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Testing Coefficients of Autoregressive Conditional Heteroskedasticity Models by Graphical Approach

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Abstract

The graphical approach is applied to the autoregressive conditional heteroskedasticity time series models. After transformation, it is shown that the coefficients of GARCH model are the conditional correlation coefficients conditioned on the other components of the time series, then a new method is proposed to test the significance of the coefficients of GARCH model.

Index Terms: Time Series Chain Graph; ARCH; GARCH

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1. Introduction

Financial economists are concerned with modelling dependence in asset returns. Observations in return series of financial assets observed at daily are uncorrelated or nearly uncorrelated, the series contain higher-order dependence. The models of autoregressive conditional heteroskedasticity (ARCH)[1] form the most popular way of parameterizing this dependence. Its generalization, the Generalized ARCH (GARCH) model[2] is a more parsimonious model of the conditional variance than a high-order ARCH model, most users prefer it to the simpler ARCH alternative.

Graphical models have become an important method for the analyzing multivariate data[3-6]. Recently they have been introduced to model dependence structures among multivariate time series[7-11]. Graphs to describe multivariate time series can be classified basically into the time series chain graph, partial correlation graph and Granger causality graph.

In this paper, we express the autoregressive conditional heteroskedasticity models as time chain graphical models. In time series chain models, each vertex in a graph represents a separate variable at fixed time, and an

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edge between two vertices denotes that the two variables have causal relations or dependence. The direction of an edge between two vertices is determined by both time and partial correlation. We show that the coefficient of GARCH model deformation is the partial correlation coefficient after removing the linear effects of the other components of the time series. A new procedure with graphical approach is proposed to test the coefficients of GARCH model.

2. Time series chain graph and garch model

We introduce a time series chain graph (TSC-graph) by firstly giving the definition of a graph over a finite and nonempty set V . Let $G = (V, E)$, elements in V are called vertices, and E is a set of directed or undirected edges denoted as $a \rightarrow b, a - b$. $a \rightarrow b$ is called directed edges, while $a - b$ is called the undirected edge.

Suppose that $X = (X_1(t), X_2(t), \dots, X_d(t))'$ is a d-dimensional stationary time series. The TSC-graph of the stationary process X is the graph $G_{TS} = (V_{TS}, E_{TS})$ where $V_{TS} = V \times Z$ with $V = \{1, 2, \dots, d\}$ and Z being an integer set, and the edge set E_{TS} satisfies

- 1) $(a, t - u) \rightarrow (b, t) \notin E_{TS} \Leftrightarrow u \leq 0$ or $X_a(t - u) \perp X_b(t) \mid \bar{X}(t) \setminus \{X_a(t - u)\}$
- 2) $(a, t - u) - (b, t) \notin E_{TS} \Leftrightarrow u \neq 0$ or $X_a(t) \perp X_b(t) \mid \bar{X}_V(t) \cup \{X_{V \setminus \{a, b\}}(t)\}$

where $\bar{X}_A(t) = \{X_A(s), s < t\}$ denotes the past of the subprocess $X_A = \{X_A(t)\}$ at time t . Here it involves conditional orthogonality. For random vectors X, Y and Z , we call X and Y to be conditionally orthogonal given Z , denoted by $X \perp Y \mid Z$, if X and Y are uncorrelated after the linear effects of Z have been removed.

For stationary ARCH(q) model

$$\begin{cases} X_t = \sigma_t \varepsilon_t \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i X_{t-i}^2 \end{cases} \quad (2.1)$$

where $\{\varepsilon_t\}$ is i.i.d. Normal random with mean zero and variance 1 and $\alpha_0 > 0, \alpha_i \geq 0 (i = 1, \dots, q)$.

Simple transformation is given as follows:

$$X_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i X_{t-i}^2 + \eta_t \quad (2.2)$$

where $\eta_t = \sigma_t^2 (\varepsilon_t^2 - 1)$ is uncorrelated time series with mean zero and variance λ_1 .

Let $Y_t = X_t^2$, then

$$Y_t = \alpha_0 + \sum_{i=1}^q \alpha_i Y_{t-i} + \eta_t \quad (2.3)$$

In order to verify the auto-regressive conditional heteroskedasticity, we need to test $H_0 : \alpha_i = 0 (i = 1, \dots, q)$ via $H_1 : \exists \alpha_i \neq 0$. So we test the significance of the coefficients for AR(q) model.

