Evaluating Water Reuse Applications under Uncertainty: A Novel Picture Fuzzy Multi Criteria Decision Making Medthod

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Abstract—Evaluation of water reuse options is also one of the applications of multi-criteria decision-making (MCDM) problems. In this paper, we refer to a new method for selecting the best water reuse option in the available options by using picture fuzzy MCDM.

Index Terms—Multi-criteria decision-making, picture fuzzy, dissimilarity measure, water reuse.

I. INTRODUCTION

Reuse of water refers to the treatment and rehabilitation of non-traditional or deteriorated water for beneficial purposes [10]. Water reuse is synonymous with using reclaimed water, which can provide an option to reduce water scarcity, especially under the new reality of climate change and the increase in human activities. Water reuse has become widespread all over the world to solve the depletion of water resources, leading to the reduced water supply. Evaluation of water reuse applications is a weight replacement process and the most appropriate selection of water reuse applications. From this, the assessment involves analyzing many criteria with social, technical, economic, political, environmental and technical aspects to ensure sustainable decision making [21]. The challenge with water reuse application evaluation (WRAE) is that alternatives are diverse in nature, and often have conflicting criteria. The fuzzy set theory [20] is a very effective method for solving such contradictory and uncertain problems.

Fuzzy set theory was introduced in 1965 by L. A. Zadeh. Immediately, it became a useful method to study the problems of imprecision and uncertainty. Since then, a lot of new theories treating imprecision and uncertainty have been introduced. For instance, intuitionistic fuzzy set was introduced in 1986, which is a generalization of the notion of a fuzzy set. While fuzzy set gives the degree of membership of an element in a given set, intuitionistic fuzzy set gives a degree of membership and a degree of non-membership. Picture fuzzy set [4] is an extension of the crisp set, fuzzy set and intuitionistic set. A picture fuzzy set has three memberships: a degree of positive membership, a degree of negative membership, and a degree of neutral membership of an element in this set. This approach is widely used by researchers in both theory and application. Dinh et al. investigate the distance and dissimilarity measure on picture fuzzy set and apply it in pattern recognition [13]. Hoa and Thong (2017) improved fuzzy clustering algorithms using picture fuzzy sets and applications for geographic data clustering [11]. Son [7, 8] given an application of picture fuzzy set in the problems of clustering. Dinh et al. introduced the picture fuzzy database and some examples on picture fuzzy database [12].

We often use decision-making methods because of the uncertainty and complexity of the nature of decision-making. By the MCDM methods, we can determine the best alternative from multiple alternatives with respect to some criteria. In recent times, the choice of suppliers has increasingly played an important role in both academia and industry. Therefore, there are many MCDM techniques developed for the supplier selection supplier selection [2, 3, 5, 9, 19, 22]. However, the above methods are limited to use in set theory. Therefore, it is difficult to encounter problems of uncertain or incomplete data. There are some authors who have proposed MCDM methods using fuzzy set theory or intuitionistic fuzzy set for the supplier selection [4, 6, 9, 14 ,16, 17, 18,21].

With the considered criteria for water reuse options, there are usually three levels. For example, the public acceptability attribute has three levels: agreement, disagreement, and neutrality; freshwater saving has also three levels: low, middle, and high. Therefore, we use the multi criteria decision making method based on picture fuzzy set to select the best alternative in evaluating water reuse applications. The rest of paper is organized as follows: section 2, we recall the concept of picture fuzzy set and some operators of two picture fuzzy sets. In section 3, we propose a new MCDM method using the similarity measure of picture fuzzy sets. Finally, we apply the proposed method for evaluating water reuse application in section 4.

II. PRELIMINARITY

In this section, we recall the definition of picture fuzzy sets and dissimilarity measure of them. After that, we introduce a new dissimilarity measure of picture fuzzy sets.

A. Picture fuzzy sets

Here we recall the concept of picture fuzzy sets.

Definition 1 [4]. Let U be a universal set. A Picture fuzzy set (PFS) A on the U is $A = \left\{ \left(u, \mu_A(u), \eta_A(u), \gamma_A(u) \right) \mid u \in U \right\}$ where $\mu_A(u)$ is called the "degree of positive membership of u in A", $\eta_A(u)$ is called the "degree of neutral membership of u in A" and $\gamma_A(u)$ is called the "degree of negative membership of u in A" where $\mu_A(u), \eta_A(u), \gamma_A(u) \in [0,1]$ satisfy the following condition:

$$0 \le \mu_A(u) + \eta_A(u) + \gamma_A(u) \le 1, u \in U.$$

The family of all picture fuzzy set in U is denoted by $PFS(U)$.

