section, we first review the autofocusing method suggested in [6], and the proposed method.

3. 1 Autofocusing approach

The autofocusing approach avoids the need for preliminary DOA estimates by noticing the fact that the signal subspace eigenvectors span the same subspace as the array manifold at each frequency and so, it uses them to construct the focusing matrix, instead of using the array steering vectors, which require the preliminary knowledge of the DOAs.

Suppose the focusing frequency is f_0 , then the focusing matrix of the autofocusing approach is

$$\mathbf{T}_{auto}(f_i) = \frac{1}{\sqrt{J}}\mathbf{U}(f_0)\mathbf{U}^H(f_i)$$

where $J = i_{\max} - i_{\min} + 1$ is the actual number of frequency bins during bandwidth B and $\mathbf{U}(f_i)$ is a $N \times N$ unitary whose columns are the eigentvectors of the autocorrelation matrix $\mathbf{R}_{xx}(i)$. It is assumed that all the sources have non-zero energy at all frequency bins considered. It is to be noted that this matrix belongs to the same class of SST [9] matrices. Since the eigenvectors of $\mathbf{R}_{xx}(i)$ can be grouped into signal space and noise subspace eigenvectors, $\mathbf{U}(f_i)$ can be represented as

$$\mathbf{U}(f_i) = [\mathbf{U}_{ss}(f_i)\mathbf{U}_N(f_i)] \tag{10}$$

where $U_{ss}(f_i)$ is a $N \times D$ matrix whose columns represent the D orthonormal eigenvectors of $\mathbf{R}_{xx}(i)$ corresponding to the D largest eigenvalues, and $\mathbf{U}_N(f_i)$ is a $N \times (N - D)$ matrix whose columns represent the remaining (*N*-*D*) orthonormal eigenvectors of $\mathbf{R}_{xx}(i)$ .Multiply $\mathbf{X}(i)$ by the focusing matrix and define the transformed vector

$$\mathbf{Y}(i) = \mathbf{T}_{auto}(f_i)\mathbf{X}(i) \text{ ,} \tag{11}$$

then the autocorrelation of $\mathbf{Y}(i)$ is

$$\begin{aligned}\mathbf{R}_{yy}(i) &= E(\mathbf{Y}(i)\mathbf{Y}^H(i)) \\ &= \mathbf{T}_{auto}(f_i)\mathbf{A}(f_i)\mathbf{R}_{ss}(i)\mathbf{A}^H(f_i)\mathbf{T}^H{}_{auto}(f_i) + \frac{1}{J}\sigma_n{}^2\mathbf{I},\end{aligned} \tag{12}$$

substituting (10) into (12) and consider that $\mathbf{U}^H{}_N(f_i)\mathbf{A}(f_i) = 0$,we get

$$\mathbf{R}_{yy}(i) = \frac{1}{J}\mathbf{U}_{ss}(f_0)\tilde{\mathbf{R}}(i)\mathbf{U}^H{}_{ss}(f_0) + \frac{1}{J}\sigma_n{}^2\mathbf{I} \tag{13}$$

where

$$\tilde{\mathbf{R}}(i) = \mathbf{U}^H{}_{ss}(f_i)\mathbf{A}(f_i)\mathbf{R}_{ss}(i)\mathbf{A}^H(f_i)\mathbf{U}_{ss}(f_i) \tag{14}$$

then the coherently combined autocorrelation matrix is

$$\begin{aligned}R_{coh} &= \sum_{i=i_{\min}}^{i_{\max}}\mathbf{R}_{yy}(i) = \frac{1}{J}\mathbf{U}_{ss}(f_0)\Big(\sum_{i=i_{\min}}^{i_{\max}}\tilde{\mathbf{R}}(i)\Big)\mathbf{U}^H{}_{ss}(f_0) + \sigma_n{}^2\mathbf{I} \\ &= \mathbf{U}_{ss}(f_0)\mathbf{R}_s\mathbf{U}^H{}_{ss}(f_0) + \sigma_n{}^2\mathbf{I}\end{aligned} \tag{15}$$

where

$$\mathbf{R}_s = \frac{1}{J}\sum_{i=i_{\min}}^{i_{\max}}\tilde{\mathbf{R}}(i) = \frac{1}{J}\sum_{i=i_{\min}}^{i_{\max}}\mathbf{U}^H{}_{ss}(f_i)\mathbf{A}(f_i)\mathbf{R}_{ss}(i)\mathbf{A}^H(f_i)\mathbf{U}_{ss}(f_i) \tag{16}$$

Then, using (15) the autofocusing approach extracts the DOA estimate by a narrowband MUSIC algorithm. However, the MUSIC algorithm requires calibration of the array. Bad calibration of array may lead to inaccurate DOA estimates.

