I.J. Modern Education and Computer Science, **2020, 4, 27-39**
Published Online August 2020 in MECS (http://www.mecs-press.org/)
DOI: 10.5815/ijmecs.2020.04.03

Mean-Field Theory in Hopfield Neural Network for Doing 2 Satisfiability Logic Programming

Saratha Sathasivam
School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM, Penang Malaysia
Email: saratha@usm.my

Shehab Abdulhabib Alzaeemi
School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM, Penang Malaysia
Email: shehab_alzaeemi@yahoo.com

Muraly Velavan
School of General & Foundation Studies, AIMST University, 08100 Bedong, Kedah, Malaysia
Email: muraly@aimst.edu.my

Received: 20 March 2020; Accepted: 08 May 2020; Published: 08 August 2020

Abstract: The artificial neural network system's dynamical behaviors are greatly dependent on the construction of the network. Artificial Neural Network's outputs suffered from a shortage of interpretability and variation lead to severely limited the practical usability of artificial neural networks for doing the logical program. The goal for implementing a logical program in Hopfield neural network rotates rounding minimizing the energy function of the network to reaching the best global solution which ordinarily fetches local minimum solution also. Nevertheless, this problem can be overcome by utilizing the hyperbolic tangent activation function and the Boltzmann Machine in the Hopfield neural network. The foremost purpose of this article is to explore the solution quality obtained from the Hopfield neural network to solve 2 Satisfiability logic (2SAT) by using the Mean-Field Theory algorithm. We want for replacing the real unstable prompt local field for the separate neurons into the network by its average local field utility. By using the solution to the deterministic Mean-Field Theory (MFT) equation, the system will derive the training algorithms in which time-consuming stochastic measures of collections are rearranged. By evaluating the outputs of global minima ratio (zM), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) with computer processing unit (CPU) time as benchmarks, we find that the MFT theory successfully captures the best global solutions by relaxation effects energy function.

Index Terms*:* Logic program, Neural networks, Mean field theory, 2 Satisfiability.

1. Introduction

The real prototype of contemporary artificial neural network motivated by the biologicals nervousness system in order to extract computational ability from human brains [1]. Hopfield Neural Network (HNN) is considered as the well-known one neural networks used for optimization [2]. This is because it possesses a property that allows the network to converge to a local minimum of a network that defines energy dynamics which can subsequently be applied to the optimization through energy minimization. HNN has demonstrated the capability of finding solutions to a difficult optimization problem [3, 4]. It has brought major progress in the area of computational modeling and optimization with their ability to resolve complex real-world mathematical applications [5]. However, the HNN sometimes fails to (correspond) converge to the accurate pattern and, when it does; the solutions obtained are far from optimal [1, 6]. Kowalski [7] developed the main concept of mathematical and computational as well and informal logic as a programming language in HNN for the representation and analysis of a given problem. The ability to generate intelligence through programming logic makes the development of an artificial intelligence system very appropriate for ANN [8].

Logic programming can therefore be regarded as a problem from the standing view of combinatorial optimization. To accomplish that, Wan Abdullah [9] implemented a method for computing the network's weight corresponds to the system's propositional logic. Sathasivam [10] used the Wan Abdullah method to calculate the Horn logic programming synaptic weights in HNN. The retrieved neuron states of the system were validated by implementing the Lyapunov energy dynamics. The study on the effectiveness of the relaxation method in upgrading Horn logic computing in the getting of the desired final state of the neuron in HNN was credited to Sathasivam [11]. In another development, Sathasivam [12] proposed the notion of a stochastic method in carrying out logical program in HNN. The proposed model has been shown to be used in lowering the neuron oscillations during the recovery stage performed by HNN. In order for improving the model, Velavan et al. [13] proposed Mean-field theory by combining the benefit of BM and HTAF in HNN. The proposed theory has been proven the effective ability for doing horn logic in HNN. In practice, the proposed method is limited to non-systematic Horn logic rule.

The goal in performing logic programming based on the energy minimization scheme is to achieve the best ratio of global minimum. However, there is no guarantee to find the best minimum in the network. To achieve this, a learning algorithm based on the Boltzmann Machine (BM) concept and Hyperbolic Tangent Activation Function (HTAF) was derived to accelerate the performance of doing logic programming in Hopfield Neural Network (HNN) by using Mean Field Theory (MFT). The global minima ratio, hamming distances computational time, root mean square error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) were used to measure the effectiveness of the proposed method. Later the developed models are tested by simulated data sets. The simulation results obtain agreed with the proposed theory.

In this paper the global optimization ability of Mean-Field Theory (MFT) to facilitate the retrieval phase of Hopfield Neural Network (HNN) is used in carrying out 2 Satisfiability logic programming (2SAT). Therefore, we propose a new hybrid computational model by incorporating the Mean-Field Theory algorithm (MFT) to foster the retrieval phase of HNN in optimizing 2 Satisfiability (MFTHNN2SAT) in attaining better accuracy of the global minimum ratio, sensitivity, and robustness for network. The contributions of the present study are to improve the retrieval phase of HNN for optimal 2SAT logic programming. Secondly, the proposed hybrid model by incorporating the Mean-Field Theory algorithm (MFT) into the HNN as a single computational model for optimal 2SAT in Boolean satisfiability representation. Finally, to explore the feasibility of the MFTHNN2SAT model and measure its performance in terms of accuracy with the existing models for HNN2SAT.

In the following section, we discussed some of the related theories of HNN and MFT. In this section, we will be focusing on the algorithms used in integrating MFT in HNN.

