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# Construction of Periodic Complementary Multiphase Sequences Based on Perfect Sequences

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**Abstract**

This paper provides a construction method of periodic complementary multiphase sequences. The proposed method is based on perfect sequences possessing ideal periodic auto-correlation properties. By interleaving any two different perfect sequences with the same length, a kernel set of periodic complementary sequence with multiphase elements can be generated. Compared with the known periodic complementary binary sequences, the presented periodic complementary multiphase sequences may obtain much more lengths of element sequences, which will assure that the generated sequences can provide a more flexible choice of parameters for communication systems.

**Index Terms:** periodic complementary multiphase sequences; perfect sequences; interleaving technique; correlation properties



## 1. Introduction

Different from unitary sequences whose correlation properties are limited by theoretical bounds like Welch bound [1], complementary sequences with each sequence composed of a flock of element sequences can possess ideal correlation properties, that is, the sum of auto-correlation functions (ACFs) of element sequences is zero everywhere except at zero shift and the sum of cross-correlation functions (CCFs) of element sequences for any two complementary sequences is zero at any shift [2]. Complementary sequences were first studied by Golay [3], where thus sequences were also called as complementary pairs. Based on the idea of complementary pairs, binary and multiphase complementary sequence sets where the number of element sequences was not limited to be equal to 2 were investigated in [4-5], respectively.

For complementary pairs and complementary sequence sets, aperiodic correlation properties had received more attention. Actually, periodic correlation properties for complementary sequences have the same importance and periodic complementary (PC) sequences can provide a more flexible parameter choice than aperiodic complementary (AC) sequences. The binary PC sequences were first studied by Bomer and Antweiler on the

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basis of perfect binary arrays [6]. In comparison with AC sequences, the computer search shows that the performance of flock size and length of element sequence of binary PC sequences is better. Similar to PC sequences only considering periodic correlation properties, Z-periodic complementary (ZPC) sequences [7-8] pay more attention to Z-complementary (ZC) sequences [9-12] only with periodic correlation properties. Those ZPC sequences may have important application in channel estimation of multi-antenna systems. Although ZPC sequences have larger set size than PC sequences, PC sequences possess ideal periodic correlation properties while correlation values of ZPC sequences are equal to zero only within zero correlation zone (ZCZ).

In this paper, we propose a design of multiphase PC sequences on the basis of perfect sequences and interleaving technique. Different from binary PC sequences, the proposed multiphase PC sequences have more usable lengths of element sequences. For example, when the length of element sequence is smaller than 50, binary PC sequences only possess lengths of element sequence from the set {2, 4, 8, 10, 16, 20, 26, 32, 34, 36, 40} while the length of element sequence of the designed multiphase PC sequences may be any even number which is larger than 4. As a result, communication systems employing multiphase PC sequences can have a more flexible choice of parameters.

This paper is organized as follows. Section II introduces the notations and definitions required for the subsequent sections. Then, we provide the construction algorithm of multiphase PC sequences on the basis of the interleaved perfect sequences and prove periodic correlation properties of the constructed sequences in Section III. In order to show how the proposed design algorithm works, a simple construction example using Zadoff-Chu perfect sequences is given in Section IV. Finally, Section V summarizes the results.

## 2. Preliminary Knowledge

Let $\mathbf{x} = \{x(0), x(1), \cdots, x(L-1)\}$ and $\mathbf{y} = \{y(0), y(1), \cdots, y(L-1)\}$ denote two sequences with length $L$, then the periodic CCF $\rho_{\mathbf{x},\mathbf{y}}(\tau)$ between $\mathbf{x}$ and $\mathbf{y}$ can be expressed as

$$\rho_{\mathbf{x},\mathbf{y}}(\tau) = \sum_{l=0}^{L-1} x(l) y^*(l+\tau)_L , \tag{1}$$

where the symbol * denotes a complex conjugate and the notation $(\cdot)_L$ denotes a modulo $L$ operation. If $\mathbf{x} = \mathbf{y}$, the above equation becomes the periodic ACF.