Definition 2. The picture fuzzy set $B = \left\{ \left(u, \mu_B(u), \eta_B(u), \gamma_B(u) \right) \mid u \in U \right\}$ is called the subset of the picture fuzzy set $A = \left\{ \left(u, \mu_A(u), \eta_A(u), \gamma_A(u) \right) \mid u \in U \right\}$ iff $\mu_B(u) \le \mu_A(u)$ $\eta_B(u) \le \eta_A(u)$ and $\gamma_B(u) \ge \gamma_A(u)$ for all $u \in U$.

B. Dissimilarity measure of picture fuzzy sets

In this section, we recall concept of dissimilarity measure for picture fuzzy sets.

Definition 3. Given $U = \left\{ u_1, u_2, ..., u_n \right\}$ is an universe set. For any $A, B \in PFS(U)$. A function $Dis : PFS(U) \times PFS(U) \to [0,1]$ is a dissimilarity measure between PFS-sets if it satisfies follow properties:

PF-Diss 1: $Dis(A, B) = Dis(B, A)$;

PF-Diss 2: $Dis(A, A) = 0$

PF-Diss 3: If $A \subset B \subset C$ then
$Dis(A, C) \ge \max \left\{ Dis(A, B), Dis(B, C) \right\}$.

Now, we can identify some new dissimilarity measures between picture fuzzy sets.

Given $U = \left\{ u_1, u_2, ..., u_n \right\}$ is an universe set. For any $A, B \in PFS(U)$, we denote

$$T_A(u_i) = \mu_A(u_i) + \eta_A(u_i) - \gamma_A(u_i) \quad (1)$$

$$T_B(u_i) = \mu_B(u_i) + \eta_B(u_i) - \gamma_B(u_i) \quad (2)$$

$$S_A(u_i) = \eta_A(u_i) - \gamma_A(u_i) \quad (3)$$

$$S_B(u_i) = \eta_B(u_i) - \gamma_B(u_i) \quad (4)$$

and

$$D_i(A, B) = \frac{|T_A(u_i) - T_A(u_i)| + |S_A(u_i) - S_A(u_i)|}{4} \quad (5)$$

for all $i = 1, 2, ..., n$.

Definition 4. Given $U = \left\{ u_1, u_2, ..., u_n \right\}$ is a universal set. For any $A, B \in PFS(U)$, a dissimilarity measure $Dis : PFS(U) \times PFS(U) \to [0,1]$ is defined by

$$DM_T(A, B) = \frac{1}{n} \sum_{i=1}^{n} D_i(A, B) \quad (6)$$

Theorem 1. $DM_T(A, B)$ in the eq.(6) is a dissimilarity measure on $PFS(U)$.

Proof.
From eq.(1), eq.(2), eq.(3) and eq.(4), we have

$$-1 \le T_A(u_i), T_B(u_i), S_A(u_i), S_B(u_i) \le 1$$

for all $i = 1, 2, ..., n$.

Hence, according to Eq.(5) we have $D_i(A, B) \in [0,1]$.

So that, from Eq.(6) we have $DM_T(A, B) \in [0,1]$ for all $A, B \in PFS(U)$.

(PF-Diss 1). It is obviously.
(PF-Diss 2). It is obviously.
(PF-Diss 3). If $A \subset B \subset C$ then
$$\begin{cases} \mu_A(u_i) \le \mu_B(u_i) \le \mu_C(u_i) \\ \eta_A(u_i) \le \eta_B(u_i) \le \eta_C(u_i) \\ \gamma_A(u_i) \ge \gamma_B(u_i) \ge \gamma_C(u_i) \end{cases}$$

for all $u_i \in U$.

So that $T_A(u_i) \le T_B(u_i) \le T_C(u_i)$
and
$S_A(u_i) \le S_B(u_i) \le S_C(u_i)$
Hence
$|T_A(u_i) - T_C(u_i)| \ge \max \left\{ |T_A(u_i) - T_B(u_i)|, |T_B(u_i) - T_C(u_i)| \right\}$
and
$|S_A(u_i) - S_C(u_i)| \ge \max \left\{ |S_A(u_i) - S_B(u_i)|, |S_B(u_i) - S_C(u_i)| \right\}$

Hence $DM_T(A, C) \ge \max \left\{ DM_T(A, B), DM_T(B, C) \right\}$.