2. Research Methodology

There are four major steps involved in this paper:

a) Reviewing and analysing the existing model of logic programming in HN proposed by Abdullah [9]. Hence, the fundamental formulations of MFT paradigm will be reviewed.
b) Formulating a new MFT algorithm for solving 2 SAT problem. The newly proposed MFT will be the integrated with logic programming in HN.
c) Testing the proposed model by incoporating the simulated data set The performance evaluation metrics involved are global minima ratio, CPU time, RMSE, MAE and MAPE.

3. Hopfield Neural Network (HNN)

HNN is essentially been utilized as a CAM (Content Addressable Memory) and solves the combinatory optimisation problem. An example of the discrete HNN construction with three neurons [14] is represented in Fig.1. below. Besides this, one of the peculiarities of HNN is dole out representation. The HNN's memory will be stored as a pattern and it overlaps simultaneously one other different memories over the alike sets of process neurons. In that, one neuron is updated in each cycle this called asynchronous control in HNN. This helps each neuron during the processing gain its own judgments matching to its local position and store at CAM and retrieve it when required [15, 16]. HNN include N neurons are identified as literals: $x_i(t), i = 1, 2, \ldots\ldots N$ with bipolar form $x_i \in [-1,1]$, comply with the dynamics of the neuron state $x_i \to \mathrm{sgn}(h_i)$ where sgn is the signum function. The energy function (Lyapunov function) and local field for HNN in the second order is given by the following equations:

$$H = -\frac{1}{2}\sum_i \sum_j J_{[ij]}^{(2)} S_i S_j - \sum_i J_i^{(1)} S_i \tag{1}$$

$$h_i(t) = \sum_j J_{[ij]}^{(2)} S_j(t) + J_i^{(1)} \tag{2}$$

where $J_{ij} = J[ij]$ for *i, j* distinct, and $J_{ii} = 0$ for any *i* equal *j*. The update rule of the neuron state in the network specified as:

$$x_i(t+1) \rightarrow \text{sgn}[h_i(t)] \tag{3}$$

Sathasivam relaxation method which leads the Lyapunov function decreases monotonically is given by [11]:

$$\frac{dh_i^{new}}{dt} = R\frac{dh_i}{dt} \tag{4}$$

where h_i is the local field at HNN and R signifies the relaxation rate [11].

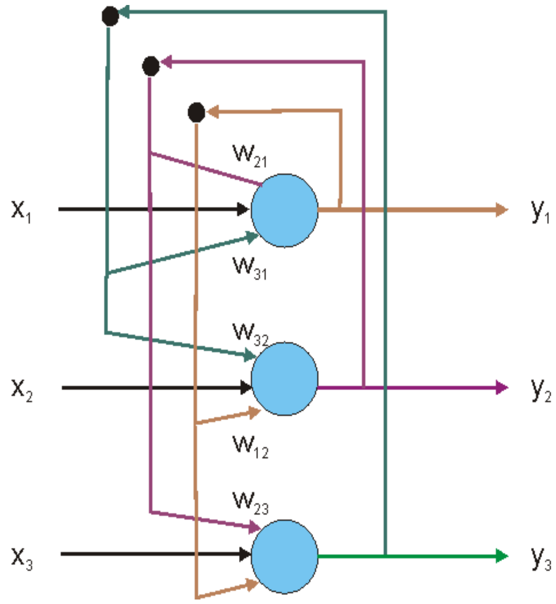

Fig.1. Discrete Hopfield neural network with three neurons [13]

4. Mean Field Theory (MFT)

The mean-field theory into the Boltzmann machine utilized Peterson and Anderson [17] with *n* units and, they derivatives MFT equations as follows:

$$< s_i >= \tanh((\sum_{j=1}^{n} J_{ij} < s_j > + I_i) / T) \tag{5}$$

where $s_j = (s_1 \ldots s_n) \in \{-1, 1\}^n$ is output values, J_{ij} is the synaptic weights from unit *j* to the unit *i*, I_i is the unit threshold *i*, and (T> 0) is the parameter of temperature. In the process of adding, the weight matrix *J= (Jij)* is a symmetric matrix in MFT which is similar to the weight matrix in HNN, and $< s_i >$ describes the mean value of the output value *si* at the time Boltzmann machine. The distribution of Boltzmann of the state converges by using the following equation:

$$P(s) = \frac{1}{z}\exp(-\frac{F(S)}{T}) \tag{6}$$

where $Z = \sum_{\{s\}} exp(-F(s)/T)$ at a given temperature *T* by the force*F(S)*.. Hereby, we represent $\sum_{\{s\}}$ as the total sum all configuration of the corresponding system. Peterson and Anderson [17] also suggested the iterative method as shown in the following equation:

$$x_i(t+1) = \tanh((\sum_{j=1}^{n} J_{ij} x_j(t) + I_i) / T) \tag{7}$$

whereas t is discrete time as {0, 1, 2, …}. In this paper, this discrete-time of the recurrent neural network is named as the MFTHNN, in this model MFTHNN, updated every neuron at the same time-synchronous and asynchronous anywhere only the selected random neurons that are amongst n units is updating by:

$$v_j^{approx} =< v_j \geq \sum_i J_{ji} < x_i > \tag{8}$$

Consequently, computing the average states of the neuron by implanted in a stochastic machine as shown in the following equation:

$$< x_i >= \tanh(\frac{1}{2T}\sum_i J_{ji} < x_j >) \tag{9}$$

The following Table 1. shows some related research regarding MFT.