The interleaving operation between $\mathbf{x}$ and $\mathbf{y}$ can be given as follows,

$$\mathbf{x} \otimes \mathbf{y} = \{x(0), y(0), x(1), y(1), \cdots, x(L-1), y(L-1)\} . \tag{2}$$

If a unitary sequence $\mathbf{z} = \{z(0), z(1), \cdots, z(L-1)\}$ has ideal auto-correlation properties, that is, its out-of-phase periodic ACF is equal to zero, then the sequence $\mathbf{z}$ can be called a perfect sequence.

Let $\Phi = \{\mathbf{a}_i, 0 \le i \le M-1\}$ be a sequence set with set size $M$. Each sequence $\mathbf{a}_i$ includes a flock of element sequences of length $L$, written as $\mathbf{a}_i = \{\mathbf{a}_{i,r}, 0 \le r \le N-1\}$ with each element sequence $\mathbf{a}_{i,r} = \{a_{i,r}(0), a_{i,r}(1), \cdots, a_{i,r}(L-1)\}$. Then, the sequence $\mathbf{a}_i = \{\mathbf{a}_{i,r}, 0 \le r \le N-1\}$ is called a PC sequence with flock size $N$ if the sum $\Psi_{\mathbf{a}_i,\mathbf{a}_i}(\tau)$ of the out-of-phase periodic ACF of $N$ element sequences is zero, namely

$$\Psi_{\mathbf{a}_i,\mathbf{a}_i}(\tau) = \sum_{r=0}^{N-1} \rho_{\mathbf{a}_{i,r},\mathbf{a}_{i,r}}(\tau) = \begin{cases} \sum_{r=0}^{N-1} E_{\mathbf{a}_{i,r}}, & (\tau)_L = 0 \\ 0, & (\tau)_L \neq 0 \end{cases}, \tag{3}$$

where the notation $E_{\mathbf{a}_{i,r}}$ denotes the energy of element sequence $\mathbf{a}_{i,r}$.

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If any two sequences $\mathbf{a}_i$ and $\mathbf{a}_j$ of $\mathcal{A}$ satisfy the following equation, then the set $\mathcal{A}$ becomes a PC sequence set with set size $M$, flock size $N$ and element sequence length $L$,

$$\psi_{\mathbf{a}_i,\mathbf{a}_j}(\tau) = \sum_{r=0}^{N-1} \phi_{\mathbf{a}_{i,r},\mathbf{a}_{j,r}}(\tau) = 0, \quad \forall \tau . \tag{4}$$

In a matrix form, the PC sequence set $\mathcal{A}$ can be expressed as

$$\mathcal{A} = \begin{bmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \\ \vdots \\ \mathbf{a}_{M-1} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{0,0} & \mathbf{a}_{0,1} & \cdots & \mathbf{a}_{0,N-1} \\ \mathbf{a}_{1,0} & \mathbf{a}_{1,1} & \cdots & \mathbf{a}_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{a}_{M-1,0} & \mathbf{a}_{M-1,1} & \cdots & \mathbf{a}_{M-1,N-1} \end{bmatrix}. \tag{5}$$

It is well known that the set size of PC sequence set is smaller than or equal to its flock size, namely $M \le N$. If $M = 2$, two PC sequences become PC mates. In general, a PC sequence set can be expanded from PC mates by interleaving technique, concatenation and so on [13]. Hence, the construction of PC mates has great significance, especially in the case of $M = N = 2$ since the PC mates in this case achieve the theoretical bound of PC sequences.

## 3. Construction Algorithm of Multiphase PC Sequences Based on Perfect Sequences

The interleaving technique had been significantly employed to construct many kinds of sequences, such as AC sequences [4, 13], ZC sequences [9, 11], ZCZ sequences [14-20] and so on. In this section, we will propose a construction of multiphase PC mates by interleaving any two perfect sequences with the same length. The construction algorithm can be given as follows.