It means PF-Diss 3 satisfy. □

Now, we assign u_i is the weight ω_i, such that $\sum_{i=1}^{n} \omega_i = 1$. We can define a dissimilarity measure of two picture fuzzy sets as follows:

Definition 5. Given $U = \{u_1, u_2, ..., u_n\}$ is a universal set. For any $A, B \in PFS(U)$, a dissimilarity measure $Dis : PFS(U) \times PFS(U) \to [0,1]$ is defined by

$$DM_T^{\omega}(A, B) = \sum_{i=1}^{n} \omega_i D_i(A, B) \qquad (7)$$

Definition 6. Given $U = \{u_1, u_2, ..., u_n\}$ is a universal set. For any $A, B \in PFS(U)$, a dissimilarity measure $Dis : PFS(U) \times PFS(U) \to [0,1]$ is defined by

$$DM_P^{\omega}(A, B) = \sum_{i=1}^{n} \omega_i D_i^P(A, B) \qquad (8)$$

Where

$$D_i^P(A, B) = \frac{|T_A(u_i) - T_A(u_i)|^P + |S_A(u_i) - S_A(u_i)|^P}{4} \qquad (9)$$

for all $i = 1, 2, ..., n$, $p \in N^*$

Theorem 2 Given $U = \{u_1, u_2, ..., u_n\}$ is a universal set. For any $A, B \in PFS(U)$. We have

a) $DM_T^{\omega}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \omega_i D_i(A, B)$

b) $DM_P^{\omega}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \omega_i D_i^P(A, B)$

are the dissimilarity measure between picture fuzzy sets.
Proof.

a)   We have

From eq.(1), eq.(2), eq.(3) and eq.(4), we have
$$-1 \leq T_A(u_i), T_B(u_i), S_A(u_i), S_B(u_i) \leq 1$$
for all $i = 1, 2, ..., n$.

Hence, according to Eq.(5) we have $D_i(A, B) \in [0,1]$.

So that, from Eq.(7) we have $DM_T^{\omega}(A, B) \in [0,1]$ for all $A, B \in PFS(U)$.

(PF-Diss 1). It is obviously.

(PF-Diss 2). It is obviously.

(PF-Diss 3). If $A \subset B \subset C$ then
$$\begin{cases} \mu_A(u_i) \leq \mu_B(u_i) \leq \mu_C(u_i) \\ \eta_A(u_i) \leq \eta_B(u_i) \leq \eta_C(u_i) \\ \gamma_A(u_i) \geq \gamma_B(u_i) \geq \gamma_C(u_i) \end{cases}$$

for all $u_i \in U$.

So that $T_A(u_i) \leq T_B(u_i) \leq T_C(u_i)$
and
$S_A(u_i) \leq S_B(u_i) \leq S_C(u_i)$
Hence
$$|T_A(u_i) - T_C(u_i)| \geq \max\{|T_A(u_i) - T_B(u_i)|, |T_B(u_i) - T_C(u_i)|\}$$
and
$$|S_A(u_i) - S_C(u_i)| \geq \max\{|S_A(u_i) - S_B(u_i)|, |S_B(u_i) - S_C(u_i)|\}$$

Hence $DM_T^{\omega}(A, C) \geq \max\{DM_T^{\omega}(A, B), DM_T^{\omega}(B, C)\}$.

It means PF-Diss 3 satisfy.

b)   We have

From eq.(1), eq.(2), eq.(3) and eq.(4), we have
$$-1 \leq T_A(u_i), T_B(u_i), S_A(u_i), S_B(u_i) \leq 1$$
for all $i = 1, 2, ..., n$.

Hence, according to Eq.(5) we have $D_i(A, B) \in [0,1]$.

So that, from Eq.(7) we have $DM_P^{\omega}(A, B) \in [0,1]$ for all $A, B \in PFS(U)$.

(PF-Diss 1). It is obviously.

(PF-Diss 2). It is obviously.

(PF-Diss 3). If $A \subset B \subset C$ then
$$\begin{cases} \mu_A(u_i) \leq \mu_B(u_i) \leq \mu_C(u_i) \\ \eta_A(u_i) \leq \eta_B(u_i) \leq \eta_C(u_i) \\ \gamma_A(u_i) \geq \gamma_B(u_i) \geq \gamma_C(u_i) \end{cases}$$

for all $u_i \in U$.