Table 1. Literature review related research regarding MFT

Author	Method	Summary and Findings
Peterson & Anderson [17]	Neuron firing mechanism by probabilistic rule via Mean Field equation.	The neuron activation is regulated by applying the probabilistic principle entrenched in MFT functions. The work was one of the earliest works on MFT application in neural network.
Pedersen et al. [18]	MFT of photo induced surface relief.	The simulation has an excellent agreement with the theoretical results of surface reliefs recorded under various polarization configurations. Hence, the proposed MFT provides a straight forward model of photo induced surface reliefs for Side-Chain Azobenzene Polymers.
Kotliar et al. [19]	MFT in calculating electronic structure.	MFT has been applied in electronic structure calculations of strongly correlated materials where the one-electron description breaks down. The results were promising, where MFT outperformed the direct method.
Park et al. [20]	Cluster dynamical MFT.	The nature of the Mott transition in the Hubbard model at half-filling has been investigated by using clusters dynamical MFT. Thus, the cluster dynamical MFT has outshined the single-site dynamical MFT in term of performance metrics.
Yurtseven & Senol [21]	MFT to compute Temperature-Pressure (*T-P*) diagram for oxygen.	The MFT has successfully applied to study behaviour of the transitions between the liquid and solid phases in oxygen gas. The *T-P* diagram was constructed and achieved equilibrium at *T*=147.25 K and *P*=8.10 GPa by using MFT.
Arsenault, von Lilienfeld & Millis [22]	Dynamical MFT using machine learning paradigm.	Dynamical MFT approximates the solution to an interacting fermion system in terms of the solution of an auxiliary quantum impurity problem. Hence, the median relative difference for the quasi-particle weight has been computed by dynamical MFT.
Javanainen & Ruostekoski [23]	MFT of standard optics.	The work investigated the numerical simulation of the light propagation in a dense cold atomic by incorporating MFT. The results were promising in term of accuracy, complexity and computational time.
Rubenstein et al. [24]	Self-consistent MFT.	The self-consistent MFT has improved the prediction of protein-peptide recognition, identification of enzyme substrates, and empower the enzymes targeted towards alternative substrates. Thus, better accuracy and faster computation were achieved by self-consistent MFT in predicting the mechanism biological processes.

Consequently, for doing 2SAT logical rule in HNN, we adjust the proceedings in compute the local field of the network. We will utilize the equations from equation (6) to equation (8) for computing the average local field for the new network called MFTHNN-2SAT that is higher efficient into increase the network estimation to find the best model for the corresponding logical programming.

5. Satisfiability Logic Programming In Mean Field Theory Hopfield Neural Network (Mfthnn-2sat)

A. 2 Satisfiability

This section accentuates 2 Satisfiability abbreviations 2SAT. Technically, 2SAT defined as the Conjunctive Normal Form (CNF) with a combination of clause wherever each contains precisely two literals in per clause [25]. Therefore, 2SAT program preserve allows the bipolar value of each literal (variable) of 1 or -1. The three basic features of 2SAT in the CNF formula (Conjunctive Normal Form) summarized into:

1. Every SAT formula contains an array of n literals $l_1, l_2, ..., l_n$ in each clause. This means 2SAT limited by $n = 2$.
2. The Boolean Formula of a set of m clauses as:

$$P = c_1 \wedge c_2 \wedge ... \wedge c_m , \exists m \tag{10}$$

3. The set of literals $l_{k,i}$ in 2SAT considered 2 literals in each clause c_k by the logic operator OR ($\vee$) as the formula:

$$c_k = (l_{k,1} \vee l_{k,2}), \forall 1 \le k \le m \tag{11}$$

In this paper we will integrate the 2SAT logical rule with MFT in HNN in do logical program. We will also examine our model MFTHNN-2SAT performance and compared with previous proposed HNN-2SAT model by Kasihmuddin et al [26]. Contemplate in the follow logic programming P_{2SAT} in the form of logical program:

$$P_{2SAT} = A; B \leftarrow, D \leftarrow C, E \leftarrow F \tag{12}$$

whereas ($;$) refers to the ($\vee$), $\leftarrow$ is a symbol of implication, and ($,$) is the ($\wedge$). Transforming the equation (12) into CNF form of Boolean Algebra [27]:

$$P_{2SAT} = \left(A \vee B\right) \wedge \left(\neg C \vee D\right) \wedge \left(E \vee \neg F\right) \tag{13}$$

where $\vee$ is the Disjunction (OR), $\neg$ is the negation of the variables, and $\wedge$ is the Conjunction (AND).

B. Logic Programming

The idea to do logic programming in artificial neural network is a comparatively new notion. The whole notion of logical program in neural network is to achieve the final state of neurons which functions according to well organized predetermined of logic rule [28], consists of neurons interconnected neuron S_i and involved input layers and output layers and it doesn't have any hidden layer involved. S_{ij} is the connection between S_i and S_j where all neurons are linked by Synaptic weight, J_{ij} . The update of neuron as follows:

$$S_i = \begin{cases} 1, if \sum_{j=1}^{N} J_{ij} S_j > \xi \\ -1, otherwise \end{cases} \tag{14}$$

where ξ is a threshold. P_{2SAT} can be displayed in MFTHNN to assign the neurons for each literal. The main purpose of the model is for minimising the logic inconsistencies by minimising the subsequent cost function:

$$E_{P_{2SAT}} = \sum_{i=1}^{NC} \prod_{j=1}^{NV} L_{ij} \tag{15}$$

whereas NC is clauses number and NV is variables number and the contradictions of logic clause L_{ij} as follows:

$$L_{ij} = \begin{cases} \frac{1}{2}\left(1 - S_x\right), if \ \neg x \\ \frac{1}{2}\left(1 + S_x\right), otherwise \end{cases} \tag{16}$$

