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# BER Performance Optimization of the SFBC-OFDM System for Economical Receiver Design with Imperfect Channel Estimation

Md. Jakaria Rahimi<sup>a\*</sup>, Md. Shaikh Abrar Kabir<sup>a</sup>, Azhar Niaz<sup>a</sup>, Md. Jahidul Islam<sup>a</sup>,Oli Lowna Baroi<sup>a</sup>

<sup>a</sup>Department of Electrical and Electronic Engineering, Ahsanullah University of Science & Technology, Dhaka-1208, Bangladesh

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# Abstract

In this paper, the bit error rate (BER) performance of SFBC-OFDM systems for frequency selective fading channels is observed for various antenna orientations and modulation schemes. The objective is to find out a suitable configuration with minimum number of receiving antenna that requires minimum signal power level at the receiver to provide reliable voice and video communication. We have considered both M-ary phase shift keying (MPSK) and M-ary quadrature amplitude modulation (MQAM) in the performance analysis considering both perfect and imperfect channel state information (CSI). The authors have expressed the BER under imperfect channel estimation condition as a function of BER under perfect channel condition in this paper. The finding shows, for a BTS with 4 transmitting antenna and MS with 2 receiving antenna BPSK performs better for both perfect and imperfect CSI. Maximum permissible channel estimation error increases with the usage of more receiving antenna at the expense of increased cost.

**Index Terms:** Space-frequency block coding (SFBC), Orthogonal frequency division multiplexing (OFDM), Rayleigh fading, Bit error rate (BER), Channel state information (CSI).

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# 1. Introduction

The inter-symbol interference (ISI) due to Multipath fading channel is one of the major problem in the modern broadband high-speed wireless communication system. Orthogonal Frequency Division Multiplexing (OFDM) mitigates the ISI problem by converting a frequency-selective channel into many parallel frequency-

\* Corresponding author. E-mail address: mjrahimi@gmail.com flat subchannels [1]. MIMO offers the benefits of spatial multiplexing and transmits diversity which enhances the function of the OFDM system [2]. Recently, space-frequency coded OFDM are being used to exploit the diversities in space and frequency. Considering the numerical analysis of the systems, SFBC-OFDM has been presented and appraised [3]-[11].

The major source of high data rate application includes mainly different forms of video over the wireless communication channels. Throughout the world, 82 percent of traffic will occur by IP video traffic by 2021 [Source: Cisco VNI, 2017]. So, we tried to optimize the performance for video data only. For reliable transmission of high data rate, video traffic bit error rate (BER) criterion is monitored. BER should be lower than  $10^{-6}$  for reliable transmission of a video signal [12].

In our research, the authors have firstly investigated how the BER performance is affected by various parameters considering both perfect and imperfect CSI. Secondly, the authors have expressed the BER under imperfect channel estimation condition as a function of BER under perfect channel condition. Finally, the value of the signal to noise ratio (SNR) at which the BER value reaches the threshold level of  $10^{-6}$  for different antenna configurations and modulation schemes is monitored. For a low-cost receiver design, we need to choose less number of the antenna to get comparable yet compromised performance. Battery's lifetime is an important issue to notice. Lower SNR will certainly increase the lifetime. So, we have tried to choose the techniques with lower SNR. We have simulated the results for both perfect CSI and imperfect CSI. In imperfect CSI, we have also observed how performance is deteriorated when the value of channel estimation error ( $\sigma e^2$ ) isincreased.

#### 2. OFDM system model with SFBC

In Fig.1, we represent a block diagram of OFDM system with SFBC with  $N_T$  transmitter and  $N_R$  receiver antennae. Here data streams are first converted from series to parallel. We assume our model with N subcarriers and the number of sub-bands is Ns where Ns = N/q. Here, q is the symbol period. After that, we modulate our data using M-ary QAM M-ary PSK. Here, M is allocated bit number of this modulation. Therefore, as the input to the SFBC encoder, we provide a signal  $S = \{s [0], s [1], \dots, s [N_t - 1]\}$ . Here,  $N_t$  is equal to the multiplication of N and Rc, where Rc is SFBC code rate.