Let $\mathbf{s}_1$ and $\mathbf{s}_2$ denote any two perfect sequences with sequence length $L_0$, written as $\mathbf{s}_1 = (s_1(0), s_1(1), \cdots, s_1(L_0 - 1))$ and $\mathbf{s}_2 = (s_2(0), s_2(1), \cdots, s_2(L_0 - 1))$. Then, multiphase PC mates in the form of matrix can be constructed as

$$\mathcal{A} = \begin{bmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{0,0} & \mathbf{a}_{0,1} \\ \mathbf{a}_{1,0} & \mathbf{a}_{1,1} \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1 \otimes \mathbf{s}_2 & \mathbf{s}_1 \otimes (-\mathbf{s}_2) \\ \underline{\mathbf{s}_1 \otimes (-\mathbf{s}_2)} & -(\underline{\mathbf{s}_1 \otimes \mathbf{s}_2}) \end{bmatrix}, \tag{6}$$

where the two notations of $-\mathbf{s}_1$ and $\underline{\mathbf{s}_1}$ respectively represent the negation sequence and conjugate reverse sequence of the perfect sequence $\mathbf{s}_1$, that is, when $\mathbf{s}_1 = (s_1(0), s_1(1), \cdots, s_1(L_0 - 1))$, we have

$$-\mathbf{s}_1 = (-s_1(0), -s_1(1), \cdots, -s_1(L_0 - 1)), \tag{7}$$

and

$$\underline{\mathbf{s}_1} = (s_1^*(L_0 - 1), s_1^*(L_0 - 2), \cdots, s_1^*(0)). \tag{8}$$

It is well known that there exists one binary perfect sequence {1, 1, -1, 1} and the others are multiphase sequences. Hence, the proposed PC sequence is multiphase one.

From (6), it is obvious that the length of element sequences of the constructed multiphase PC mates with flock size $N = 2$ is equal to $2L_0$. Since there exists two perfect sequences with the same length at least when the perfect sequence length is larger than or equal to 3, the length of element sequence of the designed multiphase PC mates may be any even number which is larger than 4. In terms of the point, multiphase PC sequences will have better performance than binary PC sequences.

The constructed multiphase PC mates possess ideal periodic ACF and CCF properties, namely $\rho_{\mathbf{a}_0,\mathbf{a}_0}(\tau)=\rho_{\mathbf{a}_1,\mathbf{a}_1}(\tau)=0, \tau\neq 0$ and $\rho_{\mathbf{a}_0,\mathbf{a}_1}(\tau)=0, \forall\tau$, which can be proved as follows.

***Proof***: We will discuss periodic ACF and CCF properties of the constructed multiphase PC mates, respectively.

*1) Periodic ACF properties.*

For the multiphase PC sequence $\mathbf{a}_0$, its periodic ACF can be calculated as

$$\rho_{\mathbf{a}_0,\mathbf{a}_0}(\tau)=\theta_{\mathbf{a}_{0,0},\mathbf{a}_{0,0}}(\tau)+\theta_{\mathbf{a}_{0,1},\mathbf{a}_{0,1}}(\tau). \tag{9}$$

We will deal with (9) in terms of two cases of even number and odd number of shift $\tau$, namely $\tau=2\tau'+1$ and $\tau=2\tau'$, where $0\le\tau'\le L_0-1$.

When $\tau=2\tau'+1$, we can obtain that

$$\begin{aligned}
&\rho_{\mathbf{a}_0,\mathbf{a}_0}(2\tau'+1)\\
&=\theta_{\mathbf{a}_{0,0},\mathbf{a}_{0,0}}(2\tau'+1)+\theta_{\mathbf{a}_{0,1},\mathbf{a}_{0,1}}(2\tau'+1)\\
&=\left[\theta_{\mathbf{s}_1,\mathbf{s}_2}(\tau')+\theta_{\mathbf{s}_1,\mathbf{s}_2}(\tau'+1)\right]+\left[\theta_{\mathbf{s}_1,-\mathbf{s}_2}(\tau')+\theta_{\mathbf{s}_1,-\mathbf{s}_2}(\tau'+1)\right]\\
&=\left[\theta_{\mathbf{s}_1,\mathbf{s}_2}(\tau')+\theta_{\mathbf{s}_1,\mathbf{s}_2}(\tau'+1)\right]+\left[-\theta_{\mathbf{s}_1,\mathbf{s}_2}(\tau')-\theta_{\mathbf{s}_1,\mathbf{s}_2}(\tau'+1)\right]\\
&=0
\end{aligned} \tag{10}$$