The synaptic weight for the neurons in MFTHNN is determined by utilizing the Wan Abdullah method [29] where the cost function $E_{P_{2SAT}}$ is comparing with the final energy function $H_{P_{2SAT}}$:

$$H_{P_{2SAT}} = -\frac{1}{2}\sum_{i=1, i\neq j}^{N}\sum_{j=1, i\neq j}^{N} J_{ij}^{(2)} S_i S_j - \sum_{i=1}^{N} J_i^{(1)} S_j \tag{17}$$

By using the local field $h_i(t)$ in the following equation to compute the neurons final state in the follow equation:

$$h_i(t) = \sum_{j=1}^{N} J_{ij}^{(2)} S_j + J_i^{(1)} \tag{18}$$

where the neuron final state is given a follows:

$$S_i = \begin{cases} 1, if \ \ h_i(t) \geq 0 \\ -1, if \ \ h_i(t) < 0 \end{cases} \tag{19}$$

Therefore, the optimized neuron state can be obtained by checking

$$\left| H_{P_{2SAT}}^{\min} - H_{P_{2SAT}} \right| \leq \partial \tag{20}$$

where ∂ is the tolerance value that defined by user [10, 16, 23].

C. 2 Satisfiability Logic Programming in Mean Field Theory Hopfield Neural Network (MFTHNN-2SAT)

HNN is been utilized in an assignment for doing the 2SAT logical model because of its ability for solving constrained optimisation problems. The procedure below shows how combining MFT in the Hopfield neural network to do 2SAT logic programming. From our simulated data set, we want to evaluate the effectiveness of doing MFTHNN-2SAT. Few parameters which will be used to measure the effectiveness will be discussed in the following section.

Input Phase

Step 1

Interpret each 2SAT clauses in the logical programming that given in equation (11) in elementary Boolean algebraic form as shown in the follow equation:

$$P_{2SAT} = (A \vee B) \wedge (\neg C \vee D) \wedge (E \vee \neg F) \tag{21}$$

Step 2

Initialization of the neurons to individually ground neurons.

Step 3

Initialization every synaptic weights values to zero.

Step 4

Determine the cost function $E_{P_{2SAT}}$ which is be linked with the negation clauses, multiplication performed a conjunction logical connective and addition performed a disjunction connective.

Step 5

Compute the synaptic weights of the neurons $J_{[ij]}^{(2)}$ and $J_{[i]}^{(1)}$ by compare the energy function and cost function based on WA method as shown in the table below:

Table 2. Derived synaptic weight

J_{AB}	-0.25
J_{CD}	-0.25
J_{EF}	-0.25
J_{A}	0.25
J_{B}	0.25
J_{C}	0.25
J_{D}	0.25
J_{E}	0.25
J_{F}	0.25

Synaptic weight in Table 2. will be stored as CAM as a building block MFTHNN during the retrieval phase as the “correct” synaptic weight configuration that agrees to logic rule.

Step 6

Calculate the global energy supposed to be, $H_{\min}$ and local field $h_i(t)$ of MFTHNN-2SAT in equation (2).

Step 7

Produce randomly primary states of neurons based on the probabilistic rule as shown in the following equation:

$$S = \begin{cases} +1, & P(h) \\ -1, & 1-(P(h) \end{cases} \tag{22}$$

where the probability $P\left(h_j\right)$ by:

$$p\left(h_i(t)\right) = \frac{1}{1+\exp(-h_i / T)} \tag{23}$$

where T is the temperature.

Step 8

Estimate the average prompt local field of MFTHNN-2SAT as shown in the following equation:

$$\begin{aligned} \left\langle s_i \right\rangle &= (+1)P\left(h_i\right) + (-1)\left[1-P\left(h_i\right)\right] \\ &= 2P\left(h_i\right) - 1 = tanh\left(h_i / 2T\right) \end{aligned} \tag{24}$$

where T refers to temperature.

Step 9

Updating the status of neurons via utilizing the Hyperbolic Tangent Activation Function (HTAF) as shown in the following equations:

$$g\left(h_i / 2T\right) = \frac{1-e^{-h_i/T}}{1+e^{-h_i/T}} \tag{25}$$

Where

$$\langle s_i \rangle = \begin{cases} +1 & for \ g(h_i / 2T) \geq 0 \\ -1 & for \ g(h_i / 2T) < 0 \end{cases} \tag{26}$$

Step 10

Calculating the final energy for each neuron when the neurons are in the stable state.

Output Phase

Step 11

The global solutions decision, $D_{MFT}(i)$ is given as follows [31]:

$$D_{MFT}(i) = \begin{cases} |\Delta E_i| \leq 0.001, & z = 1 \\ |\Delta E_i| > 0.001, & y = 1 \end{cases} \tag{27}$$

where z is the global minima, and y is the local minima.