So that
$T_A(u_i) \leq T_B(u_i) \leq T_C(u_i)$
and
$S_A(u_i) \leq S_B(u_i) \leq S_C(u_i)$
Hence
$$|T_A(u_i) - T_C(u_i)| \geq \max\{|T_A(u_i) - T_B(u_i)|, |T_B(u_i) - T_C(u_i)|\}$$
and
$$|S_A(u_i) - S_C(u_i)| \geq \max\{|S_A(u_i) - S_B(u_i)|, |S_B(u_i) - S_C(u_i)|\}$$

Hence $DM_P^{\omega}(A, C) \geq \max\{DM_P^{\omega}(A, B), DM_P^{\omega}(B, C)\}$.

It means PF-Diss 3 satisfy. □

III. THE PROPOSED MCDM METHOD

In this section, we propose a new method for the multi-criteria decision making problems which based on the dissimilarity of picture fuzzy sets. The multi-criteria decision making problem is determined the best alternative from the concepts of the compromise solution. The best compromise solution is the alternative which obtains the farthest dissimilarity measure from each alternative to the perfect choice. The procedures of the proposed method can be expressed as follows. Let $A = \{A_1, A_2, ..., A_m\}$ be the set of alternatives. Let $C = \{C_1, C_2, ..., C_n\}$ be the set of criteria with the weights of each criteria C_j is ω_j where $j = 1, 2, ..., n$ and $\sum_{j=1}^{n} \omega_j = 1$. The picture fuzzy decision making matrix $D = [d_{ij}]$ in which $d_{ij} = (d_{ij}^1, d_{ij}^2, d_{ij}^3)$ $\in PFS(C_j)$, where $j = 1, 2, ..., n$ and $i = 1, 2, ..., m$ as follow

$$
\begin{array}{c}
\begin{array}{cccc} C_1 & C_2 & ... & C_n \end{array} \\
\begin{array}{c} A_1 \\ A_2 \\ ... \\ A_m \end{array}
\begin{bmatrix}
d_{11} & d_{12} & ... & d_{1n} \\
d_{21} & d_{22} & ... & d_{1n} \\
... & ... & ... & ... \\
d_{m1} & d_{m2} & ... & d_{mn}
\end{bmatrix}
\end{array}
$$

The proposed method is presented with these following steps.

Step 1. Normalization the decision matrix

In this step, we construct the picture fuzzy decision making matrix. For instance, for all $j = 1, 2, ..., n$ and $i = 1, 2, ..., m$, we calculate

$$
\begin{array}{c}
C_j \\
\begin{array}{c} A_1 \\ A_2 \\ ... \\ A_m \end{array}
\begin{bmatrix}
c_{1j}^1 & c_{1j}^2 & c_{1j}^3 \\
c_{2j}^1 & c_{2j}^2 & c_{2j}^3 \\
... & & \\
c_{mj}^1 & c_{mj}^2 & c_{mj}^3
\end{bmatrix}
\end{array}
$$

$$
\xrightarrow{d_{ij}^k = \frac{c_{ij}^k}{\sum_{k=1}^{3} c_{ij}^k}, k = 1,2,3}
\begin{array}{c}
C_j \\
\begin{array}{c} A_1 \\ A_2 \\ ... \\ A_m \end{array}
\begin{bmatrix}
d_{1j}^1 & d_{1j}^2 & d_{1j}^3 \\
d_{2j}^1 & d_{2j}^2 & d_{2j}^3 \\
... & & \\
d_{mj}^1 & d_{mj}^2 & d_{mj}^3
\end{bmatrix}
\end{array} \quad (10)
$$

Then decision making matrix $D = [d_{ij}]$, in which $d_{ij} = (d_{ij}^1, d_{ij}^2, d_{ij}^3) \in PFS(C_j)$, is a picture fuzzy decision making matrix, where $j = 1, 2, ..., n$ and $i = 1, 2, ..., m$. It means that A_i is a picture fuzzy set on the criteria set $C = \{C_1, C_2, ..., C_n\}$.

This step is ignored if matrix D is the given picture fuzzy decision making matrix.

Step 2. Determine the weight of each criteria

We determine the weight ω_j ($j = 1, 2, ..., n$) of the criteria C_j ($j = 1, 2, ..., n$) such that

$$
\sum_{j=1}^{n} \omega_j = 1.
$$

Step 3. Determine the perfect choice

In this section, we determine the perfect choice. Here, we pay attention to the benefit criteria and cost criteria. Usually, with the perfect choices we can take the picture fuzzy number (1,0,0) for the benefit criteria and (0,0,1) for the cost criteria. Note that (1,0,0) is the largest value of a picture fuzzy linguistic and (0,0,1) is the smallest value of a picture fuzzy linguistic. So that the perfect choice A_b get the picture fuzzy number $A_b(j)$ at the criteria C_j, in which $A_b(j) = (1,0,0)$ if C_j is the benefit criteria and $A_b(j) = (0,0,1)$ if C_j is the cost criteria, for all $j = 1, 2, ..., n$.