3. 2 The proposed method

In this paper, we propose a robust method for wideband DOA estimation which does not require any preliminary DOA estimation or calibration of the array during the whole estimating process with the performance similar to the autofocusing approach. The calibration of array is avoided by noticing the fact that ESPRIT algorithm estimates the DOAs needn't requiring calibration of the array and so, we can use the TLS-ESPRIT [7] to process the coherently combined autocorrelation matrix expressed by (15). Meanwhile, we suggest the method of selecting the best focusing frequency which minimizes the mean square error.

Let us consider the same signal model as described in Section II. In particular, the focusing matrix for the *i*th frequency bin considered of our method is

$$\mathbf{T}_{opt}(f_i) = \frac{1}{\sqrt{J}}\mathbf{U}(f_{opt})\mathbf{U}^H(f_i)$$

where f_{opt} is the optimal focusing frequency we propose which is selected in the following rule.

The rule of selecting f_{opt} : f_{opt} is the central frequency, that is

$$f_{opt} = \frac{f_{\max} + f_{\min}}{2} = f_{i_{mid}}$$

where i_{mid} satisfies $i_{mid} = \left[\frac{f_{opt}}{f_s / K}\right] = \left[\frac{K f_{opt}}{f_s}\right]$

After the optimal focusing frequency f_{opt} has been selected, the following step is to use the autofocusing approach to get the coherently combined autocorrelation matrix R_{coh} which will be summarized in the end of this section. When R_{coh} has been calculated, instead of using the MUSIC algorithm to get the final DOA estimates, we will apply the ESPRIT algorithm to R_{coh} in succession.

Consider an array model of the modified vision of that constructed in section II. We construct two subarrays of the model $\mathbf{X}$ by denoting the first N-1 sensors by $\mathbf{X}_u$ and the last N-1 sensors by $\mathbf{X}_v$ respectively, and then the received data vectors can be written as follows:

$$x_{un}(t) = \sum_{n=1}^{D} a_{nd} s_n(t - \tau_n(\theta_d)) + \eta_{un}(t)$$

$$x_{vn}(t) = \sum_{n=1}^{D} a_{nd} s_n\left(t - \tau_n(\theta_d) + \frac{\Delta}{c}\cos\theta_d\right) + \eta_{vn}(t)\,. \quad 1 \le n \le N-1 \tag{17}$$

Corresponding to (3)-(9), we get the DFT coefficients of (17) of the two subarrays

$$\mathbf{X}_u(i) = \hat{\mathbf{A}}(f_i)\mathbf{S}(i) + \mathbf{N}_u(i), i = i_{\min}, \ldots, i_{\max}$$

$$\mathbf{X}_v(i) = \hat{\mathbf{A}}(f_i)\mathbf{\Phi}\mathbf{S}(i) + \mathbf{N}_v(i), i = i_{\min}, \ldots, i_{\max}$$

where the subscripts u and v denote the two subarrays $\mathbf{X}_u$ and $\mathbf{X}_v$, respectively. And then we can define

$$\mathbf{Z}(i) = \tilde{\mathbf{A}}\mathbf{S}(i) + \mathbf{N}_z(i)$$

where

$$\mathbf{Z}(i) = \begin{bmatrix} \mathbf{X}_u(i) \\ \mathbf{X}_v(i) \end{bmatrix} \quad \tilde{\mathbf{A}}(i) = \begin{bmatrix} \hat{\mathbf{A}}(f_i) \\ \hat{\mathbf{A}}(f_i)\mathbf{\Phi} \end{bmatrix} \quad \mathbf{N}_z(i) = \begin{bmatrix} \mathbf{N}_u(i) \\ \mathbf{N}_v(i) \end{bmatrix}$$