Fig. 1. OFDM with SFBC in block diagram

To implement SFBC, we have utilized the generator matrix G as used in STBC which can be represented as follows

$$G = \begin{pmatrix} g_{11} & g_{12} & \dots & g_{1N_T} \\ g_{21} & g_{22} & \dots & g_{2N_T} \\ \vdots & \vdots & \dots & \vdots \\ g_{p1} & g_{p2} & \dots & g_{pN_T} \end{pmatrix}$$
(1)

Here, every element  $g_{i,j}$  of matrix G is a combination of the subset of elements of S and their conjugates. The matrix G maintains orthogonality [2], [13]. To utilize both the space and frequency diversity in SFBC message is divided into N<sub>T</sub> blocks, S<sub>1</sub>, S<sub>2</sub>, ..., S<sub>N<sub>T</sub></sub>, each has a length of N and  $\frac{N}{p}$  sub-blocks. After that, OFDM symbols are generated as X<sub>1</sub>, X<sub>2</sub>, ..., X<sub>N<sub>T</sub></sub>. In OFDM transmitter there are IFFT and add CP blocks.

To avoid inter-symbol interference (ISI) due to frequency selective fading the delay spread should be smaller than the guard time interval of the multipath channel.Here, the receiver will get a signal which is transmitted signal is convolved with and channel impulse response in time domain.

Generally, the fading channel changes slowly if we consider ideal indoor environments and in OFDM block it remains almost stable. So, we have considered the fading method stable in each OFDB and there is a change is different blocks. We have presumed real synchronization, as fading method amalgamates with different antenna pairs which create a correlation.

At the receiving side, the OFDM demodulator demodulates the signal by removing CP and doing FFT. Then SFBC decoder block decodes the signal, here we use maximum likelihood (ML) detection and sent it to demapping block. After demapping, the data streams are converted from parallel to series.



Fig. 2. The process of SFBC in OFDM [14]

In Fig.2, for transmitting x1, x2, x3 & x4 data symbols by SFBC-OFDM, we transmit x1 and x2 using one frequency but from different antennae. At the same time, we transmit  $-x2^*$  and x1\* using another frequency but from different antennae. Similarly, we transmit x3 & x4 symbols.

#### 3. BER performance of the SFBC-OFDM system

In this section, closed-form expressions for the BER of OFDM systems incorporated with SFBC having  $N_T$  transmit and  $N_R$  receive antennae are presented for both M-ary QAM and M-ary PSK modulation techniques. [15]

#### 3.1. SFBC-OFDM System with M-ary QAM

An SFBC-OFDM system with M-ary QAM with  $N_T$  transmit and  $N_R$  receive antennae is considered here. After normalization of the transmitted signal energy, the normalized instantaneous SNR can be expressed as

$$\boldsymbol{\gamma} = \frac{1}{M_T R_c} \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} \left| \boldsymbol{H}_{j,i}[\boldsymbol{k}] \right|^2 \boldsymbol{\gamma}_s \tag{2}$$

where  $H_{j,i}[k]$  represents the channel coefficient of the k-thsubchannel with the *i*-th transmit and *j*-th receive antennae, *Rc* is the code rate,  $\eta[k]$  is the noise component,  $\gamma_s = \frac{E_s}{N_0}$  with Es the symbol energy at the transmitter and  $\frac{N_0}{2}$  the variance of thereal/imaginary part of the AWGN. So, we can express the BER of MQAM-SFBC-OFDM over frequency-selective fading channels as

$$BER_{MQAM} = \frac{0.2}{N} \sum_{k=0}^{N-1} exp\left(\frac{-1.6\gamma_s \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |H_{j,i}[k]|^2}{R_c M_T (2^{\beta} - 1)}\right)$$
(3)