The concept of the mean-field theory (MFT) is focusing on one spin in the neural network and all the other spins are displaced with an average knowledge field and vacillations of the other spins are ignored. That will help to reduce the number of neurons which stuck in a local minimum and update to global minima. Also, that helps the oscillating of the network can be diminished which leads to lowered computational time and rise the number of global solutions. So, by merging the advantage of these both methods 2SAT logical program and MFT in HNN will be get the best network called MFTHNN-2SAT. The following flow chart summarizes our proposed network called MFTHNN-2SAT:

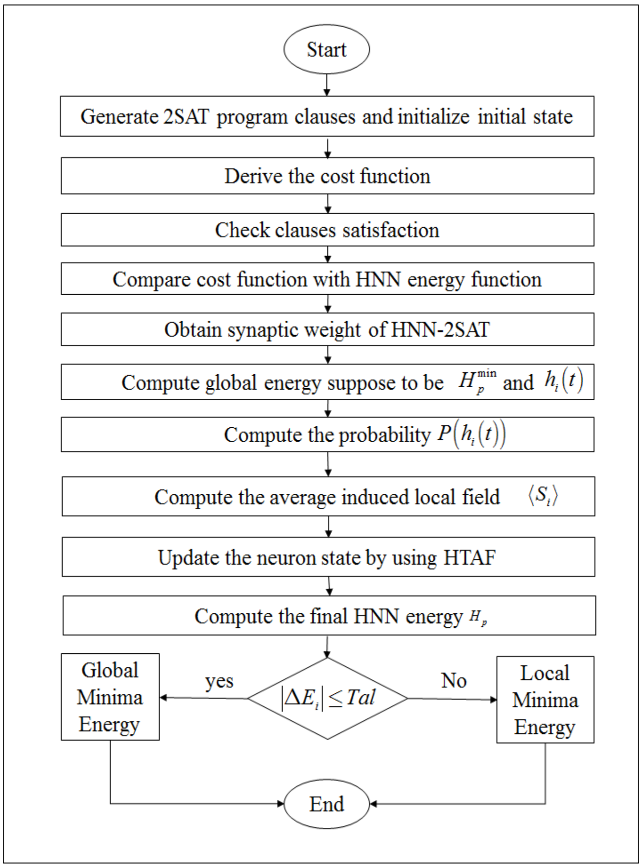

Fig.2. Flowchart of MFTHNN-2SAT

6. Experimental Results and Discussion

The simulation of the models will be performed utilizing the Dev C++ software with Microsoft Windows 7, on a computer (3.40GHz processor, 500GB hard disk, and 4096MB RAM) to comparatively analyze MFTHNN-2SAT, the parameters in Table 3. are chosen by tring and error technique.

Table 3. List of Parameters in MFTHNN-2SAT model

Parameter	Value
Relax	100
COMBMAX	100
Number of Trial	100
Number of learning	100
Number of checking	5
Activation Function	HTAF

Based on the simulations for models, the higher accuracy of HNN model has higher value of zM , and lower RMSE, MAE, and MAPE are more sensible to outliers and suitable for evaluating the accuracy:

$$zM = \frac{1}{tc}\sum_{i=1}^{n} N_P \tag{28}$$

The local minimum ratio equation of is defined as [16]:

$$L_m = 1 - zM \tag{29}$$

where c is the combination of the neuron, t is the trial, and N_P is the number of global minima energy of the propose model.

$$RMSE = \sum_{i=1}^{n} \sqrt{\frac{1}{n}(f_{\max} - f_i)^2} \tag{30}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n} \left| f_{\max} - f_i \right| \tag{31}$$

$$MAPE = \frac{100}{n}\sum_{i=1}^{n} \left| \frac{f_{\max} - f_i}{f_i} \right| \tag{32}$$

$f_{\max}$ is a total number clauses of 2SAT, f_i is the fitness of the solutions and n is the number of iteration. CPU time is the time gained by an appropriate model to achieve one performance. The CPU time in the following equation:

$$CPU\ time = Learning\ time + Retrieval\ time \tag{33}$$

zM is the ratio between global total minima energy and total simultaneous numbers is defined by Kasihmuddin *et al* and Alzaeemi *et a l* [8, 16]. When the final energy is within the limit, it is known as global minima energy. In this study, we define the *CPU time* as the total time that is taken for the network to generate maxima satisfied clauses in the logic programming by using different activation functions [31]. In order to evaluate the performance quality of the model, we used parameters such as RMSE, MAE and MAPE. By using these parameters, we can compare the errors between the proposed models with the comparison model.

As defined by Sathasivam & Abdullah [32], the best model HNN has the least *CPU time* during retrieval phase and the learning phase.

Consequently, we utilize the proceedings of doing 2SAT logical program in the MFTHNN model. By utilizing the average induced local field and simulated annealing technique of MFT to accelerating the stuck the neurons from the local minimum to global minimum through going cross energy relax loop for each neuron. After that, let the network evolves until reaching the minima of energy. Then the final state of the neurons (a stable state of the neuron) obtained is been tested when the states of neurons reside unchanged for more than five times running. The values in Table 3. obtained by trial and error technic. The following figures show the result got for simulated data set.