Since we only deal with univariate time series, V_{TS} in TSC-graph $G_{TS} = (V_{TS}, E_{TS})$ is $Z = \{\dots, -1, 0, 1, \dots\}$, each vertex represents one component of Y_t at some time. As an illustration, Fig. 1 presents a TSC graph G_{TS} for ARCH(2) time series Y_t , where $t-2 \rightarrow t$ denotes that Y_{t-2} is causal for Y_t and an absent edge between $t-1$ and t implies that there does not exist a causal relation between Y_{t-1} and Y_t . The test of coefficients based on graphical approach for AR model has been discussed in detail [12].

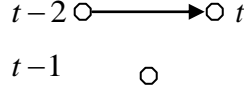


Fig. 1 The graph for ARCH(2)

For stationary GARCH(p,q) model

$$\begin{cases} X_t = \sigma_t \varepsilon_t \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i X_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 \end{cases} \quad (2.4)$$

where $\{\varepsilon_t\}$ is i.i.d. Normal random with mean zero, variance 1 and $\alpha_0 > 0, \alpha_i, \beta_j \geq 0 (i=1, \dots, q, j=1, \dots, p)$.

$$\text{Let } Y_t = X_t^2, \varphi_i = \begin{cases} \alpha_i + \beta_i, 1 \leq i \leq \min(p, q) \\ \alpha_i, p < i \leq q \\ \beta_i, q < i \leq p \end{cases}$$

Simple transformation is given as follows:

$$Y_t = \alpha_0 + \sum_{i=1}^m \varphi_i Y_{t-i} - \sum_{i=1}^p \beta_i \eta_{t-i} + \eta_t \quad (2.5)$$

where $m = \max(p, q)$, $\eta_t = \sigma_t^2 (\varepsilon_t^2 - 1)$ is uncorrelated time series with mean zero and variance λ_2 . The most popular GARCH model in applications has been the GARCH(1,1) model. For stationary GARCH(1,1)

$$\begin{cases} X_t = \sigma_t \varepsilon_t \\ \sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{cases} \quad (2.6)$$

Simple transformation is given as follows:

$$Y_t = \alpha_0 + \varphi_1 Y_{t-1} - \beta_1 \eta_{t-1} + \eta_t \quad (2.7)$$

where $\varphi_1 = \alpha_1 + \beta_1$.

Let $y_{1,t} = Y_t, y_{2,t} = \eta_t$, Fig. 2 presents a TSC graph G_{TS} for model (2.7), Vertex i in V represents time series $y_{i,t}$, while vertex $(i, t-u)$ in TSC-graph $V_{TS} = V \times Z$ denotes the component $y_{i,t-u}$. We know from the definition of the TSC-graph that $(1, t-1) \rightarrow (1, t)$ denotes that Y_{t-1} is causal for Y_t and an absent edge between $(2, t-1)$ and $(1, t)$ implies that η_{t-1} is noncausal for Y_t .

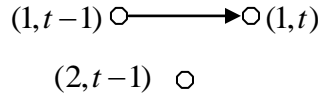


Fig. 2 The graph for GARCH(1,1)

To verify the conditional heteroskedasticity, we need to test $H_0 : \varphi_i = 0$ for GARCH(p,q) model. So we need to test the significance of the coefficients for model (2.5).

3. III. parametric tests by graphical models

Let $\pi = \{\mu + \alpha U + \beta V \mid \mu, \alpha, \beta \in R\}$ be a linear subspace spanned by random variables U and V . The best linear predictor, denoted by $Proj_{\pi} X$, of X on U and V is defined as

$$E(X - Proj_{\pi} X)^2 = \min_{\mu, \alpha, \beta} E(X - \mu - \alpha U - \beta V)^2 \quad (3.1)$$

$Proj_{\pi} X$ is called the linear projector of X on π . The linear property holds for $Proj_{\pi} X$.

Proposition 3.1

$$Proj_{\pi}(aX + bY) = aProj_{\pi} X + bProj_{\pi} Y \quad (3.2)$$

The definition of the linear projector $Proj_{\pi} X$ can be generalized to a general linear subspace π spanned by random variables X_1, X_2, \dots, X_n . With $Proj_{\pi} X$, we readily define the partial covariance as follows:

$$Cov_L(X, Y \mid \pi) = E(X - Proj_{\pi} X)(Y - Proj_{\pi} Y)$$

Theorem 3.1 For model (2.5)

$$1) (1, t-k) \rightarrow (1, t) \in E_{TS} \Leftrightarrow k \in \{1, \dots, m\} \text{ and } \varphi_k \neq 0. \quad (3.3)$$

$$2) (2, t-k) \rightarrow (1, t) \in E_{TS} \Leftrightarrow k \in \{1, \dots, p\} \text{ and } \beta_k \neq 0. \quad (3.4)$$