Finally, we can express the average BER as,

$$\overline{BER}_{MQAM} = \mathbf{0}.\,\mathbf{2} \times \left(\frac{R_C M_T (2^\beta - 1) + 1.6\gamma_s}{R_C M_T (2^\beta - 1)}\right)^{-M_R M_T} \tag{4}$$

If we take channel estimation errors into consideration, the received signal can be written as

$$\mathbf{r} = \mathbf{\hat{H}}\mathbf{s} + \mathbf{e}\mathbf{s} + \mathbf{W} \tag{5}$$

where a minimum mean-squared error (MMSE) channel estimate of the channel is considered,  $\hat{H} = H - e$ , with the estimation error at the receiver. The MMSE has the following properties:

$$E\left[\left(H-\widehat{H}\right)\left(H-\widehat{H}\right)^{\dagger}|\widehat{H}\right] = \sigma_{e}^{2}I_{N}N_{R},$$
$$E\left[H|\widehat{H}\right] = \widehat{H},$$

where (.)<sup>*†*</sup> denotes the complex conjugate transpose operation,  $I_N N_R$  is the  $N N_R \times N N_R$  identity matrix and  $\sigma_e^2$  indicates the channel estimation quality.

So, we can express the instantaneous SNR of SFBC-OFDM over frequency-selective fading channels under imperfect CSI at the receiver as

$$\gamma = \frac{\sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |\hat{H}_{j,i}[k]|^2 \gamma_s}{R_C M_T (1 + \gamma_s \sigma_e^2)}$$
(6)

Therefore, the expression for the BER of MQAM-SFBC-OFDM with imperfect CSI is given by

$$BER_{MQAM} = \frac{0.2}{N} \sum_{k=0}^{N-1} exp\left(\sqrt{\frac{-1.6\gamma_s \sum_{j=1}^{M_R} \sum_{i=1}^{M_T} |\hat{H}_{j,i}[k]|^2}{R_c M_T (2^{\beta} - 1)(1 + \gamma_s \sigma_e^2)}}\right)$$
(7)

Then, we can show the average BER expression similarly as the perfect CSI case as below

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$$\overline{BER}_{MQAM} = \mathbf{0}.2 \times \left(\frac{R_C M_T (2^\beta - 1)(1 + \sigma_e^2 \gamma s) + 1.6 \gamma_s}{R_C M_T (2^\beta - 1)(1 + \sigma_e^2 \gamma s)}\right)^{-M_R M_T}$$
(8)

$$\overline{BER}_{MQAM(imperfect)} = 0.2 \times \left( \left( \frac{\overline{BER}_{MQAM(perfect)}}{0.2} \right)^{\frac{-1}{M_R M_T}} - \frac{1.6\sigma_e^2 \gamma_s^2}{R_c M_T (2^\beta - 1)(1 + \sigma_e^2 \gamma_s)} \right)^{-M_R M_T}$$
(9)

# 3.2. MPSK-SFBC-OFDM System

The same approach as used for the MQAM case, the BER of MPSK-SFBC-OFDM can be expressed as

$$BER_{MPSK} = \frac{1}{N\beta} \sum_{k=0}^{N-1} exp\left(\frac{-7\gamma_s \sum_{j=1}^{M_R} \sum_{k=1}^{M_T} |H_{j,i}[k]|^2}{R_c M_T (2^{1.9\beta} + 1)}\right)$$
(10)

Similarly, as MQAM case, we can obtain a closed-form expression for the average BER:

$$\overline{BER}_{MPSK} = \mathbf{0} \cdot \mathbf{2} \times \left(\frac{R_C M_T (2^{1.9\beta} + 1) + 7\gamma_s}{R_C M_T (2^{1.9\beta} + 1)}\right)^{-M_R M_T}$$
(11)