$$\mathbf{\Phi} = diag(e^{j\gamma_1}, \ldots, e^{j\gamma_D})$$

where $\gamma_d = 2\pi f_d \Delta \frac{\cos\theta_d}{c}$ .And $\mathbf{\Phi}$ is a unitary matrix that relates the measurements between subarray $\mathbf{X}_u$ and $\mathbf{X}_v$. In the absence of noise, the signal subspace can be obtained by collecting a sufficient number of measurements and finding any set of D linearly independent measurement vectors. These vectors span the D-dimensional subspace spanned by $\tilde{\mathbf{A}}(i)$. Since we have got $\mathbf{Z}(i)$, we would be able to obtain the transformed vector $\mathbf{W}(i)$ which is computed from (11), that is

$$\mathbf{W}(i) = \mathbf{T}_{opt}(f_i)\mathbf{Z}(i)\,.$$

And then the signal subspace can be obtained from knowledge of the covariance of $\mathbf{W}(i)$

$$\mathbf{R}_{ww}(i) = \mathbf{T}_{opt}(f_i)\tilde{\mathbf{A}}(f_i)\mathbf{R}_{ss}(i)\tilde{\mathbf{A}}^H(f_i)\mathbf{T}^H{}_{opt}(f_i) + \frac{1}{J}\sigma_n{}^2\mathbf{I}\,.$$

Furthermore, according to (12)-(16), we can obtain $\mathbf{R}_{ww}$ similar to (15) in expression. Furthermore, the 2(N-1)-D smallest eigenvalues of $\mathbf{R}_{ww}$ are equal to $\sigma_n{}^2$. The D eigenvectors corresponding to the D largest eigenvalues are used to obtain $\mathbf{E}_s = [\mathbf{e}_1 | \ldots | \mathbf{e}_D]$, where $\mathrm{span}\{\mathbf{E}_s\} = \mathrm{span}\{\tilde{\mathbf{A}}\}$, the signal subspace. Furthermore, $\mathbf{E}_s$ can be decomposed into $\mathbf{E}_u$ and $\mathbf{E}_v$ so that

$$\mathbf{E}_s = \begin{bmatrix} \mathbf{E}_u \\ \mathbf{E}_v \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{G} \\ \mathbf{A}\mathbf{\Phi}\mathbf{G} \end{bmatrix}$$

where $\mathbf{G}$ a unique, nonsingular matrix is satisfies $\mathbf{E}_s = \mathbf{A}\mathbf{G}$. Since $\mathbf{E}_u$ and $\mathbf{E}_v$ share a same common space, the rank of $\mathbf{E}_{uv} \triangleq [\mathbf{E}_u \,|\, \mathbf{E}_v]$ is D, which implies there exists a unique rank D matrix $\mathbf{F}$ such that

$$\mathbf{0} = [\mathbf{E}_u \,|\, \mathbf{E}_v]\mathbf{F} = \mathbf{E}_u\mathbf{F} + \mathbf{E}_v\mathbf{F} = \mathbf{A}\mathbf{G}\mathbf{F}_u + \mathbf{A}\mathbf{\Phi}\mathbf{G}\mathbf{F}_v .$$

Defining $\mathbf{\Psi} \triangleq -\mathbf{F}_u[\mathbf{F}_v]^{-1}$ and then we get $\mathbf{G}\mathbf{\Psi}\mathbf{G}^{-1} = \mathbf{\Phi}$. Therefore, the eigenvalues of $\mathbf{\Psi}$ must be equal to $\mathbf{\Phi}$.

At the end of this section, we give the summary of our method as follows.

1) *Obtain the received data* $\mathbf{X}$ *from the output of the array of N sensors.*
2) *Compute the DFT coefficients of the received data* $\mathbf{X}$ *and determine* $i_{\min}$, $i_{\max}$ *and* f_{opt}.
3) *Construct the focusing matrices* $\mathbf{T}_{opt}(f_i), i_{\min} \le i \le i_{\max}$.
4) *Compute the transformed vector* $\mathbf{W}(i)$ *and get* $\mathbf{R}_{ww}$*, denoted as* $\hat{\mathbf{R}}_{ww}$.
5) *Compute the eigendecomposition of* $\hat{\mathbf{R}}_{ww}$

$$\hat{\mathbf{R}}_{ww}\hat{\mathbf{E}} = \hat{\mathbf{E}}\mathbf{\Lambda}$$

where $\mathbf{\Lambda} = diag(\lambda_1, \ldots, \lambda_{2D}), \lambda_1 \ge \ldots \ge \lambda_{2N-2}$, and $\hat{\mathbf{E}} = [\mathbf{e}_1 \,|\, \ldots \,|\, \mathbf{e}_{2N-2}]$.