1) **Proof.** We know from the definition of the TSC-graph that

$$(1, t-k) \rightarrow (1, t) \notin E_{TS} \Leftrightarrow Y_{t-k} \perp Y_t \mid \pi^{(1,-k)}$$

which is equivalent to $Cov_L(Y_{t-k}, Y_t \mid \pi^{(1,-k)}) = 0$, where $\pi^{(1,-k)}$ is a linear subspace spanned by $\{Z_s, s < t\} - \{Y_{t-k}\}$ with $Z_t = (Y_t, \eta_t)'$. As $Proj_{\pi^{(1,-k)}} \eta_t = 0$, it follows from (3.2) and the linear property

$$\text{of } Proj_{\pi} Y \text{ that } Proj_{\pi^{(1,-k)}} Y_t = \sum_{\substack{j=1 \\ j \neq k}}^m \varphi_j Y_{t-j} - \sum_{j=1}^p \beta_j \eta_{t-j} + \varphi_k \cdot Proj_{\pi^{(1,-k)}} Y_{t-k}$$

Then we have

$$\begin{aligned} \gamma_{k \mid \pi^{(1,-k)}} &= Cov_L(Y_{t-k}, Y_t \mid \pi^{(1,-k)}) \\ &= E(Y_t - Proj_{\pi^{(1,-k)}} Y_t)(Y_{t-k} - Proj_{\pi^{(1,-k)}} Y_{t-k}) \\ &= E\{(\varphi_k (Y_{t-k} - Proj_{\pi^{(1,-k)}} Y_{t-k}) + \eta_t)(Y_{t-k} - Proj_{\pi^{(1,-k)}} Y_{t-k})\} \\ &= \varphi_k E\{(Y_{t-k} - Proj_{\pi^{(1,-k)}} Y_{t-k})^2\} \\ &= \varphi_k Var(Y_{t-k} \mid \pi^{(1,-k)}) \end{aligned} \quad (3.5)$$

Then (3.3) follows from (3.5) directly.

2) **Proof.** We know from the definition of the TSC-graph that

$$(2, t-k) \rightarrow (1, t) \notin E_{TS} \Leftrightarrow \eta_{t-k} \perp Y_t \mid \pi^{(2,-k)}$$

which is equivalent to $Cov_L(\eta_{t-k}, Y_t \mid \pi^{(2,-k)}) = 0$, where $\pi^{(2,-k)}$ is a linear subspace spanned by $\{Z_s, s < t\} - \{\eta_{t-k}\}$.

Then we have

$$\begin{aligned} \gamma_{k|\pi^{(2,-k)}} &= Cov_L(\eta_{t-k}, Y_t \mid \pi^{(2,-k)}) \\ &= E(Y_t - Proj_{\pi^{(2,-k)}} Y_t)(\eta_{t-k} - Proj_{\pi^{(2,-k)}} \eta_{t-k}) \\ &= E\{(\beta_k(Proj_{\pi^{(2,-k)}} \eta_{t-k} - \eta_{t-k}) + \eta_t)(Y_{t-k} - Proj_{\pi^{(2,-k)}} Y_{t-k})\} \\ &= -\beta_k E\{(\eta_{t-k} - Proj_{\pi^{(2,-k)}} \eta_{t-k})^2\} \\ &= -\beta_k Var(\eta_{t-k} \mid \pi^{(2,-k)}) \end{aligned} \quad (3.6)$$

Then (3.4) follows from (3.6) directly.

Theorem 3.1 shows us that Y_{t-l} is noncausal for Y_t , if and only if $\varphi_l = 0$. Therefore if we want to test $\varphi_l = 0$, we need only test if there exists a directed edge $(1, t-l) \rightarrow (1, t)$, based on which a parametric test is proposed. We know η_{t-l} is causal for Y_t , if and only if $\beta_l \neq 0$. Therefore if we want to test $\beta_l = 0$, we need only test if there exists a directed edge $(2, t-l) \rightarrow (1, t)$.

Based on sample Y_1, Y_2, \dots, Y_n from model (2.5), we only take into account the observed part of the past.

Then the estimates of $\gamma_{k|\pi^{(1,-k)}}$ and $\gamma_{k|\pi^{(2,-k)}}$ is given as follows:

$$\hat{\gamma}_{k|\pi^{(1,-k)}} = \hat{Cov}_L(Y_{u-k}, Y_u \mid \pi_u^{(1,-k)}) \quad (3.7)$$

$$\hat{\gamma}_{k|\pi^{(2,-k)}} = \hat{Cov}_L(\eta_{u-k}, Y_u \mid \pi_u^{(2,-k)}) \quad (3.8)$$

where $\pi_u^{(1,-k)}$ and $\pi_u^{(2,-k)}$ is a subspace spanned respectively by $\{Z(m+1), \dots, Z(u-1)\} - \{Y_{u-k}\}$ and $\{Z(m+1), \dots, Z(u-1)\} - \{\eta_{u-k}\}$ for some fixed u .