If we consider imperfect channels, then we can write

$$\overline{BER}_{MPSK} = \mathbf{0} \cdot \mathbf{2} \times \left( \frac{R_C M_T (2^{1.9\beta} + 1)(1 + \sigma_e^2 \gamma_s) + 7\gamma_s}{R_C M_T (2^{1.9\beta} + 1)(1 + \sigma_e^2 \gamma_s)} \right)^{-M_R M_T}$$
(12)

$$\overline{BER}_{MPSK(imperfect)} = 0.2 \times \left( \left( \frac{\overline{BER}_{MPSK(perfect)}}{0.2} \right)^{\frac{-1}{M_R M_T}} - \frac{7\sigma_e^2 \gamma_s^2}{R_c M_T (2^\beta - 1)(1 + \sigma_e^2 \gamma_s)} \right)^{-M_R M_T}$$
(13)

4. Results



Fig. 3. Comparing QAM 16 and QAM 64 for  $N_T$  =1  $\&N_R$  =1

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We have simulated considering the number of transmit and receiver antennae:  $N_T = 1$ , 2, and 4:  $N_R = 1$  and 2. For imperfect CSI, the values of  $\sigma_e^2 = 0.005$ , 0.01, 0.02, and 0.05. The number of subcarriers is N = 512. Modulation schemes for our simulations: BPSK, QPSK, 8PSK, QAM16, and QAM64.



Fig. 4. Comparing BPSK, QPSK, and 8PSK for  $N_T = 1 \& N_R = 1$ 

From Fig. 3 and Fig. 4 we can say those performances are better when the number of bit(s) allocated per symbol is less and in case of perfect CSI than imperfect CSI.

Table 1.E<sub>b</sub>/N<sub>0</sub>(dB) at BER=10<sup>-6</sup> for N<sub>R</sub> =1 & $\sigma_e^2 = 0$ 

Modulation Techniques	N <sub>T</sub> =1	N <sub>T</sub> =2	N <sub>T</sub> =4
QAM 16	56.7	33.2	22.7
QAM 64	61.1	37.6	27.2
BPSK	51.3	27.8	17.2
QPSK	53.2	29.7	19.3
8PSK	57	33.5	23

From Table 1. it is clear that increasing Transmit antenna gives us better performances. From the table it is also evident that the SNR level required for BPSK is lowest. It's nearest competitive performance is offered by QPSK. The performances of 8PSK and QAM 16 are very much similar to each other. BER performance deteriorates when we chose higher order modulation techniques like QAM 64.

From Fig. 5 and Fig. 6 it is clearly visible that the performance gets drastically improved when the number of antenna is increased from 1 to 2. The trend is observed for different modulation techniques and only the case

of BPSK and QAM 16 is presented as sample which gives the indication that increasing the number of transmit antenna and receive antenna always results in better performances.



Comparing curves using QAM 16 for N<sub>T</sub> = 4

Fig. 5. Comparing curves using QAM 16 for  $N_T = 4$ 



Fig. 6. Comparing curves using BPSK for  $N_T = 4$ 

We have found the values of  $E_b/N_0$  where BER for perfect CSI is  $10^{-6}$  using different combinations of  $N_T$  and  $N_R$ . In Fig. 7 and Fig. 8 we have plotted bar-diagram of BER at those  $E_b/N_0$  for different values of  $\sigma_e^2$ .

In both cases we can see that for a particular  $\sigma_e^2$  BER becomes less if we increase the number of N<sub>T</sub> and N<sub>R</sub>. And for a particular value of N<sub>T</sub> and N<sub>R</sub>, increasing  $\sigma_e^2$  results in a higher value of BER which means worse performance.

But in the MQAM case, BER increases slowly with the increase of  $\sigma_e^2$  whereas in MPSK case, BER increases quickly if we increase the value of  $\sigma_e^2$ .