6) *Estimate the number of sources of* $\hat{D}$.
7) *Obtain the signal subspace estimate* $\hat{\mathcal{R}}\{\mathbf{E}_s\}$ *and decompose it to obtain* $\mathbf{E}_u$ *and* $\mathbf{E}_v$*, where*

$$\mathbf{E}_s \triangleq [\mathbf{e}_1 \,|\, \ldots \,|\, \mathbf{e}_{\hat{D}}] = \begin{bmatrix} \mathbf{E}_u \\ \mathbf{E}_v \end{bmatrix}$$

8) *Compute the eigendecoposition* $(\lambda_1 \ge \ldots \ge \lambda_{2D})$

$$\mathbf{E}^*_{uv}\mathbf{E}_{uv} \triangleq \begin{bmatrix} \mathbf{E}_u^* \\ \mathbf{E}_v^* \end{bmatrix} [\mathbf{E}_u \,|\, \mathbf{E}_u] = \mathbf{E}\Lambda\mathbf{E}^*$$

and partition $\mathbf{E}$ into $D \times D$ matrices,

$$\mathbf{E} \triangleq \begin{bmatrix} \mathbf{E}_{11} & \mathbf{E}_{12} \\ \mathbf{E}_{21} & \mathbf{E}_{22} \end{bmatrix}$$

9) *Calculate the eigenvalues of* $\mathbf{\Psi} = -\mathbf{E}_{12}\mathbf{E}_{22}^{-1}$, $\hat{\phi}_k, k = 1, \ldots, D$.
10) *Estimate* $\hat{\theta}_k = f^{-1}(\hat{\phi}_k) = \cos^{-1}\{c\arg(\hat{\phi}_k)/(2\pi f_{opt}\Delta)\}$.

4. Simulation results

In this section we present computer simulation results that illustrate the performance of the proposed method. The example refers to a uniform linear array.

We considered three wideband sources having identical spectra, centered at 150Hz with relative bandwidth 66.7% and sample frequency 400Hz, impinging from 80°, 90° and 100° on a array of ten sensors spaced half a wavelength apart. The results obtained from M=100 snapshots of K=256 samples of a finite interval 64s, using the autofocusing approach and the proposed method are presented in Fig. 2. The SNR in this figure was 10 dB and the Monte Carlo times is 10. In Fig. 3, we plot the root mean square error (RMSE) of the autofocusing approach at different focusing frequency bins versus SNR. And then, the RMSE of the performance at different sample frequency is presented versus SNR in Fig. 4. At last, in Fig. 5 we compare the performance of the autofocusing approach and the proposed method versus SNR.

It is clear in Fig. 2 that three sources are distinguished both by the two methods with similar errors. The performance of the autofocusing approach at the optimal focusing frequency provides least RMSE in Fig. 3 compared with other focusing frequency bins. So, we can conclude that the RMSE of angle estimates isn't related to the sample frequency satisfying bandpass sampling theorem. Last in Fig. 5, it is cleared seen that the performance of the proposed method is similar to or a little better than the autofocusing approach at mid SNR, however, it performs worse in low and high SNR compared with the autofocusing approach, which is because the proposed method utilizes less priori information than the autofocusing approach. However, the proposed method avoids the calibration of the array during the whole processing.