Discussion of asymptotic properties and computation of $\gamma_{k|\pi^{(1,-k)}}$ and $\gamma_{k|\pi^{(2,-k)}}$ for a general GARCH(p,q) model are complicated. Next, as an illustration, we consider stationary GARCH(1,1) model. We need to test $H_0 : \alpha_1 + \beta_1 = 0$ via $H_1 : \varphi_1 > 0$ and $H_0 : \beta_1 = 0$ via $H_1 : \beta_1 > 0$.

For some fixed $u > 2$, we first compute

$$\gamma_{1|\pi^{(1,-1)}} = Cov_L(Y_{u-1}, Y_u \mid \pi_u^{(1,-1)}) \quad (3.9)$$

$$\gamma_{1|\pi^{(2,-1)}} = Cov_L(\eta_{u-1}, Y_u \mid \pi_u^{(2,-1)}) \quad (3.10)$$

It is easily shown that

$$\gamma_{1|\pi^{(1,-1)}} = \gamma_1^y - \gamma_1^{y,\eta} \quad (3.11)$$

$$\gamma_{\|\pi^{(2,-1)}} = \gamma_1^{y,\eta} - \frac{\gamma_1^y \cdot \gamma_0^{y,\eta}}{\gamma_0^y} \quad (3.12)$$

where $\gamma_k^y = \text{Cov}(Y_t, Y_{t-k}), \gamma_k^{y,\eta} = \text{Cov}(Y_t, \eta_{t-k}), k = 0, 1$.

Then we know from (3.11) and (3.12) that $\gamma_{\|\pi^{(1,-1)}}$ and $\gamma_{\|\pi^{(2,-1)}}$ can be estimated by

$$\hat{\gamma}_{\|\pi^{(1,-1)}} = \hat{\gamma}_1^y - \hat{\gamma}_1^{y,\eta} \quad (3.13)$$

$$\hat{\gamma}_{\|\pi^{(2,-1)}} = \hat{\gamma}_1^{y,\eta} - \frac{\hat{\gamma}_1^y \cdot \hat{\gamma}_0^{y,\eta}}{\hat{\gamma}_0^y} \quad (3.14)$$

where

$$\hat{\gamma}_k^y = \frac{1}{n} \sum_{t=1}^{n-k} (Y_t - \bar{Y})(Y_{t+k} - \bar{Y}), \hat{\gamma}_k^{y,\eta} = \frac{1}{n} \sum_{t=1}^{n-k} (Y_{t+k} - \bar{Y})(\eta_t - \bar{\eta}) \text{ with } \bar{Y} = \frac{1}{n} \sum_{t=1}^n Y_t, \bar{\eta} = \frac{1}{n} \sum_{t=1}^n \eta_t.$$

Let

$$A = \begin{pmatrix} \varphi_1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, B = \begin{pmatrix} -\beta_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

and $\tilde{Y}_t = (Y_t, Y_{t-1}, Y_{t-2})'$. Then the state space representation of model (2.7) is as follows:

$$\tilde{Y}_t = A\tilde{Y}_{t-1} + B\eta_{t-1} + C\eta_t \quad (3.15)$$

Theorem 3.2 For model(2.7) with $Y_t = C'\tilde{Y}_t$, suppose that $E\eta_t = 0, E\eta_t^3 = 0, E\eta_t^2 = \sigma^2 < \infty$, $E\eta_t^4 < \infty$ and $\rho(A \otimes A + \sigma^2 B \otimes B) < 1$. Then as $n \rightarrow \infty$, it holds that

$$1) \hat{\gamma}_k^y \xrightarrow{p} \gamma_k^y, n^{1/2}(\hat{\gamma}_k^y - \gamma_k^y) \xrightarrow{d} N(0, \sigma_k^2).$$

$$\text{where } \sigma_k^2 = \lim_{n \rightarrow \infty} nE(\hat{\gamma}_k^y - \gamma_k^y)^2, k = 0, 1.$$

$$2) \hat{\gamma}_k^{y,\eta} \xrightarrow{p} \gamma_k^{y,\eta}, n^{1/2}(\hat{\gamma}_k^{y,\eta} - \gamma_k^{y,\eta}) \xrightarrow{d} N(0, \sigma_{k,y,\eta}^2)$$