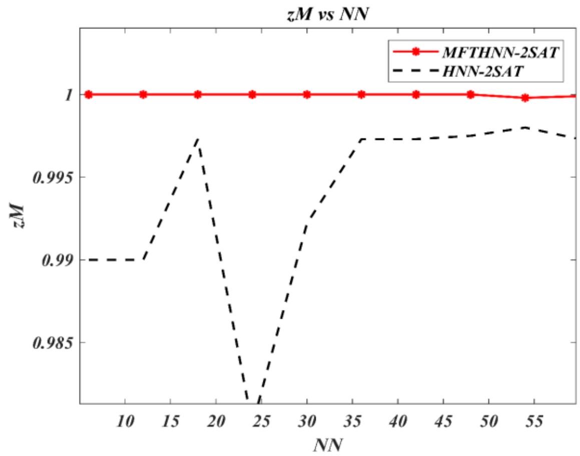

Fig.3. Global Minima Ratio (*zM*) of MFTHNN-2SAT and HNN-2SAT models

Furthermore, we can observe that from Figure 3., 98% of the neurons final state by utilizing MFTHNN-2SAT is the global minimum better than previous proposed HNN-2SAT model by Kasihmuddin et al [26]. Even increasing the network complexity by number of neurons increase doesn't affect the network stability and capacity. By using average state value, the error has been decreased rapidly. Furthermore, simulated annealing by charging the temperature manages to push the under relaxed neurons to global values.

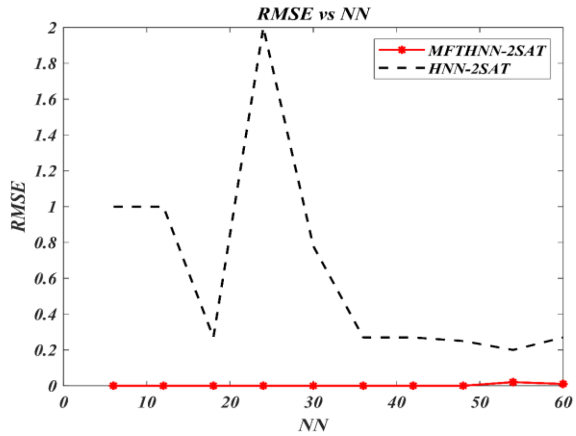

Fig.4. RMSE of MFTHNN-2SAT and HNN-2SAT models

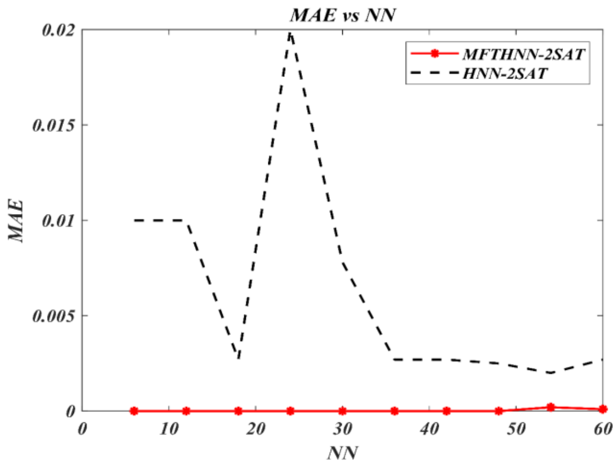

Fig.5. MAE of MFTHNN-2SAT and HNN-2SAT models

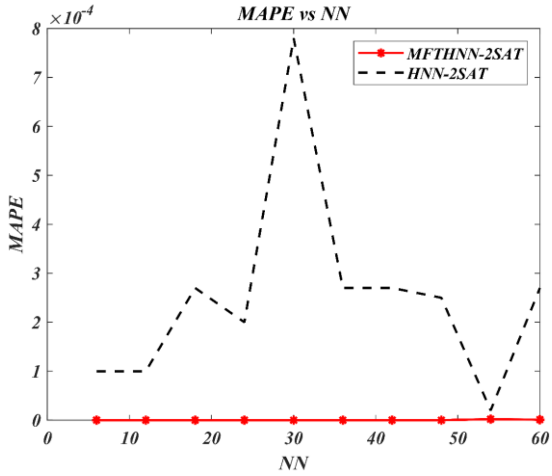

Fig.6. MAPE of MFTHNN-2SAT and HNN-2SAT models

The above Fig.4. to Fig.6. presented compare between the performance of MFTHNN-2SAT and HNN-2SAT. As observed, the errors and the timing of the system increase with increase neurons number (NN). This is because of the complexity of the system also increased since there are more local minima values involved. However, by using HTAF and Boltzmann Machines (simulated annealing), MFT able to push the neurons through energy barriers to relax into global minima. Because of this, the global minima ratio is higher and at the same time the error and the CPU time also been reduced. Besides that, by using fluctuating values of the local field in MFT, the error can be reduced immensely. As deduced from the above Figures from Fig.4. to Fig.6. the errors in MFTHNN-2SAT are near or equal to zero, due to the MFT synchronization between the data and the Hopfield network.

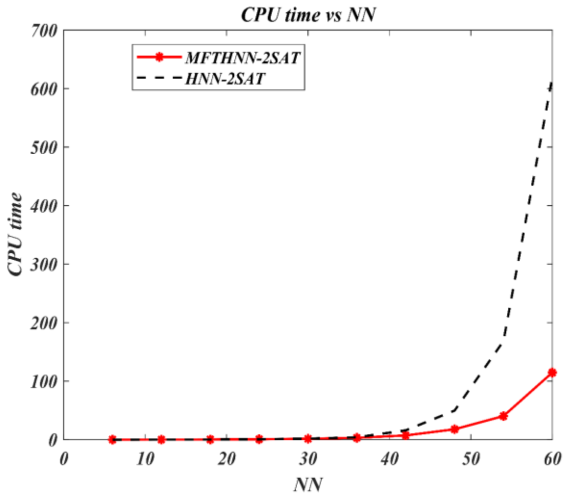

Fig.7. CPU time (second) of MFTHNN-2SAT and HNN-2SAT models

According to Fig.7. there are less or lower CPU Time is required to complete one execution of learning and testing MFTHNN-2SAT by increasing number neurons better than HNN-2SAT. As it stands, the HNN-2SAT model require a substantial amount of time to complete the learning when the complexity is higher.