5. Conclusion

In this paper, we have introduced a robust method for wideband DOA estimation which does not require any preliminary DOA estimation or calibration of the array during the whole estimating process with the performance similar to the autofocusing approach. Our method is based on the autofocusing approach and uses the ESPRIT algorithm during the narrowband processing stage. Meanwhile, we suggest the method of selecting the optimal focusing frequency which

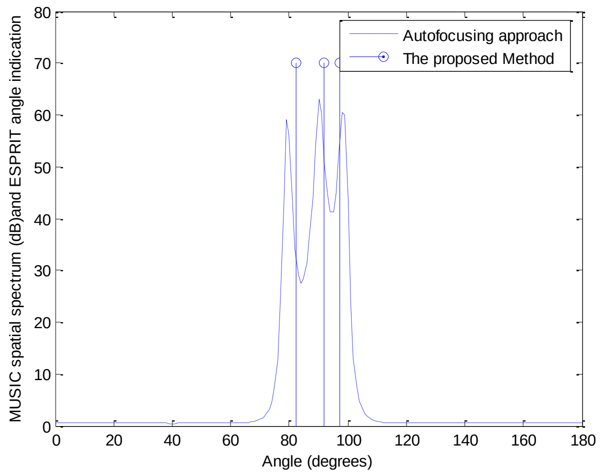

Figure 1. Comparison of the MUSIC spacial spectrum and the ESPRIT indication

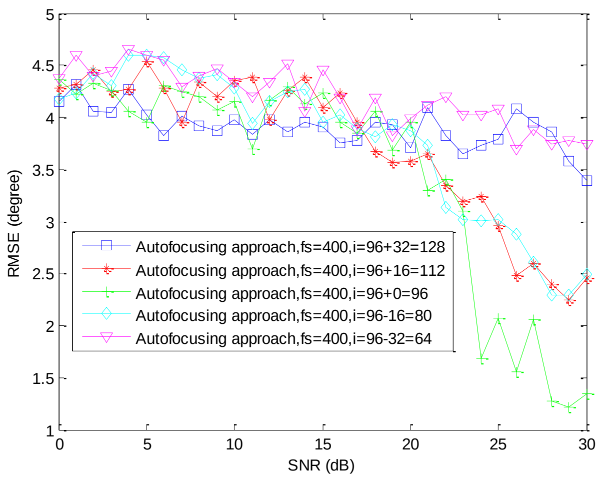

Figure 2. Comparison of RMSE of different sample frequencies of the autofocusing approach at the optimal focusing frequency vesus SNR

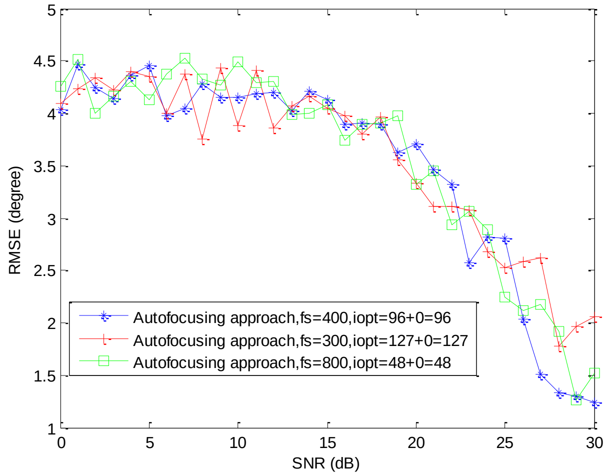

Figure 3. Comparison of RMSE of the autofocusing approach and the proposed method at the optimal focusing approach

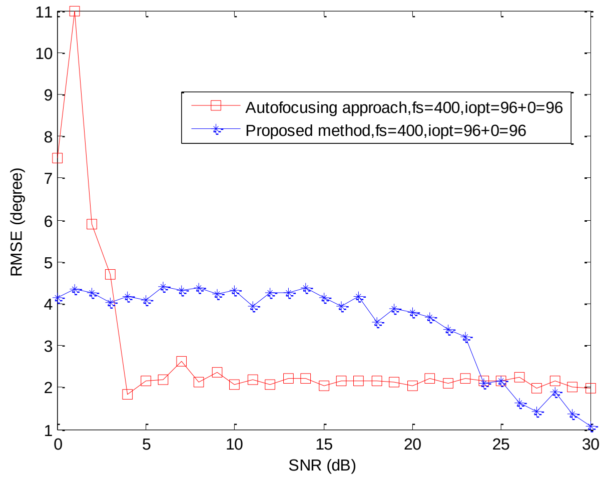

Figure 4. Comparison of RMSE of different focusing frequency bins of the autofocusing approach vesus SNR

minimizes the mean square error. The new method also leads lower computational complexity. In the future, the work will be concentrated on the relationship between the subspaces of different focusing frequency bins and analyze the computational complexity of the proposed method.

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