When $\tau=2\tau'$, we have

$$\begin{aligned}
\rho_{\mathbf{a}_0,\mathbf{a}_0}(2\tau')&=\theta_{\mathbf{a}_{0,0},\mathbf{a}_{0,0}}(2\tau')+\theta_{\mathbf{a}_{0,1},\mathbf{a}_{0,1}}(2\tau')\\
&=\left[\theta_{\mathbf{s}_1,\mathbf{s}_1}(\tau')+\theta_{\mathbf{s}_2,\mathbf{s}_2}(\tau')\right]+\left[\theta_{\mathbf{s}_1,\mathbf{s}_1}(\tau')+\theta_{\mathbf{s}_2,\mathbf{s}_2}(\tau')\right]\\
&=2\left[\theta_{\mathbf{s}_1,\mathbf{s}_1}(\tau')+\theta_{\mathbf{s}_2,\mathbf{s}_2}(\tau')\right].
\end{aligned} \tag{11}$$

The above equation shows that the periodic ACF of $\mathbf{a}_0$ in the case of $\tau=2\tau'$ is determined by the periodic ACFs of $\mathbf{s}_1$ and $\mathbf{s}_2$. Since both of $\mathbf{s}_1$ and $\mathbf{s}_2$ are perfect sequences, we can obtain $\rho_{\mathbf{a}_0,\mathbf{a}_0}(\tau)=0$ when $\tau=2\tau'$ and $\tau\neq 0$.

Combining (10) and (11), the multiphase PC sequence $\mathbf{a}_0$ can possess ideal auto-correlation properties. Similar to the case of $\mathbf{a}_0$, another multiphase PC sequence $\mathbf{a}_1$ has the same performance and its proof is omitted.

*2) Periodic CCF properties.*

The periodic CCF between $\mathbf{a}_0$ and $\mathbf{a}_1$ can be calculated as

$$\rho_{\mathbf{a}_0,\mathbf{a}_1}(\tau)=\theta_{\mathbf{a}_{0,0},\mathbf{a}_{1,0}}(\tau)+\theta_{\mathbf{a}_{0,1},\mathbf{a}_{1,1}}(\tau). \tag{12}$$

Similar to periodic ACF, the calculation of periodic CCF can be considered in two cases of $\tau = 2\mu + 1$ and $\tau = 2\mu$.

If $\tau = 2\mu + 1$, the periodic CCF satisfies that

$$\begin{aligned}
&\Psi_{\mathbf{a}_0,\mathbf{a}_1}(2\mu+1) \\
&= \theta_{\mathbf{a}_{0,0},\mathbf{a}_{1,0}}(2\mu+1) + \theta_{\mathbf{a}_{0,1},\mathbf{a}_{1,1}}(2\mu+1) \\
&= \left[\theta_{\mathbf{s}_1,\mathbf{s}_1}(\mu+1) + \theta_{\mathbf{s}_2,-\mathbf{s}_2}(\mu+2)\right] + \left[\theta_{\mathbf{s}_1,-\mathbf{s}_1}(\mu+1) + \theta_{-\mathbf{s}_2,-\mathbf{s}_2}(\mu+2)\right] \\
&= \left[\theta_{\mathbf{s}_1,\mathbf{s}_1}(\mu+1) + \theta_{\mathbf{s}_2,\mathbf{s}_2}(\mu+2)\right] + \left[-\theta_{\mathbf{s}_1,\mathbf{s}_1}(\mu+1) - \theta_{\mathbf{s}_2,\mathbf{s}_2}(\mu+2)\right] \\
&= 0
\end{aligned} \tag{13}$$

If $\tau = 2\mu$, then

$$\begin{aligned}
&\Psi_{\mathbf{a}_0,\mathbf{a}_1}(2\mu) \\
&= \theta_{\mathbf{a}_{0,0},\mathbf{a}_{1,0}}(2\mu) + \theta_{\mathbf{a}_{0,1},\mathbf{a}_{1,1}}(2\mu) \\
&= \left[\theta_{\mathbf{s}_1,-\mathbf{s}_2}(\mu-1) + \theta_{\mathbf{s}_1,\mathbf{s}_2}(\mu+1)\right] + \left[\theta_{\mathbf{s}_1,-\mathbf{s}_2}(\mu-1) + \theta_{-\mathbf{s}_1,-\mathbf{s}_2}(\mu+1)\right] \\
&= \left[-\theta_{\mathbf{s}_1,\mathbf{s}_2}(\mu-1) + \theta_{\mathbf{s}_1,\mathbf{s}_2}(\mu+1)\right] + \left[-\theta_{\mathbf{s}_1,\mathbf{s}_2}(\mu-1) + \theta_{\mathbf{s}_1,\mathbf{s}_2}(\mu+1)\right] \\
&= 0
\end{aligned} \tag{14}$$

From (13) and (14), it is obvious that the constructed multiphase PC mates possess ideal periodic CCF properties.