From the above Figures from Fig.3. to Fig.7. we can notice from the previous result that during the network becomes more complexity by increasing the number of neurons, the Mean-Field Theory in HNN succeeds to achieve a better results with a higher global minimum ratio, lower error by lower computation time. This is because of the simulated annealing processes supported by the Mean-Field Theory whereas the neurons are forcible to skip the energy barricade to relaxation region in the global solutions via altering the temperature values. By utilizing the MFT method, the neurons in the network are capable to relax and up to a global minimum state and don't stuck in local minimum states. This Mean-Field Theory approximation often converts exactly in the termination of infinite range communications, where each spin communicates with all of the others. This MFT approximate usually converts exactly in termination of unlimited field communications, whereas all spins communicate with all of the others. This is due to the inducing local field of MFT in HNN is the sum of very many terms in the structure of the network, and can apply a central limit theorem.

Simulation results obtained from MFTHNN-2 SAT seem to be more stable, less computational complexity compare with other method, HNN-2SAT. MFT performance more robust and stable in term of CPU time, global minimum error and error measurements then the other method. The usage of average local field proved again that by reducing the spin fluctuation and split, the network performance can be increased.

7. Conclusion

The principle of MFT is that was focused on one spin and all the other spins are replaced with an average background field. The fluctuations of the other spins are ignored. This will decrease the number of neurons get stuck in local minima values. The computational time can be decreased and number of global solutions will be increased too.

In this paper, we improved the retrieval phase of HNN for optimal 2SAT logic programming by incorporating the Mean-Field Theory (MFT) into HNN as a single computational model for optimal 2SAT in Boolean satisfiability representation. The explored the feasibility of the MFTHNN2SAT model and measure its performance in terms of accuracy with the existing models for HNN2SAT. Mean-Field Theory implemented in Hopfield neural network in implementing 2SAT logical program has been confirmed efficient in stimulating the computational capacity of neuro symbolic integration better than HNN-2SAT as shown from the results in Fig.3. to fig.7. The results of computer simulation in the aspects of *zM*, RMSE, MAE, MAPE, and CPU time also agreed with the proposed theory. We can conclude the MFTHNN-2SAT well performed compared with HNN-2SAT. In future, MFT can be integrated with metaheuristic algorithms to accelerate the performance as the network gets larger.

Acknowledgment

This research is supported by Research Universiti Grant (RUI)(1001/PMATHS/8011131) by University Sains Malaysia.

References

[1] R. Rojas, (2013). *Neural networks: A systematic introduction.* Springer Science & Business Media.
[2] J.J. Hopfield, & D.W. Tank, (1985). Neural computation of decisions in optimization problems. *Biological Cybernatics*, 52, 141-152.
[3] S. A. S. Alzaeemi, & S. Sathasivam, (2018). Hopfield neural network in agent based modeling. *MOJ App Bio Biomech*, *2*(6), 334-341.
[4] S. Sathasivam, M. Mamat, M. Mansor, & M. S. M. Kasihmuddin, (2020). Hybrid Discrete Hopfield Neural Network based Modified Clonal Selection Algorithm for VLSI Circuit Verification. *Pertanika Journal of Science & Technology*, *28*(1).
[5] S. Haykin, (1992). *Neural Networks: A Comprehensive Foundation*. New York: Macmillan College Publishing.
[6] S. Sathasivam, (2015). Acceleration Technique for Neuro Symbolic Integration. *Applied Mathematical Sciences*, *9*(9), 409-417.
[7] R. A. Kowalski, (1979). *Logic for Problem Solving*. New York: Elsevier Science Publishing.
[8] S. A. Alzaeemi, M. A. Mansor, M. S. M. Kasihmuddin, & S. Sathasivam, (2019, December). Comparing the logic programming between Hopfield neural network and radial basis function neural network. In *AIP Conference Proceedings* (Vol. 2184, No. 1, p. 060044). AIP Publishing LLC.
[9] W. A. T. Wan Abdullah, (1991). Neural Network Logic. Benhar, O., Bosio, C., del Giudice, P., & Tabet, E. (eds.), Neural Networks: From Biology to High Energy Physics, *ETS Editrice*, Pisa, pp.135-142
[10] S. Sathasivam, (2010). Upgrading logic programming in Hopfield network. *Sains Malaysiana*, *39*(1), 115-118.
[11] S. Sathasivam, (2009, August). Energy relaxation for Hopfield Network with the new learning rule. In *AIP Conference Proceedings* (Vol. 1159, No. 1, pp. 118-122). American Institute of Physics.
[12] S. Sathasivam, N. P. Fen, & M. Velavan, (2013, June). Boltzmann machine and reverse analysis method. In *2013 IEEE 7th International Power Engineering and Optimization Conference (PEOCO)* (pp. 52-56). IEEE.
[13] M. Velavan, Z. R. bin Yahya, M. N. bin Abdul Halif, & S. Sathasivam, (2016). Mean field theory in doing logic programming using Hopfield network. *Modern Applied Science*, *10*(1), 154.
[14] J. J. Hopfield, (1982). Neural networks and physical systems with emergent collective computational abilities. *Proceedings National Academy of Science USA*, *79*, 2554-2558. http://dx.doi.org/10.1073/pnas.79.8.2554
[15] Y. Ding, L. Dong, L. Wang, & G. Wu, (2010a). A High Order Neural Network to Solve Crossbar Switch Problem. In: Wong, K.W. et al. (eds.) *ICONIP 2010, Part II, LNCS 6444*, Heidelberg: Springer pp. 692–699.
[16] M.S.M. Kasihmuddin, M.A. Mansor, M.F.M. Basir, S. Sathasivam, Discrete mutation Hopfield neural network in propositional satisfiability. *Mathematics* 2018, *7*, 1133.
[17] C. Peterson, & J. Anderson, (1988). A Mean Field Theory Learning Algorithm for Neural Networks. *Complex System 1*, 995-1019.
[18] T. G. Pedersen, P. M. Johansen, N. C. R. Holme, P. S. Ramanujam, & S. Hvilsted, (1998). Mean-field theory of photo induced formation of surface reliefs in side-chain azobenzene polymers. Physical review letters, 80(1), 89.
[19] G. Kotliar, S. Y. Savrasov, K. Haule, V. S. Oudovenko, O. Parcollet, & C. A. Marianetti, (2006). Electronic structure calculations with dynamical mean-field theory. Reviews of Modern Physics, 78(3), 865.
[20] H., Park, K., Haule, & G. Kotliar, (2008). Cluster dynamical mean field theory of the Mott transition. Physical review letters, 101(18), 186403.