Fig. 7. Comparison of performances between perfect & imperfect CSI for QAM 64





Fig. 8. Comparison of performances between perfect & imperfect CSI for QPSK

Modulation Technique	$\begin{array}{l} \text{Perfect} \\ ({\sigma_e}^2 = 0) \end{array}$	$\sigma_e^2 = 0.005$	$\sigma_e^2 = 0.01$	$\sigma_e^2=0.02$
BPSK	9.8	10	10.3	10.8
QPSK	11.8	12.5	13.4	15.9
8PSK	15.6	19	00	00
QAM16	15.2	20	œ	œ
QAM64	19.7	$\infty$	ω	ω

Here  $\infty$  means curves become parallel to x-axis before reaching BER=10<sup>-6</sup>. Here we can see that as we increase the number of bit(s) allocated and  $\sigma_e^2$  we require higher  $E_b/N_0$  and at some points curves don't even reach BER=10<sup>-6</sup>. We can also observe here that BPSK and QPSK give better performances than others.



Fig. 9. Comparing BPSK, QPSK, 8PSK, QAM 16 and QAM 64 for  $\sigma_e^2=0.02$ ,  $N_T = 4$ , and  $N_R = 1$ 

From Fig. 9 we can say for  $N_R = 1$  if  $\sigma_e^2 = 0.02$  even BER for BPSK can't reach  $10^{-6}$ .

# 5. Conclusions

From our analysis, we have found that for increasing the number of transmitters and receivers, SNR decreases for BER=10<sup>-6</sup>. BPSK technique is always better. If we can reduce the number of receiving antennae, the cost can be reduced. But the performances surely get deteriorated. So we have found that BPSK with 2 receivers can be the best technique after comparing with QPSK, 8PSK, 16QAM and 64QAM modulation techniques for different antenna orientations. It is because the performance drastically gets better with 2 receivers than 1 receiver. For  $N_R = 1$ ,  $\sigma_e^2$  should be less than 0.02 as even BER for BPSK can't reach 10<sup>-6</sup> at  $\sigma_e^2$ =0.02. But with  $N_R$  =2, we can allow more channel estimation error. More receiver antennae will result in better performances but the battery consumption and hardware cost will surely go high. In our work we have expressed BER with imperfect CSI as a function of BER with perfect CSI where the parameters those are responsible for performance deterioration are separated. It surely helps the researchers to focus on these parameters to measure the penalty due to imperfect CSI.

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# **Authors' Profiles**



**Md. JAKARIA RAHIMI** received his B.Sc (Hons) and M.Sc majoring in Electrical and Electronic from Bangladesh University of Engineering and Technology (BUET). He is currently working as an Assistant Professor at the Faculty of Electrical and Electronic Engineering at Ahsanullah University of Science and Technology. His research interests include Speech processing and Digital signal processing. He has been pursuing his Ph.D. degree since 2016 from BUET.



**Md. SHAIKH ABRAR KABIR** received his B.Sc (Hons) majoring in Electrical and Electronic from Ahsanullah University of Science and Technology in 2018. He is currently working as a Lecturer in the Faculty of Electrical and Electronic Engineering at the same University. His research interests include Speech processing, Wireless communications, and Electronics.



**AZHAR NIAZ** received his B.Sc (Hons) majoring in Electrical and Electronic from Ahsanullah University of Science and Technology in 2018. His research interests include Speech processing, communications, and Electronics.



**Md. JAHIDUL ISLAM** received his B.Sc (Hons) majoring in Electrical and Electronic from Ahsanullah University of Science and Technology in 2018. His research interests include Speech processing, communications, and Electronics.



**OLI LOWNA BAROI** received her B.Sc (Hons) majoring in Electrical and Electronic from Ahsanullah University of Science and Technology in 2018. She is currently working as a Lecturer in the Faculty of Electrical and Electronic Engineering at the same University. Her research interests include Speech processing, communications, and Electronics.

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