According to the above analysis, the ideal periodic correlation performance of the proposed multiphase PC mates possess is proved.

Q.E.D.

## 4. A Construction Example

In order to show how the presented construction algorithm of multiphase PC mates works, this section will provides a simple example.

The Zadoff-Chu sequences in [21] are used as perfect sequences to generate multiphase PC mates. Let the sequence length of Zadoff-Chu sequences $L_0$ be equal to 5. Then there exist four perfect sequences in total. We employ two sequences with indexes 1 and 2, respectively. The two Zadoff-Chu sequences can be expressed as

$$\mathbf{s}_1 = \left(1, e^{j\frac{2}{5}\pi}, e^{j\frac{6}{5}\pi}, e^{j\frac{2}{5}\pi}, 1\right), \tag{15}$$

and

$$\mathbf{s}_2 = \left(1, e^{j\frac{4}{5}\pi}, e^{j\frac{2}{5}\pi}, e^{j\frac{4}{5}\pi}, 1\right). \tag{16}$$

In terms of (6), we can obtain multiphase PC mates with element sequence length equal to 10 as follows.

$$\mathbf{a}_{0,0} = \left[1,1,e^{j\frac{2}{5}\pi},e^{j\frac{4}{5}\pi},e^{j\frac{6}{5}\pi},e^{j\frac{2}{5}\pi},e^{j\frac{2}{5}\pi},e^{j\frac{2}{5}\pi},1,1\right]. \tag{17}$$

$$\mathbf{a}_{0,1} = \left[1,-1,e^{j\frac{2}{5}\pi},-e^{j\frac{4}{5}\pi},e^{j\frac{6}{5}\pi},-e^{j\frac{2}{5}\pi},e^{j\frac{2}{5}\pi},-e^{j\frac{4}{5}\pi},1,-1\right]. \tag{18}$$

$$\mathbf{a}_{1,0} = \left[-1,1,-e^{-j\frac{4}{5}\pi},e^{-j\frac{2}{5}\pi},-e^{-j\frac{2}{5}\pi},e^{-j\frac{6}{5}\pi},-e^{-j\frac{4}{5}\pi},e^{-j\frac{2}{5}\pi},-1,1\right]. \tag{19}$$

$$\mathbf{a}_{1,1} = \left[-1,-1,-e^{-j\frac{4}{5}\pi},-e^{-j\frac{2}{5}\pi},-e^{-j\frac{2}{5}\pi},-e^{-j\frac{6}{5}\pi},-e^{-j\frac{4}{5}\pi},-e^{-j\frac{2}{5}\pi},-1,-1\right]. \tag{20}$$

Fig. 1 and 2 give the distribution of periodic ACF and CCF of the generated multiphase PC mates, respectively. According to Fig. 1, the sum of real parts of periodic ACFs of $\mathbf{a}_{0,0}$ and $\mathbf{a}_{0,1}$ is equal to zero except for $\tau = 0$ while the sum of imaginary parts of periodic ACFs of $\mathbf{a}_{0,0}$ and $\mathbf{a}_{0,1}$ is always equal to zero. As a result, the proposed multiphase PC sequence $\mathbf{a}_0$ has ideal periodic auto-correlation properties. Similarly, we can see that the proposed multiphase PC mates $\mathbf{a}_0$ and $\mathbf{a}_1$ have ideal periodic cross-correlation properties from Fig. 2.