[21] H. Yurtseven, & M. G. Şenol, (2015). Calculation of the T–P phase diagram for oxygen using the mean field theory. Calphad, 51, 272-281.
[22] L. F. Arsenault, O. A. von Lilienfeld, & A. J. Millis, (2015). Machine learning for many-body physics: efficient solution of dynamical mean-field theory. arXiv preprint arXiv:1506.08858.
[23] J. Javanainen, & J. Ruostekoski, (2016). Light propagation beyond the mean-field theory of standard optics. Optics express, 24(2), 993-1001.
[24] A. B. Rubenstein, M. A. Pethe, & S. D.. Khare, (2017). MFPred: Rapid and accurate prediction of protein-peptide recognition multispecificity using self-consistent mean field theory. PLoS Computational Biology, 13(6), e1005614.
[25] M. S. M. Kasihmuddin, M. A. Mansor, & S. Sathasivam, (2017). Robust Artificial Bee Colony in the Hopfield Network for 2-Satisfiability Problem. *Pertanika Journal of Science & Technology*, *25*(2).
[26] M. S. M. Kasihmuddin, M. A. Mansor, S. Alzaeemi, M. F. M. Basir, and S. Sathasivam. "Quality Solution of Logic Programming in Hopfield Neural Network." In *Journal of Physics: Conference Series*, vol. 1366, no. 1, p. 012094. IOP Publishing, 2019.
[27] M. Mansor, Z. Jamaludin, M. Kasihmuddin, S. Alzaeemi, F. Basir, S. Sathasivam (2020). Systematic Boolean Satisfiability Programming in Radial Basis Function Neural Network. *Processes*, *8*(2), 214.
[28] M. S. M. Kasihmuddin, M. A. Mansor, & S. Sathasivam, (2018). Discrete Hopfield Neural Network in Restricted Maximum k-Satisfiability Logic Programming. *Sains Malaysiana*, *47*(6), 1327-1335.
[29] M. S. M. Kasihmuddin, S. Sathasivam, & M. A. Mansor, (2017, August). Hybrid genetic algorithm in the Hopfield network for maximum 2-satisfiability problem. In *AIP Conference Proceedings* (Vol. 1870, No. 1, p. 050001). AIP Publishing LLC.
[30] L. C. Kho, M. S. M. Kasihmuddin, M. A. Mansor, & S. Sathasivam, (2019, December). 2 satisfiability logical rule by using ant colony optimization in Hopfield Neural Network. In *AIP Conference Proceedings* (Vol. 2184, No. 1, p. 060009). AIP Publishing LLC.
[31] S. Sathasivam, & M. Velavan, (2014). Boltzmann Machine and Hyperbolic Activation Function in Higher Order Network. *Modern Applied Science*, *8*(3), 154-160.
[32] Sathasivam, S. & Abdullah, W.A.T. (2011). Logic mining in neural network: reverse analysis method. Computing, 91(2), 119-133.

Authors' Profiles

Saratha Sathasivam working as Associate Professor at the School of Mathematical Sciences, Universiti Sains Malaysia. She received her MSc (Math) and BSc(Ed) from Universiti Sains Malaysia. She received her Ph.D from Universiti Malaya, Malaysia. Her current research interest are neural networks, agent based modeling and numerical methods**.**

Shehab Abdulhabib Alzaeemi received a Bachelor Degree of Education (Science) from Taiz Universiti in 2004 and Master of Science (Mathematics) from Universiti Sains Malaysia in 2016 and now student PhD in Universiti Sains Malaysia. He was fellow under the Academic Staff Training System of Sana'a Community College from 2005-2014. His research interests mainly focus on neural network, logic programming, and data mining.

Muraly Velavan is a senior lecturer and also the Deputy Director in the School of General & Foundation Studies, AIMST University. He received his MSc at Universiti Malaysia Perlis. His current research interest are neural networks and agent based modeling.

How to cite this paper: Saratha Sathasivam, Shehab Abdulhabib Alzaeemi, Muraly Velavan, " Mean-Field Theory in Hopfield Neural Network for Doing 2 Satisfiability Logic Programming", International Journal of Modern Education and Computer Science(IJMECS), Vol.12, No.4, pp. 27-39, 2020.DOI: 10.5815/ijmecs.2020.04.03