$$\text{where } \sigma_{k,y,\eta}^2 = \lim_{n \rightarrow \infty} nE(\hat{\gamma}_k^{y,\eta} - \gamma_k^{y,\eta})^2, k = 0, 1.$$

$$3) n^{1/2}(\hat{\gamma}_{\|\pi^{(1,-1)}} - \gamma_{\|\pi^{(1,-1)}}) \xrightarrow{d} N(0, \sigma_2^2)$$

$$\text{where } \sigma_2^2 = \theta'W\theta \text{ with } W = \lim_{n \rightarrow \infty} nE(\hat{\gamma} - \gamma)(\hat{\gamma} - \gamma)',$$

$$\text{where } \hat{\gamma} = (\hat{\gamma}_0^y, \hat{\gamma}_1^y, \hat{\gamma}_0^{y,\eta}, \hat{\gamma}_1^{y,\eta}), \gamma = (\gamma_0^y, \gamma_1^y, \gamma_0^{y,\eta}, \gamma_1^{y,\eta}).$$

$$\text{and } \theta = (0, 1, 0, -1)'$$

$$4) n^{1/2}(\hat{\gamma}_{\|\pi^{(2,-1)}} - \gamma_{\|\pi^{(2,-1)}}) \xrightarrow{d} N(0, \sigma_3^2)$$

$$\text{where } \sigma_3^2 = \zeta'W\zeta, \text{ and } \zeta = (\zeta_1, \zeta_2, \zeta_3, \zeta_4)' \text{ with}$$

$$\zeta_1 = -\frac{\hat{\gamma}_1^y \cdot \hat{\gamma}_0^{y,\eta}}{(\hat{\gamma}_0^y)^2}, \quad \zeta_2 = -\frac{\hat{\gamma}_1^{y,\eta}}{\hat{\gamma}_0^y}, \quad \zeta_3 = -\frac{\hat{\gamma}_1^y}{\hat{\gamma}_0^y}, \quad \zeta_4 = 1.$$

Theorem 3.2 can be directly proved by Shao[13] and Liu[14]. So we omit it here. Denote

$$T_{1,n} = \frac{n^{1/2} \hat{\gamma}_{\|\pi^{(1,-1)}}}{\sigma_2}.$$

We know that under $H_0 : \alpha_1 = \beta_1 = 0$, $\gamma_{\|\pi^{(1,-1)}} = 0$. Hence it follows from Theorem 3.2 that

$T_{1,n} \xrightarrow{d} N(0,1)$, as $n \rightarrow \infty$. Therefore we reject $H_0 : \alpha_1 = \beta_1 = 0$ when $T_{1,n} > \Phi^{-1}(1-\alpha)$. Denote

$$T_{2,n} = \frac{n^{1/2} \hat{\gamma}_{\|\pi^{(2,-1)}}}{\sigma_3}.$$

We know that under $H_0 : \beta_1 = 0$, $\gamma_{\|\pi^{(2,-1)}} = 0$. Hence it follows from Theorem 3.2 that

$T_{2,n} \xrightarrow{d} N(0,1)$, as $n \rightarrow \infty$. Therefore we reject $H_0 : \beta_1 = 0$ when $T_{2,n} < -\Phi^{-1}(1-\alpha)$.

We have to calculate $\gamma_{\|\pi^{(1,-1)}}$ and $\gamma_{\|\pi^{(2,-1)}}$ which involves calculating η_t , η_t can be estimated by firstly estimating parameters α_1 and β_1 , then recursively by

Step 1. Regress Y_t on Y_{t-1} , $t = 2, 3, \dots, n$ and obtain \hat{a}_0, \hat{a}_1 , and residuals

$$e_t = Y_t - \hat{a}_0 - \hat{a}_1 Y_{t-1}, t = 2, 3, \dots, n.$$

Step 2. Regress e_{t-1} on Y_{t-1} , $t = 2, 3, \dots, n$ and obtain \hat{b}_0, \hat{b}_1 and residuals

$$v_t = e_{t-1} - \hat{b}_0 - \hat{b}_1 Y_{t-1}, t = 3, 4, \dots, n.$$

Step 3. $\hat{\gamma}_{\|\pi^{(2,-1)}}$ is given by

$$\hat{\gamma}_{\|\pi^{(2,-1)}} = \frac{1}{n-3} \sum_{t=4}^n e_t v_t$$

4. Conclusion

The new test method based on the graphical approach for the conditional heteroskedasticity model is proposed in the paper. For an illustration, GARCH(1,1) is expressed the time series chain graphs and the test of the significance of the coefficients is given. Compared to the traditional test method, our methods are intuitive and very simple in computations. The new method will have important implication in the financial market.

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