In addition, it should be noted that only two perfect sequences in Zadoff-Chu set with set size 4 and sequence length 5 are used. If different two Zadoff-Chu perfect sequences are employed, different multiphase PC mates will be generated. When the set size of the used perfect sequence set is huge, a large number of different multiphase PC mates can be designed. For example, there exist $L_0 - 1$ Zadoff-Chu perfect sequences in total if the sequence length $L_0$ is a prime number. Then, we can obtain multiple multiphase PC mates for a given perfect sequence set, which can also ensure a more flexible usage of multiphase PC mates.

## 5. Conclusion

We propose a design scheme of multiphase PC sequences based on interleaved perfect sequences. In terms of length of element sequences of PC sequences, the proposed multiphase PC sequences possess better performance than traditional binary PC sequences. When Zadoff-Chu perfect sequences are used in the presented construction algorithm, the length of element sequence of the designed multiphase PC sequences may be any even number which is larger than 4. As a result, communication systems employing the proposed multiphase PC sequences can have a more flexible choice of parameters.

## References


[1] L. R.Welch, "Lower bounds on the maximum cross correlation of signals," IEEE Trans. on Information Theory, vol. 20, no. 3, pp. 397-399, 1974.

[2] H. H. Chen, The Next Generation CDMA Technologies. Chichester, John Wiley & Sons. Ltd., 2007.

[3] M. J. E. Golay, "Complementary series," IRE. Trans. Inf. Theory, vol. IT-7, pp.82-87, Apr. 1961.

[4] C. C. Tseng and C. L. Liu, "Complementary sets of sequences," IEEE Trans. Inf. Theory, vol. IT-18, no. 5, pp. 644-652, Sep. 1972.

[5] R. Sivaswamy, "Multi-phase complementary codes," IEEE Trans. Inf. Theory, vol. 24, pp. 546-552, Sep. 1978.

[6] L. Bomer, M. Antweiler, "Periodic complementary binary sequences," IEEE Transactions on Information Theory, vol 36, no. 6, pp. 1487-1494, 1990.
[7] W. N. Yuan, Y. F. Tu, and P. Z. Fan, "Optimal training sequences for cyclic-prefix-based single-carrier multi-antenna systems with space-time block-coding, " IEEE Trans. on Wireless Commun., vol. 7, no. 11, pp. 4047-4050, 2008.
[8] Y. F. Tu, P. Z. Fan, L. Hao and X. H. Tang, "A simple method for generating optimal Z-periodic complementary sequence set based on phase shift," IEEE Signal Process. Lett., vol.17, no. 10, pp. 891-893, 2010.
[9] P. Z. Fan, W. N. Yuan, and Y. F. Tu, "Z-complementary binary sequences," IEEE Signal Process. Lett., vol. 14, no. 8, pp. 509-512, Aug. 2007.
[10] L. F. Feng, P. Z. Fan, X. H. Tang, and K.-K. Loo, "Generalized pairwise Z-complementary codes," IEEE Signal Process. Lett., vol. 15, pp. 377-380, 2008.
[11] Z. Y. Zhang, W. Chen, F. X. Zeng, H. Wu, and Y. H. Zhong, "Z-complementary sets based on sequences with periodic and aperiodic zero correlation zone," EURASIP J. Wireless Comm. And Networking, Volume 2009 (2009), Article ID 418026, 9 pages. doi:10.1155/2009/418026.
[12] X. D. Li, P. Z. Fan, X. H. Tang, and L. Hao, "Quadriphase Z-complementary sequences," IEICE trans. Fundamentals, E93-A, no. 11, pp. 2251-2257, Nov. 2010.
[13] P. Z. Fan and M. Darnell, Sequence Design for Communications Applications, London, U. K.: Wiley, 1996.
[14] S. Matsufuji, N. Kuroyanagi, N. Suehiro, and P. Z. Fan, "Two types of polyphase sequence sets for approximately synchronized CDMA systems," IEICE Trans. Fundamentals, vol. E86-A, no. 1, pp. 229–234, 2003.
[15] T. Hayashi, "Zero-correlation zone sequence set constructed from a perfect sequence," IEICE Trans. Fundamentals, vol. E90-A, no. 5, pp. 1107–1111, May 2007.
[16] T. Hayashi, "A novel class of zero-correlation zone sequence set constructed from a perfect sequence," IEICE Trans. Fundamentals, vol. E91-A, no. 4, pp. 1233–1237, April 2008.
[17] T. Hayashi, "A class of zero-correlation zone sequence set using a perfect sequence," IEEE Signal Process. Lett., vol. 16, no.4, pp. 331-334, 2009.
[18] Z. C. Zhou, X. H. Tang, and G. Gong, "A new class of sequences with zero or low correlation zone based on interleaving technique," IEEE Trans. Inf. Theory, vol. 54, no. 9, pp. 4267-4273, Sept. 2008.
[19] X. H. Tang, and W. H. Mow, "A new systematic construction of zero correlation zone sequences based on interleaved perfect sequences," IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5729-5734, Dec. 2008.
[20] Z. C. Zhou, Z. Pan, and X. H. Tang, "New families of optimal zero correlation zone sequences based on interleaved technique and perfect sequences," IEICE Trans. Fundamentals, vol. E91-A, no. 12, pp. 3691–3697, Dec. 2008.
[21] D. C. Chu, "Polyphase codes with good periodic correlation properties," IEEE Trans. on Inf. Theory, vol. 18, no. 4, pp. 531-532, 1972.