Step 4. Calculate the dissimilarity measure of the each alternative to the perfect choice

The dissimilarity measure of the each alternative and the perfect choice $\mathrm{Dis}(A_i, A_b)$.

Step 5. Ranking the alternatives

Now, we can rank the alternatives based on the dissimilarity measure of the each alternative and the perfect choice as follows

$$A_i \prec A_k \text{ iff } \mathrm{Dis}(A_i, A_b) \leq \mathrm{Dis}(A_k, A_b) \qquad (11)$$

IV. OUR PROPOSED METHOD FOR EVALUATING WATER REUSE APPLICATIONS

In this section, we use our method proposed in section 3 for evaluating water reuse application. Data is taken from reference Pan et al [15]. This data is summarized as follows (Table 1, Table 2):

- Framework: The proposed has been applied to the City of Penticton (CoP), British Columbia (BC), Canada. The municipal water is used for residential, commercial, institutional and industrial sectors. The wastewater generated is collected and treated by the an advanced wastewater treatment plant by using biological nutrient removed technology. The treated water is partially reused and the remaining is discharged to a lake.

- Specific criteria: C_1: public acceptability (PA); C_2: freshwater saving (FS); C_3: life cycle cost (LCC); C_4: human health risk (HHR); and C_5: the local governments' polices (GP).

We consider that C_1, C_2, C_5 are the benefit criteria and C_3, C_4 are the cost criteria.

- Alternative water reuses: A_1: toilet flushing (TF); A_2: vegetable watering in gardens (VW); A_3: flower watering in gardens (FW); A_4: Agricultural irrigation (AI); A_5: Public parks watering (PPW); A_6:

Golf course watering (GCW); and A_7: drinking water (DW).

Now, we present the process of our method for this evaluating water reuse applications.

Step 1. Normalization the decision matrix.

From the Eq.(10), we obtain the normalization decision matrix (Table 3).

Step 2. Determine the weight of criteria

The weight of criteria C_j is $\omega_j = 0.2$ for all $j = 1, 2, \ldots, 5$.

Step 3. Determine the perfect choice

The perfect choice is

$$A_b = (A_b(1), A_b(2), A_b(3), A_b(4), A_b(5))$$

where

$$A_b(1) = A_b(2) = A_b(5) = (1, 0, 0)$$

and

$$A_b(3) = A_b(4) = (0, 0, 1).$$

Step 4. Calculate the dissimilarity measure of the each alternative to the perfect choice

Using Eq.(9) with $p = 1$, then dissimilarity measure of the each alternative and the perfect choice is calculated by Eq.(8). The results are shown in Table 4.

Step 5. Ranking the alternatives

We use Eq.(11) to rank the alternatives based on the dissimilarity measure of the each alternative and the perfect choice (Table 4).

Table 1. Data of public acceptability and freshwater saving

Alternatives	C_1: Public acceptability			C_2: Freshwater saving (ML/year)		
	Agree	Neutrality	Disagree	Low	Mid	High
TF	80	9	11	428.8	536	643.2
VW	63.5	13	23.5	2624.8	3281	3937.2
FW	84.5	10	5.5	3192.5	3990.6	4788.8
AI	74.5	10	15.5	3192.5	3990.6	4788.8
PPW	85.5	8	6.5	886.3	1107.9	1329.5
GCW	88.5	7	4.5	361.8	452.3	542.7
DW	24	14	62	3192.5	3990.6	4788.8

Table 2. Data of life cycle cost, human health risk and government's polices

Alternatives	C_3 : life cycle cost			C_4 : human health risk			C_5 : governments' polices
	Low	Mid	High	Low	Mid	High	
TF (A_1)	1555358	1944198	2333038	7.10E-12	7.51E-12	8.30E-12	M (0.5,0.4,0.1)
VW(A_2)	1637219	2046524	2455829	1.83E-11	1.89E-11	2.03E-11	L(0.2,0.5,0.3)
FW(A_3)	834019	1042524	1251028	1.78E-11	1.84E-11	1.99E-11	H(0.8,0.1,0.05)
AI(A_4)	146660	183326	219991	9.07E-12	1.00E-11	1.26E-11	M(0.5,0.4,0.1)
PPW(A_5)	635529	794411	953293	9.34E-12	9.77E-12	1.07E-11	H(0.8,0.1,0.05)
GCW(A_6)	78219	97774	117328	8.43E-12	8.87E-12	9.83E-12	M(0.5,0.4,0.1)
DW(A_7)	1197674	1497092	1796511	2.76E-08	4.01E-08	1.00E-07	VL(0.1,0,0.9)