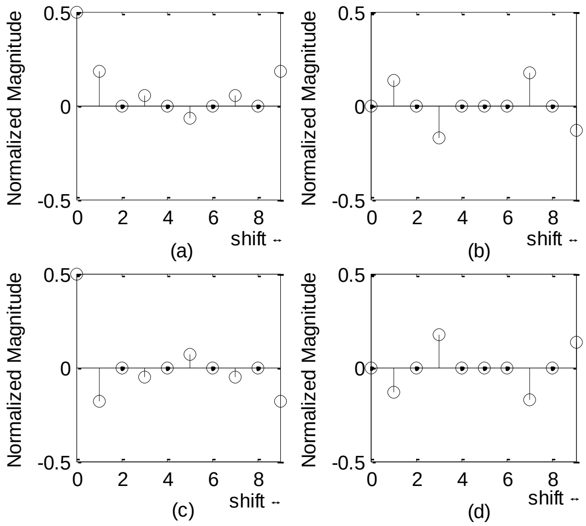


Figure 1. The distribution of periodic ACF of the proposed multiphase PC sequence $\mathbf{a}_0$. (a). The real part of periodic ACF of $\mathbf{a}_{0,0}$; (b). The imaginary part of periodic ACF of $\mathbf{a}_{0,0}$; (c). The real part of periodic ACF of $\mathbf{a}_{0,1}$; (d). The imaginary part of periodic ACF of $\mathbf{a}_{0,1}$.

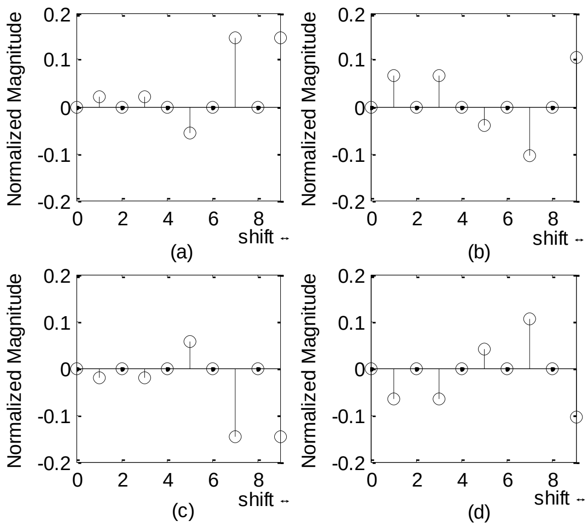


Figure 2. The distribution of periodic CCF between the proposed multiphase PC mates $\mathbf{a}_0$ and $\mathbf{a}_1$. (a). The real part of periodic CCF between $\mathbf{a}_{0,0}$ and $\mathbf{a}_{1,0}$; (b). The imaginary part of periodic CCF between $\mathbf{a}_{0,0}$ and $\mathbf{a}_{1,0}$; (c). The real part of periodic CCF between $\mathbf{a}_{0,1}$ and $\mathbf{a}_{1,1}$; (d). The imaginary part between periodic CCF of $\mathbf{a}_{0,1}$ and $\mathbf{a}_{1,1}$