Table 3. Decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	(0.8,0.09, 0.11)	(0.266667,0.333333, 0.4)	(0.266667,0.333333,0.4)	(0.309908,0.327804,0.362287)	(0.5,0.4,0.1)
A_2	(0.635,0.13,0.235)	(0.266667,0.333333,0.4)	(0.266667,0.333333,0.4)	(0.318261,0.328696,0.353043)	(0.2,0.5,0.3)
A_3	(0.845,0.1,0.055)	(0.266666,0.333331,0.400003)	(0.266667,0.333333,0.4)	(0.317291,0.327986,0.354724)	(0.8,0.1,0.05)
A_4	(0.745,0.1,0.155)	(0.266666,0.333331,0.400003)	(0.266666,0.333334,0.4)	(0.286391,0.315756,0.397853)	(0.5,0.4,0.1)
A_5	(0.855,0.08,0.065)	(0.266661,0.333333,0.400006)	(0.266667,0.333333,0.4)	(0.313318,0.327742,0.35894)	(0.8,0.1,0.05)
A_6	(0.885,0.07,0.045)	(0.266657,0.333358,0.399985)	(0.266667,0.333333,0.399999)	(0.310726,0.326944,0.36233)	(0.5,0.4,0.1)
A_7	(0.24,0.14,0.14)	(0.266666,0.333331,0.400003)	(0.266667,0.333333,0.4)	(0.16458,0.239117,0.596303)	(0.1,0,0.9)

Table 4. Ranking of alternatives with dissimilarity

Alternatives	$DM_1(A_i, A_b)$	Rank
TF	0.377652	4
VW	0.451457	6
FW	0.348657	1
AI	0.380642	5
PPW	0.348826	2
GCW	0.363685	3
DW	0.579784	7

Now, we give some results when use our method with the difference weight vectors (using Eq.(9) with $p = 2$. For instance, with ω_1 we consider health-risk's weight more important than others; with ω_2 we ignore the Government policy criteria; and with ω_3 we dismiss the public acceptability criteria. These present in Table 5. Finally, we also recall the results cited in Pan et al. 2018 in Table 6.

Table 5. Ranking of alternatives with dissimilarity and weights

Alternatives	$\omega_1 = (0.1,0.2,0.2,0.4,0.1)$		$\omega_2 = (0.25,0.25,0.25,0.25,0)$		$\omega_2 = (0,0.25,0.25,0.25,0.25)$	
	$DM_2(A_i, A_b)$	Rank	$DM_2(A_i, A_b)$	Rank	$DM_2(A_i, A_b)$	Rank
TF	0.655163	5	0.604568	4	0.636108	4
VW	0.71036	7	0.65549	6	0.716843	6
FW	0.635177	2	0.589318	3	0.596128	2
AI	0.643393	4	0.609762	5	0.624569	3
PPW	0.633343	1	0.589102	2	0.594673	1
GCW	0.645413	3	0.580045	1	0.636155	5
DW	0.704258	6	0.713761	7	0.800051	7

Table 6. Our method's result and the results cited in Pan et al.[15].

Alt.	Our method with weight ω	Pro-economy	Pro-social	Pro-environment	WRAE with a generalized parameter
TF	4	5	5	5	5
VW	6	6	6	6	6
FW	1	2	1	1	1
AI	5	4	4	3	4
PPW	2	1	2	2	2
GCW	3	3	3	4	3
DW	7	7	7	7	7

V. CONCLUSION

In this paper, we define some new dissimilarity measures of picture fuzzy sets ((Eq.2), Eq.(3) and Eq.(4)). After that, we introduce the MCDM with use the dissimilarity measure of picture fuzzy sets. Finally, we apply the proposed method to evaluate the water reuse applications. We also cite the results were determined in [15] to compare with our method. If the weight vector changes, the results will be also changed. Observe that, our method is easier to understand than the method of Pan [15]. Theory of picture fuzzy set is suitable for data-structured problems such as the problem of water reuse as seen in this paper.

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